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NORMAL SUBGROUP AS A CATALYST TO NANO TOPOLOGY

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Abstract: The aim of this paper is to evolve a nano topological structure from a finite group and here we define the nano approximations via normal subgroup of a group and its properties were also discussed based on choice of Normal subgroups. Algebraically and topologically, structural equivalences are based on the two renowned maps isomorphism between groups and homeomorphism in topological spaces. This induces us to create a link between groups and Nano topology induced by group which in turn has its own impact on well known theorems such as Fundamental theorem of homomorphism on groups and Second isomorphism theorem.

Keywords and Phrases: Normal subgroup, cosets, Nano topology induced by group, Fundamental theorem of homomorphism in Nano topological spaces.

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1. Introduction

The abstract concept of a finite group [7] was first formulated by A. Cayley. Lellis Thivagar et al [10] interjected a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. The topology recommended here is named so because of its size, Since it has atmost five elements in it. It is an expanding research area which stimulates explorations on both real-world applications. A key concept in the nano topological model is the equivalence relation. Equivalence classes are the building blocks for the construction of lower approximation, upper approximation, boundary. It soon invoked a natural question concerning possible connection between nano topology and algebraic systems. It is a natural query to ask what does happen if we substitute an algebraic system instead of the universe set. Thus, we propose a new method to generate nano topology by defining lower and upper approximations with respect to the normal subgroups. In recent years, there has been a fast growing interest in this new emerging theory, ranging from work in pure theory such as topological and algebraic foundations, to diverse areas of applications.

2. Preliminaries

The following recalls necessary concepts and preliminaries required in the sequel of our work.

Definition 2.1. [10]: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup\limits_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\} \right\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{ \bigcup\limits_{X \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [10]: Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$

(ii) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_R(X)\}$ as the nano topological space.

Definition 2.3. [7]: A non-empty set G together with a binary operation *: GXG $\to G$ is called a group if the following conditions are satisfied.

(i) For all a,b$\in G$, a*b$\in G$ is associative

(ii) For all a,b,c $\in G$,(a*b)*c = a *(b*c)

(iii) There exists $e \in G$ such that a*e = e*a=a for all a$\in G$

(iv) For any element $a \in G$ there exists an element a'$\in G$ such that a*a' = a'*a = e.

Definition 2.4. [7]: A subset H of a group is called a sub-group of G if H forms a group with respect to the binary operation in G.

Definition 2.5. [7]: Let H be a subgroup of a group G. Let $a \in G$. Then the set aH = $\{ah/h \in H\}$ is called the left coset of H defined by a in G.
Similarly Ha = $\{ha/h \in H\}$ is called the right coset of H defined by a.
Every element of G belongs to one and only one left(right) coset and so the left(right) coset forms a partition of G.

Definition 2.6. [7]: A subgroup H of G is called a normal subgroup of G if aH = Ha for all $a \in G$.

Definition 2.7. [7]: A group G is said to be abelian if ab = ba for all a,b $\in$G.

Theorem 2.8. [7]: Every subgroup of an abelian group is a normal subgroup.

Definition 2.9. [7] Let G and G' be two groups. A mapping f: $G \to G'$ is called an homomorphism from G to G' provided $f(xy) = f(x)f(y)$ for all $x, y \in G$.

Definition 2.10. [7] Let G and G' be two groups. A mapping f: $G \to G'$ is called an isomorphism, if

(i) f is a homomorphism

(ii) f is 1-1 and onto

3. Nano Topology via Group

In this section we have provoked a new concept of nano topology induced by group, which differs in the sense that the choice of universal set here is a finite group. By considering, A as a subgroup of the universe G and the possible inclusions between the normal subgroup N and A, their impact on the nano topology were also examined.

Definition 3.1. Let G be a group with identity element 'e' and N be a normal subgroup of a group G, and for each $x \in G$, xN induces a equivalence relation on G and A be a nonempty subset of G then

(i) $\underline{N}(A) = \{x \in G : xN \subseteq A\}$

(ii) $\overline{N}(A) = \{x \in G : xN \cap A \neq \emptyset\}$

(iii) $B_N(A) = \overline{N}(A) - \underline{N}(A)$

Then $\underline{N}(A), \overline{N}(A)$ and $B_N(A)$ are lower, upper approximations and boundary of A with respect to the normal subgroup N respectively.

Definition 3.2. Let G be a group, N be a normal subgroup of G and $\tau_N(A) = \{G, \emptyset, \underline{N}(A), \overline{N}(A), B_N(A)\}$ where A$\subseteq G$,and $\tau_N(A)$ satisfies the following axioms.

(i) G and $\emptyset$ are in $\tau_N(A)$

(ii) The union of elements of any subcollection of $\tau_N(A)$ is in $\tau_N(A)$

(iii) The intersection of the elements of any finite subcollection of $\tau_N(A)$ is in $\tau_N(A)$

That is, $\tau_N(A)$ forms a topology on G called the nano topology on G with respect to A. We call [G, $\tau_N(A)$] as the nano topological space induced by the group G.

Definition 3.3. Let G be a group with identity element 'e' and N be a normal subgroup of a group G and A be a nonempty subset of G.

(i) If $\underline{N}(A) = \emptyset$ and $\overline{N}(A) = G$, then $\tau_N(A) = \{G, \emptyset\}$, the indiscrete nano topology on G.

(ii) If $\underline{N}(A) = \overline{N}(A) = A$, then the nano topology, $\tau_N(A) = \{G, \emptyset, \underline{N}(A)\}$.

(iii) If $\underline{N}(A) = \emptyset$ and $\overline{N}(A) \neq G$, then $\tau_N(A) = \{G, \emptyset, \overline{N}(A)\}$.

(iv) If $\underline{N}(A) \neq \emptyset$ and $\overline{N}(A) = G$, then $\tau_N(A) = \{G, \emptyset, \underline{N}(A), B_N(A)\}$.

(v) If $\underline{N}(A) \neq \overline{N}(A)$ where $\underline{N}(A) \neq \emptyset$, then $\tau_N(A) = \{G, \emptyset, \underline{N}(A), \overline{N}(A), B_N(A)\}$ is the discrete nano topology on G.

Example 3.4. Let G $=\{1, i, -1, -i\}$ be an abelian group with multiplication as a binary operation with the following table.

$*$	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

Cayley Table

Let N $=\{1, -1\}$ be any subgroup of G and since G is an abelian group, N is also an normal subgroup of G and A $= \{-i\} \subseteq G$ then $\underline{N}(A) = \emptyset$, $\overline{N}(A) = \{i, -i\}$, $B_N(A) = \emptyset$. Hence $\tau_N(A) = \{G, \emptyset, \{i, -i\}\}$

Proposition 3.5. Let [G,$\tau_H(A)$] and [G,$\tau_N(A)$] be the nano topological space induced by group G, $A \subseteq G$ then

(i) N $\subseteq H \Rightarrow \underline{N}(A) \subseteq \underline{H}(A)$

(ii) N $\subseteq H \Rightarrow \overline{N}(A) \subseteq \overline{H}(A)$

Theorem 3.6. Let H and N be the normal subgroups of G, and A be any nonempty subset of G, [G,$\tau_H(A)$] and [G,$\tau_N(A)$] be the nano topological spaces on G then

(i) $\overline{H \cap N}(A) \subseteq \overline{H}(A) \cap \overline{N}(A) \subseteq \overline{H}(A) \cup \overline{N}(A)$

(ii) $\underline{H \cap N}(A) \supseteq \underline{H}(A) \cup \underline{N}(A) \supseteq \underline{H}(A) \cap \underline{N}(A)$

Proof. (i) For all c $\in \overline{H \cap N}(A) \Leftrightarrow c(H \cap N) \cap A \neq \emptyset \Leftrightarrow$ there exist a $\in c(H \cap N) \cap$ A$\Leftrightarrow$ there exist $a \in c(H \cap N)$ and $a \in A \Leftrightarrow$ there exist $a \in cH \cap cN$ and $a \in A$ $\Leftrightarrow$ there exist $a \in cH, a \in A$ and $a \in cN, a \in A$, $\Leftrightarrow$ there exist $a \in cH \cap A$ and $a \in cN \cap A \Rightarrow cH \cap A \neq \emptyset$ and $cN \cap A \neq \emptyset \Leftrightarrow c \in \overline{H}(A)$ and $c \in \overline{N}(A) \Leftrightarrow c \in \overline{H}(A) \cap \overline{N}(A)$. Thus it holds that $\overline{H \cap N}(A) \subseteq \overline{H}(A) \cap \overline{N}(A)$. And also $\overline{H}(A) \cap \overline{N}(A) \subseteq \overline{H}(A) \cup \overline{N}(A)$. Thus proved. (ii) For all c $\in \underline{H}(A) \cup \underline{N}(A) \Leftrightarrow c \in \underline{H}(A)$ or

$\underline{N}(A) \Leftrightarrow cH \subseteq A$ or $cN \subseteq A \Rightarrow cH \cap cN \subseteq A \Leftrightarrow c(H \cap N) \subseteq A \Leftrightarrow c \in \underline{H \cap N}(A)$. Thus we get $\underline{H \cap N}(A) \supseteq \underline{H}(A) \cup \underline{N}(A)$. And also $\underline{H}(A) \cup \underline{N}(A) \supseteq \underline{H}(A) \cap \underline{N}(A)$. Thus $\underline{H \cap N}(A) \supseteq \underline{H}(A) \cup \underline{N}(A) \supseteq \underline{H}(A) \cap \underline{N}(A)$.

Theorem 3.7. Let [G,$\tau_N(A)$] be the nano topological space induced by group G with respect to $A \subseteq G$ then $\overline{N}(A) = AN$

Proof. Let x be any element of $\overline{N}(A)$; then xN$\cap A \neq \emptyset$. This means that there exists $y \in G$ such that $y \in xN$ and $y \in A$. Since N is a normal subgroup of G, then $x \in yN$ and $y \in A$ and so $x \in AN$, Hence $\overline{N}(A) \subseteq AN$. On the other hand, let x be any element of AN; then there exist $a \in A$ and $n \in N$ such that x=an. Thus a= $xn^{-1} \in xN$. By fact that N is a normal subgroup of G, we have a=$xn^{-1} \in xN$, which implies a$\in xN \cap A \neq \emptyset$. Thus $x \in \overline{N}(A)$. Hence $AN \subseteq \overline{N}(A)$. Therefore, we have $\overline{N}(A) = AN$.

Proposition 3.8. Let H and N be normal subgroups of G. If A is a subgroup of G then $\overline{H}(A)\overline{N}(A) \subseteq \overline{HN}(A)$.

Theorem 3.9. Let [G,$\tau_H(A)$] and [G,$\tau_N(A)$] be nano topological spaces on G, $A \subseteq G$ then $\overline{H}(A)\overline{N}(A) = \overline{HN}(A)$

Proof. By Proposition 4.8, we have $\overline{H}(A)\overline{N}(A) \subseteq \overline{HN}(A)$. Let x be any element in $\overline{NH}(A)$; it follows from the definition of $\overline{NH}(A)$ that x(HN)$\cap A \neq \emptyset$, which implies that there exists $y \in G$ such that $y \in x(HN) \cap A$ and so $y \in x(HN)$ and $y \in A$. Hence there exists $a \in H, b \in N$ such that $y = x$ab. Since $y = xab \in xaN$ and $y \in A$, We have $xaN \cap A \neq \emptyset$, which implies xa $\in \overline{N}(A)$. Thus $x \in \overline{N}(A)a^{-1}$, since $a \in H \subseteq \overline{H}(A)$, it follows $x \in \overline{N}(A)\overline{H}(A)$. Hence $\overline{H}(A)\overline{N}(A) \supseteq \overline{HN}(A)$. Thus $\overline{H}(A)\overline{N}(A) = \overline{HN}(A)$.

Theorem 3.10. Let [G,$\tau_H(A)$] and [G,$\tau_N(A)$] be nano topological spaces on G. If A is a subgroup of G, then $\underline{H}(A)\underline{N}(A) = \underline{HN}(A)$.

Proof. First we have to prove that, if $\underline{H}(A) \neq \emptyset$, then $\underline{H}(A) = A$. For any $x \in \underline{H}(A)$, by the definition we have $xH \subseteq A$. Thus $x = x$e $\in A$, where e is the identity element of H. Since A is a subgroup of G, it follows that $x^{-1} \in A$. Hence H $= x^{-1}xH \subseteq AA \subseteq A$. It is obvious that $\underline{H}(A) \subseteq A$ is true. We only need to prove $A \subseteq \underline{H}(A)$. Let $a \in A$. By H$\subseteq A$, we have aH$\subseteq AA \subseteq A$ which implies $a \in \underline{H}(A)$. Thus $A \subseteq \underline{H}(A)$. So we get $\underline{H}(A)$=A. Similarly, $\underline{N}(A) \neq \emptyset$, then $\underline{N}(A) = A$.
If $\underline{H}(A) \neq \emptyset$ and $\underline{N}(A) \neq \emptyset$, by the above proof we have $\underline{H}(A) = A$ and $\underline{N}(A) = A$. Then $\underline{H}(A)$ and $\underline{N}(A)$ are subgroups of G. This means that the identity element $e \in \underline{H}(A)$ and $e \in \underline{N}(A)$. So we have $H \subseteq A$ and $N \subseteq A$, and so $HN \subseteq A$. Hence $e \in \underline{HN}(A) \neq \emptyset$. Similar to the proof of $\underline{H}(A) = A$, we have $\underline{HN}(A) = A$. Now we are to prove that, if $\underline{H}(A) = \emptyset$, then $\underline{HN}(A) = \emptyset$. Since $\underline{H}(A) = \emptyset$, we

have $aH \subset A$ for any $a \in G$, which implies $aH \subset NA$. Thus $a \notin \underline{HN}(A)$. Hence $\underline{HN}(A) = \emptyset$. Since $\underline{H}(A) = \emptyset$, we have $aH \subsetneq A$ for any $a \in G$, which implies $aH \subsetneq NA$. Thus $a \notin \underline{HN}(A)$. Hence $\underline{HN}(A) = \emptyset$.

Theorem 3.11. Let N be a normal subgroup of a group G and A a subgroup of G such that $N \supset A$, then the nano topology $\tau_N(A) = \{G, \emptyset, \overline{N}(A)\}$.
Proof. First, we are to prove the fact, if $N \supset A$, then xN$\supset A$ for any $x \in G$. Assume that there exists $x \in G$ such that $xN \subseteq A$; then $x = x$e $\in A$, where $e \in N$ is the identity element. Since A is a subgroup of G, it follows that $x^{-1} \in A$. Thus $x^{-1}xN \subseteq AA \subseteq A$. That is, $N \subseteq A$. This is a contradiction. Hence, $\forall x \in G$, $xN \supset A$. So we get $\underline{N}(A) = \emptyset$. By the definition of $\overline{N}(A)$, for all $x \in G$, $xN \cap A \neq \emptyset$. Hence $\overline{N}(A) = A$. Thus the nano topology [G,$\tau_N(A)$] $= \{G, \emptyset, \overline{N}(A)\}$.

Theorem 3.12. Let N be a normal subgroup of a group G and A a subgroup of G such that $N \subseteq A$ then the nano topology on G, $\tau_N(A) = \{G, \emptyset, \underline{N}(A)\}$.
Proof. If N$\subseteq A$, then $xN \subseteq AA \subseteq A$ for any $x \in A$. This implies $x \in \underline{N}(A)$. Thus $A \subseteq \underline{N}(A)$. Hence $\underline{N}(A) = A$. Also, by the definition of $\overline{N}(A)$, for all $x \in N$, $xN \cap A \neq \emptyset$. Hence $\underline{N}(A) = \overline{N}(A) = A$. Thus the nano topology $\tau_N(A) = \{G, \emptyset, \underline{N}(A)\}$.

Remark 3.13. By considering N as a normal subgroup of G and A to be any arbitrary subset of G and by taking into account the possible inclusions between N and A we have framed the structure of the nano topology induced on G.

Theorem 3.14. Let N be a normal subgroup of G and A be any subset of G such that $N \subset A$, then the nano topology is either $\tau_N(A) = \{G, \emptyset, \underline{N}(A), B_N(A)\}$ or $\tau_N(A) = \{G, \emptyset, \underline{N}(A), \overline{N}(A), B_N(A)\}$.

Theorem 3.15. Let N be a normal subgroup of a group G and A be any subset of G such that $N \supset A$,then the nano topology $\tau_N(A) = \{G, \emptyset, \overline{N}(A)\}$.

Theorem 3.16. Let N be a normal subgroup of G and A be any subset of G, such that A $= N^c$, then the nano topology $\tau_N(A) = \{G, \emptyset, \overline{N}(A)\}$.

Theorem 3.17. Let N be a normal subgroup of a group G and A be any subset of G, having atleast one element of xN $\forall x \in G$, then nano topology is either the indiscrete nano topology on G or $\tau_N(A) = \{G, \emptyset, \overline{N}(A)\}$.

4. Nano Topology and its Structural Equivalence in Groups

Algebraically and topologically speaking structural equivalence is based on the existence of isomorphic groups and homeomorphic topological spaces. In this section we have made an attempt to connect these structural equivalence property to create a link between isomorphic groups and nano topology induced by the group.

Remark 4.1. Suppose f: $G \to G'$ is a homomorphism and A and A' are subgroups (normal subgroups) of G and G', respectively then $f(A)$ and $f^{-1}(A')$ are subgroups (normal subgroups) of G' and G respectively.

Theorem 4.2. Let f be an isomorphism between groups G and G', N is a normal subgroup of G and A be a subgroup of G. Then

(i) $f(\underline{N}(A)) = \underline{f(N)}(f(A))$

(ii) $f(\overline{N}(A)) = \overline{f(N)}(f(A))$

Proof. (i). Since f is an isomorphism from G and G', N is a normal subgroup of G, and A be a subgroup of G , then $f(N)$ and $f(A)$ are a normal subgroup of G'. Since $N \subseteq A \Leftrightarrow f(N) \subseteq f(A)$, by Theorem 4.12 we can get $\underline{N}(A) =$ A $\Leftrightarrow \underline{f(N)}(f(A)) = f(A)$. Thus $f(\underline{N}(A)) = \underline{f(N)}(f(A))$.
(ii): For any element $a \in \overline{N}(A)$, by the definition $\overline{N}(A)$ we have aN$\cap A \neq \emptyset$. Since $f(aN \cap A) \subseteq f(aN) \cap f(A)$, it follows that $f(aN) \cap f(A) \neq \emptyset$, which implies $f(a)f(N) \cap f(A) \neq \emptyset$. Thus $f(a) \in \overline{f(N)}(f(A))$. Hence $f(\overline{N}(A)) \subseteq \overline{f(N)}(f(A))$. On the other hand, for any element $y \in \overline{f(N)}(f(A))$, by the definition of $\overline{f(N)}(f(A))$, we have $yf(N) \cap f(A) \neq \emptyset$. Thus there exist $n \in N$ and $a \in A$ such that yf(n) = f(a). Since N is a normal subgroup of G, then f(N) is also a normal subgroup of G'. Thus $(f(n))^{-1} \in f(N)$. Hence $y= f(a)(f(n))^{-1} = f(a)f(n^{-1}) = f(an^{-1})$. Since a = $(an)^{-1}n \in (an^{-1})N \cap A \neq \emptyset$, it follows that $an^{-1} \in \overline{N}(A)$. This means $y \in f(\overline{N}(A))$. Hence $\overline{f(N)}(f(A)) \subseteq f(\overline{N}(A))$. So we get $f(\overline{N}(A)) = \overline{f(N)}(f(A))$.

Theorem 4.3. Let G and G' be two groups. Let f be an isomorphism from G to G', N' a normal subgroup of G' and A' is a nonempty subset of G'. Then

(i) $f^{-1}(\underline{N'}(A') = \underline{f^{-1}(N')}(f^{-1}(A'))$

(ii) $f^{-1}(\overline{N'}(A') = \overline{f^{-1}(N')}(f^{-1}(A'))$

Theorem 4.4. If f: G $\to G'$ be an isomorphism, and N is a normal subgroup of G and A be any subset of G then f: [G,$\tau_N(A)$] $\to [G', \tau_{f(N)}(f(A))]$ is a nano homeomorphism.
Proof. Since f: G $\to G'$ is an isomorphism, we have f is a bijection and also $f(xy) = f(x)f(y)\ \forall x, y \in G$. Let N be any normal subgroup of G, since f is an isomorphism f(N) is also a normal subgroup of G' and also $A \subseteq G \Rightarrow f(A) \subseteq G'$. Hence by the definition of nano topology on groups there exist two nano topologies $\tau_N(A)$ on G and $\tau_{f(N)}(f(A))$ on G' respectively. Now consider

f: [G,$\tau_N(A)$] $\rightarrow$ $[G', \tau_{f(N)}(f(A))]$ where f is an isomoprhism from G to G'. To prove that f is a homeomorphism. Since the only nano open sets in $\tau_N(A)$ are $G, \emptyset, \underline{N}(A), \overline{N}(A)$ *and* $B_N(A)$ and we have proved that image of a lower and upper approximations with respect to G are also lower and upper approximations with respect to G'. Since f is 1-1 and onto f(G) = G' and $f(\emptyset) = \emptyset$. Hence it follows that f is an open map. Similarly we can prove that inverse image of every nano open set in $\tau_N(A)$ is nano open in $\tau_{f(N)}(f(A)$. Thus f is a nano homeomorphism.

Remark 4.5. This example is an assertion of the above theorem.

Example 4.6. Let G = $\{1, -1, i, -i\}$ be a cyclic group of order 4 and G' = $\{0, 1, 2, 3\}$ be a group under addition modulo 4. If f: G$\rightarrow G'$ defined by $f(1) = 0, f(-1) = 2, f(i) = 1, f(-i) = 3$ is an isomorphism. Let N = $\{1, -1\}$ be a normal subgroup of G and A = $\{-i\}$ be any subset of G. Then the nano topology induced by group on G is [G,$\tau_N(A)$] = $\{G, \emptyset, \{i, -i\}\}$. And also f(N) = $\{0, 2\}$ is a normal subgroup of G' and f(A) = $\{3\}$ is a subset of G' then nano topology induced by group on G', [G',$\tau_{f(N)}(f(A))$] = $\{G, \emptyset, \{1, 3\}\}$. Then $f^{-1}(G') = G$, $f^{-1}(\emptyset) = \{\emptyset\}$, $f^{-1}(\{1, 3\}) = \{i, -i\}$. That is inverse image of every nano open set in G' is nano open in G. Also we note that the image of every nano open set in G is nano open in G' and f is a bijection. Thus [G,$\tau_N(A)$] is homeomorphic to [G',$\tau_{f(N)}(f(A))$]. Thus f is a nano homeomorphism.

Definition 4.7. [7] Let f: G$\rightarrow G'$ be a homomorphism. Let K= $\{x/x \in G, f(x) = e'\}$. Then K is called the kernel of f and is denoted by kerf.

Theorem 4.8. [7] Let f: G$\rightarrow G'$ be a homomorphism. Then the kernel K of f is a normal subgroup of G.

Theorem 4.9. [7] (First isomorphism theorem) Let f: G$\rightarrow G'$ be an epimorphism with kernel K, then G/K and G' are isomorphic.

Theorem 4.10. [7] (Second isomorphism theorem) Let G be a group, Let H and N be subgroups of G and N be normal in G. Then the factor-groups NH/N and H/(N$\bigcap$H) are isomorphic.

Remark 4.11. The above theorem paves way to the first and second isomorphism theorem in nano topological space induced by groups.

Theorem 4.12. Let f: G$\rightarrow G'$ be an epimorphism with kernel K, then the nano topology induced by group G/K and G' are nano homeomorphic.
Proof. By First isomorphism theorem we have, If f: G$\rightarrow G'$ be an epimorphism with kernel K, then G/K and G' are isomorphic. Since we have proved in Theorem 5.4 that "If two groups are isomorphic then the corresponding induced nano

topologies are nano homeomorphic". Hence nano topology induced by group G/K and G' are nano homeomorphic.

Remark 4.13. The following is an example for first isomorphism theorem on nano topological space induced by group.

Example 4.14. Let G = $\{1, a, a^2, a^3, a^4, a^5, a^6, a^7\}$ and G' = $\{1, b, b^2, b^3\}$ be two cyclic groups of order 8 and 4 respectively. And f:G→ G' be an epimorphism defined as $f(1) = f(a^4) = 1$, $f(a) = f(a^5) = b$, $f(a^2) = f(a^6) = b^2$, $f(a^3) = f(a^7) = b^3$ with kernel K = $\{1\}$. If $\chi : G/K \to G'$ is an isomorphism defined as $\chi(1K) = 1$, $\chi(aK)$ = b, $\chi(a^2K) = b^2$, $\chi(a^3K) = b^3$, then the nano topology induced by group induced by $G/K = \{1K, aK, a^2K, a^3K\}$ with respect to the normal subgroup N =$\{1K\}$ and $A = \{aK\} = \{a, a^5\} \subseteq G/K$ is [G/K,$\tau_N(A)$] = $\{G/K, \emptyset, \{aK\}\}$ which is nano homeomorphic to the nano topology induced by G' with respect to the normal subgroup $\chi(N) = 1$, $\chi(A) = b$ [G',$\tau_{\chi(N)}(\chi(A))$] = $\{G', \emptyset, \{b\}\}$.

Theorem 4.15. Let G be a group, Let H and N be subgroups of G and N be normal in G. Then the nano topology induced by the factor-groups NH/N and H/(N∩H) are nano homeomorphic.

5. Conclusion

In this paper, we studied the properties of nano approximations based on choice of subgroups. These studies gives us a link between nano topology and group theory which yields the First and second isomorphism theorem in nano topological space induced by group. In future this work can also be extended further on rings and also in other areas.

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