South East Asian J. of Mathematics and Mathematical Sciences
Vol. 15, No. 2 (2019), pp. 59-64

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

EXPONENTIAL STABILITY OF NEUTRAL TIME DELAY DIFFERENTIAL SYSTEMS WITH LMI APPROACH

P. Baskar, K. S. Anand*, V. Umesha**

Department of Mathematics,
New Horizon college of Engineering,
Marathahalli, Bengaluru, Karnataka 560103, INDIA

E-mail : pbaskar_83@yahoo.com, padmanabhanrnsit@gmail.com

*Department of Mathematics,
A. P. S. College of Engineering,
Somanahalli, Bengaluru, Karnataka 560082, INDIA.

**Department of Mathematics,
Dayananda Sagar College of Engineering,
Shavige Malleshwara Hills, Bengaluru, Karnataka 560078, INDIA.

E-mail : umesh82@gmail.com

(Received: June 27, 2019)

Abstract: In this paper we developed the analyzed and the globally exponentially stability of Neutral Time Delay-differential systems. Based on a novel Lyapunov kravoski's functional method (LKF) and linear matrix inequality (LMI) a new delay dependent stability criterion is derived. The stability conditions which are in the form of LMI and it can be solved by the help of some standard numerical MATLAB algorithms.

Keywords and Phrases: Exponential stability, Delay-dependent stability, Linear Matrix Inequality, Lyapunov- Krasovskii functional, Time-varying delay.

2010 Mathematics Subject Classification: 93D05, 93D09, 93D20.

1. Introduction

Stability is a very basic issue in control theory and has been extensively applied to biological and engineering systems. For the effective work of any systems, stability is required. Suppose the system is in instability we will not able to expect the accurate results. There are many types of systems particularly fuzzy, neural, stochastic for each system we can provide the stability conditions in the form of LMI.

Research on the stability of time-delay systems began in the 1950s, first using frequency-domain methods and later also using time-domain methods. Frequency-domain methods determine the stability of a system from the distribution of the roots of its characteristic equation or from the solutions of a complex Lyapunov matrix function equation. They are suitable only for systems with constant delays. The main time-domain methods are the Lyapunov-Krasovskii functional and Razumikhin function methods.

Dynamical systems with time delays have been of effective interest from the past few decades. In particular, the interest in stability analysis of various neutral differential systems has been growing rapidly due to their successful applications in practical fields such as circuit theory, bio engineering, population dynamics, automatic control and so on. Current results on the stability of time delay system is divided into two categories, delay dependent and delay independent stability of time delay systems. A number of delay independent sufficient conditions for the asymptotic stability of neutral delay differential systems have been presented by various researchers (for example [1, 2]).

Now a days stability of neutral system have more attention on two categories namely stability with nonlinear uncertainties [3-8], and robust stability [9, 10]. A Lyapunov kravoski's functional method (LKF) and linear matrix inequality (LMI) a new delay dependent stability criteria is derived. By using descriptor model transformation and decomposition technique, some delay-dependent stability criteria are obtained in [9]. Many of the researchers prove the delay dependent stability criteria of neutral time delay system by the help of weighted matrices. Also most of the researchers show the asymptotical stability of neutral time delay systems but this paper also shows the same and exponential stability of neutral time delay systems without any conditions.

Notation

Throughout this paper, the notation $*$ represents the elements below the main diagonal of a symmetric matrix. A^T means the transpose of A. We say $X > Y$ if $X - Y$ is positive definite, where X and Y are symmetric matrices of same dimensions. $\| \cdot \|$ refers to the Euclidean norm for vectors. τ_M and τ_m denote the maximal and minimal eigenvalue of a matrix A respectively.

2. Problem formulation and Preliminaries

Consider a neutral delay differential system of the form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h(t)), \\ x(s) = \Phi(s), \quad \dot{x}(s) = \varphi(s), \quad s \in [-h, 0] \end{cases} \tag{1}$$

$x(t) \in \mathbb{R}^n$ is the state $\Phi(\cdot)$ and $\varphi(\cdot)$ are continuous vector valued initial functions, A, B and C are real constant matrices and $h(t)$ denotes time varying delay and it is assumed to satisfy $0 \leq h(t) \leq h_M$ and $0 \leq \dot{h}(t) \leq d \leq 1$, where h_M and d are positive constants.

Lemma 2.1.(Schur complement [1]) *Let M, P, Q be given matrices such that $Q > 0$, then*

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \leftrightarrow P + M^T Q^{-1} M < 0.$$

Lemma 2.2. *For any vectors a, $b \in \mathbb{R}^n$ and scalar $\epsilon > 0$, we have $2a^T b \leq \epsilon a^T a + \epsilon^{-1} b^T b$.*

Lemma 2.3. [5] *For any constant matrix $M \in \mathbb{R}^{n \times n} > 0$, $M = M^T > 0$ scalar $\tau > 0$, vector function $w : [0, \tau] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\Big(\int_0^\tau w(s) ds \Big) M \Big(\int_0^\tau w(s) ds \Big)^T \leq \tau \int_0^\tau w^T(s) M w(s) ds.$$

Definition 2.4. *System (1) is said to be globally exponentially stable with convergence rate α if there are two positive constants α and ρ such that $\|x(t)\| \leq \rho e^{-\alpha t}$, $t \geq 0$.*

3. Global stability results

In this section, we will perform stability analysis of uncertain neutral system with time varying delay described by (1). We can rewrite system (1) to the following descriptor system:

$$\begin{cases} \dot{x}(t) = y(t), \\ y(t) = Ax(t) + Bx(t-h(t)) + Cy(t-h(t)). \end{cases} \tag{2}$$

Theorem 3.1. *System (1) is globally exponentially stable with convergence rate $\alpha > 0$ if there exist some positive definite matrices $P_i > 0$; $i = 1, 2, 3, 4, 5, 6$ and the* *real matrices $N_j > 0$; $j = 1, 2, 3$ such that the following LMI condition is satisfied*

$$\Xi_1 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \epsilon_1 & \epsilon_2 & 0 & 0 \\ * & \varphi_{12} & \varphi_{23} & \varphi_{24} & 0 & 0 & \epsilon_3 & 0 \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 & \epsilon_4 \\ * & * & * & \varphi_{44} & \Delta_1 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & \Delta_2 & 0 & 0 \\ * & * & * & * & * & * & \Delta_3 & 0 \\ * & * & * & * & * & * & * & \Delta_4 \end{bmatrix} < 0. \tag{3}$$

Proof: This theorem can be prove by considering the Lyapunov functions are $V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$ are as follows.

We define Lyapunov functions as follows:

$$V_1(t) = e^{2\alpha t} x^T(t) P_1 x(t),\ V_2(t) = \int_{t-h}^{t} e^{2\alpha s} x^T(s) P_2 x(s) ds, \tag{4}$$

$$V_3(t) = \int_{t-h}^{t} e^{2\alpha s} y^T(s) P_3 y(s) ds,\ V_4(t) = h_M \int_{-h_M}^{0} \int_{t+\beta}^{0} e^{2\alpha s} \dot{x}^T(s) P_4 \dot{x}(s) ds d\beta, \tag{5}$$

$$V_5(t) = \int_{t-\bar{\tau}} e^{2\alpha s} \dot{x}^T(s) P_5 ds,\ V_6(t) = \bar{\tau} \int_{\bar{\tau}}^{0} \int_{t+\theta}^{0} e^{2\alpha s} x^T(s) P_6 x(s) ds d\theta. \tag{6}$$

Let us define the derivative of the Lyapunov functions is as follows

$$\dot{V}_1 = e^{2\alpha t} x^T(t) \{A P_1 + A^T P_1 + 2\alpha P_1\} x(t), \tag{7}$$

$$\dot{V}_2 = e^{2\alpha t} [x^T(t) P_2 x(t) - (1 - \dot{h}) e^{-2h\alpha} x^T(t-h) P_2 x(t-h)], \tag{8}$$

$$\dot{V}_3 = e^{2\alpha t} y^T(t) P_3 y(t) - (1 - \dot{h}) e^{-2h\alpha} y^T(t-h) P_3 y(t-h), \tag{9}$$

$$\dot{V}_4 = h_M^2 x^T(t) P_4 x(t) - H_M \int_{t-h_m}^{t} e^{2\alpha(s-t)} x^T(s) P_4 x(s) ds, \tag{10}$$

$$\dot{V}_5 = e^{2\alpha t} [x^T(t) P_5 x(t) - (1 - \bar{\tau}) e^{-2h\alpha} x^T(t-h) P_5 x(t-h)], \tag{11}$$

$$\dot{V}_6 = \tau^2 x^T(t) P_6 x(t) - \bar{\tau} \int_{t-\tau}^{-t} e^{2\alpha(s-t)} \dot{x}^T(s) P_6 \dot{x}(s) ds. \tag{12}$$

Here we using Lemma 2.2, we have

$$\begin{aligned}\dot{V} &= e^{2\alpha t}\Big[x^T(t)(AP_1 + A^T P_1 + 2\alpha P_1 + P_2 + P_5) + 2x^T P_1 Bx(t-h) \\ &+ 2x^t P_1 Cy(t-h) - (1-h)e^{-2\alpha h}x^T(t-h)P_2 x(t-h) \\ &- (1-\tau)e^{-2\alpha h}x^T(t-h)P_5 x(t-h) + y^T(t)By(t) + h_M^2 y^T(t)P_4 y(t) \\ &- (1-h)e^{-2\alpha h}y^T(t-h)P_3 y(t-h) - x^T e^{-2\alpha h}P_4 x(t) + 2x^T e^{-2\alpha h}P_4 x(t-h) \\ &- x^T e^{-2\alpha h}P_4 x(t) + 2x^T e^{-2\alpha h}P_4 x(t-h) - e^{-2\alpha h}x^T(t-h)P_4 x(t-h) \\ &+ \bar{\tau}^2 y^T(t)P_6 y(t) - x^T e^{-2\alpha h}P_6 x(t) + 2x^T e^{-2\alpha h}P_6 x(t-h) \\ &- e^{-2\alpha h}x^T(t-h)P_6 x(t-h)\Big] \leq e^{2\alpha t}\xi^T\Sigma\xi, \end{aligned} \tag{13}$$

where $\xi^T = [x^T(t) \quad x^T(t-h) \quad y^T(t) \quad y^T(t-h)]$ and

$$\Sigma = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} \\ * & * & \varphi_{33} & \varphi_{34} \\ * & * & * & \varphi_{44} \end{bmatrix} < 0.$$

By applying Lemma 2.1 in $\mathbb{R}$ with some effort, we get $\Xi_1 < 0$. Therefore, by Lyapunov-Krasovskii stability theorem, we have $\dot{V}(t) \leq 0$. Hence we concluded that the following result.Also we obtain that $\|x(t)\| \leq \rho e^{-\alpha t}$.This implies that the system (1) is globally exponentially stable with converges rate α.

4. Conclusion We have presented a sufficient condition to guarantee the exponential stability for neutral delay differential system with Constant delay. Based on the Lyapunov-Krasovskii functional theory, the delay dependent criterion has been derived to guarantee the exponential stability of neutral delay differential system with Constant delay.

References

[1] D. Yue, S. Won, Delay-dependent robust stability of stochastic systems with time-delay and nonlinear uncertainties, Electron Lett, (2001), 13.

[2] D. Yue, S. Won, O. Kwon, Delay dependent stability of neutral systems with time-delay: an LMI approach, IEE Proc - Control Theory Appl., 150(1)(2003), 23-7.

[3] Y. Y. Cao, J. Lam, Computation of robust stability bounds for time-delay systems with nonlinear time-varying perturbation, Int J Syst Sci, 31(2000), 359-65.

[4] Q. L. Han, Robust stability for a class of linear systems with time-varying delay and nonlinear perturbations, Comput Math Appl., 47(2004), 1201-9.

[5] Q. L. Han, On robust stability of linear neutral systems with non linear parameter perturbations, Proceeding of the 2004 American Control Conference, Boston, Massachusetts, (2004) 2027-32.

[6] J. H. Park, Novel robust stability criterion for a class of neutral systems with mixed delays and Non linear perturbations, Appl Math Comput, 161(2005), 413-21.

[7] J. Zhang, S. Peng, Q. Jiqing, Robust stability criteria for uncertain neutral system with time delay and nonlinear uncertainties, Chaos Solitons Fract., 38(2008), 160-7.

[8] Z. Zou, Y. Wang, New stability criterion for a class of linear systems with time-varying delay and non linear perturbations, IEE Proc (Control Theory Appl), 153 (5)(2006), 623-6.

[9] Q. L. Han, A descriptor system approach to robust stability of uncertain neutral systems with discrete and distributed delays, IEEE Trans Autom Control, 49(2004), 1791-6.

[10] X. G. Liu, M. Wu, R. Martin, M. L. Tang, Stability analysis for neutral systems with mixed delays, J Comput Appl Math, 202(2007), 478-97.