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NOTE ON NANO HYPER CONNECTED SPACES

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Abstract: The objective of this paper is to define nano hyper connected space and study its properties.This paper gives a charcterization of nano hyper connected spaces,using the concept of nano pre open sets and nano semi open sets. Further some new weak nano functions are defined which preserve nano hyperconnected property.

Keywords and Phrases: Nano topology, nano hyper connected space,nano ultra connected, nano feebly continuous functions, nano somewhat nearly continuous function.

2010 Mathematics Subject Classification: 54G05.

1. Introduction

Lellis Thivagar et al [1] introduced nano topological space with respect to a subset X of an finite universe which is defined interms of lower and upper approximations of X. The elements of the nano topological space are called nano open sets. Connectedness is one of the core concepts of topology. Here we introduce nano hyper connectedness and derived their charaterizations in terms of nano semi open sets, nano pre open sets and nano α open sets. In this paper we have introduced a new class of functions on nano topological spaces called nano feebly continuous functions, nano somewhat nearly continuous function and establish the relation between them.

2. Preliminaries

Definition 2.1. [4]: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalance relation on $\mathcal{U}$ named as the indiscernibility relation.Elements belonging to the same equivalance class are said to indiscernible with one another.The pair ($\mathcal{U}$,R) is said to be the approximation space.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $(\mathcal{U},\tau_R(X))$. That is
$L_R(X) = \cup\{R(x) : R(x) \subseteq X\}$ where R(x)denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is
$U_R(X) = \cup\{R(x) : R(x) \cap X \neq \emptyset\}$

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.That is
$B_R(X)= \mathcal{U}_R(X) - L_R(X)$.

Definition 2.2. [4]: Let $\mathcal{U}$ be the universe, R be an equivalance relation on $\mathcal{U}$ and $\tau_R(X)$ =\{ $\mathcal{U}$, $\emptyset$, $L_R(X)$, $U_R(X)$, $B_R(X)$\} where X $\subseteq \mathcal{U}$. Then $\tau_R(X)$ satisfies the following axioms

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on $\mathcal{U}$ called the nanotopology on $\mathcal{U}$ with repect to X. We call $(\mathcal{U},\tau_R(X))$ as the nanotopological space. The elements of $\tau_R(x)$ are called as nano open sets.

Definition 2.3. [4]:Let $\mathcal{U}$ be a nonempty finite universe and X $\subseteq \mathcal{U}$

(i) If $L_R(X)=\emptyset$ and $U_R(X)=\mathcal{U}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset\}$ the indiscrete nano topology on $\mathcal{U}$

(ii) If $L_R(X)= U_R(X) =$ X then the nano topology $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X)\}$

(iii) If $L_R(X)=\emptyset$ and $U_R(X) \neq \mathcal{U}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, U_R(X)\}$

(iv) If $L_R(X) \neq \emptyset$ and $U_R(X)=\mathcal{U}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), B_R(X)\}$

(v) If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathcal{U}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ is the discrete nano topology on $\mathcal{U}$

Definition 2.4. [4]: If ($\mathcal{U}$,$\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq \mathcal{U}$ and if A $\subseteq \mathcal{U}$,then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A).That is Nint(A)is the largest nano-open subset of A.The nano closure of A is defined as the intersection of all nanoclosed sets containing A and it is denoted by Ncl(A).That is Ncl(A)is the smallest nano closed set containing A.

Definition 2.5. [5]:A nano topological space ($\mathcal{U}$,$\tau_R(X)$) is extremally disconnected, if the nano closure of each nano-open set is nano open in $\mathcal{U}$.

Definition 2.6. [5] Let ($\mathcal{U}$,$\tau_R(X)$) be a nano topological space and A$\subseteq \mathcal{U}$. Then A is said to be

(i) nano semi open if A $\subseteq \mathcal{N}cl(\mathcal{N}int(A))$

(ii) nano pre-open if A $\subseteq \mathcal{N}int(\mathcal{N}cl(A))$

(iii) nano α-open if A $\subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(A))$

NSO($\mathcal{U}$,X), NPO($\mathcal{U}$,X), NαO($\mathcal{U}$,X) respectively denote the families of all nano semi-open, nano pre-open and nano α-open subset of $\mathcal{U}$.

Definition 2.7. :Let X be a topological space. A separation of X is a pair A and B of disjoint nonempty open subsets of X union is X. The space X is said to be connected if there does not exist a separation of X.

Definition 2.8. : Let E be a subspace of a topological space X. E is said to be separated if there exist two disjoint nonempty opensets(closed sets) A and B such that $A \cup B$=E,$A \cap \overline{B} = \emptyset$ and $\overline{A} \cap B = \emptyset$

Definition 2.9. : Let X be a nonempty set. Let $\mathcal{F}$ be a non-empty family of subsets of X. Then $\mathcal{F}$ is said to be a filter on X if the following conditions are satisfied

(i) $\emptyset \notin \mathcal{F}$

(ii) If F $\in \mathcal{F}$ and F$\subset$ H then H $\in \mathcal{F}$

(iii) If F, H $\in \mathcal{F}$ then F $\cap$ H $\in \mathcal{F}$.

Definition 2.10 :A function f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V}, \tau_R(Y))$ is said to be

(i) nano continuous if $f^{-1}(V)$ is nano open in $\mathcal{U}$ for every nano open set V of $\mathcal{V}$.

(ii) nano semi continuous if $f^{-1}(V)$ is nano semi open in $\mathcal{U}$ for every nano open set V of $\mathcal{V}$.

(iii) nano open if the image of every nano open set in $\mathcal{U}$ is nano open in $\mathcal{V}$

3. Nano Hyper Connected Space

Here we introduce nano Hyper connected space and characterize its properties.

Definition 3.1 : A nano topological space is hyper connected if intersection of any two non empty nano open sets is non empty.

Example 3.2 : Let $\mathcal{U} = \{a, b, c, d\}$ and $\mathcal{U}/R = \{\{a, b, c\}, \{d\}\}$. Let X =$\{a, b\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b, c\}\}$ here $\mathcal{U}$ and $\{a, b, c\}$ are nonempty and their intersection is non empty. Therefore$(\mathcal{U},\tau_R(X))$ is a nano hyper connected space.

Proposition 3.3 : In a nano topological space$(\mathcal{U},\tau_R(X))$, if $\tau_R(X) = \{\mathcal{U}, \emptyset\}$ then$(\mathcal{U},\tau_R(X))$ is nano hyper connected.

Proposition 3.4 : In a nano topological space$(\mathcal{U},\tau_R(X))$,if $\tau_R(X)$ =$\{\mathcal{U}, \emptyset, L_R(X)\}$ then $(\mathcal{U},\tau_R(X))$ is nano hyper connected.

Proof : In $\tau_R(X)$ the non empty nano open sets are $\mathcal{U}$ and $L_R(X)$,
$\mathcal{U} \cap L_R(X) = L_R(X) \neq \emptyset$.Therefore $(\mathcal{U}, \tau_R(X))$is nano hyper connected.

Proposition 3.5 : In a nano topological space$(\mathcal{U},\tau_R(X))$,if $\tau_R(X)$ =$\{\mathcal{U}, \emptyset, U_R(X)\}$ then $(\mathcal{U},\tau_R(X))$ is nano hyper connected.
Proof : In $\tau_R(X)$ the non empty nano open sets are $\mathcal{U}$ and $U_R(X)$,
$\mathcal{U} \cap U_R(X) = U_R(X) \neq \emptyset$.Therefore $(\mathcal{U}, \tau_R(X))$is nano hyper connected.

Remark 3.6 :

(i) In a nano topological space $(\mathcal{U},\tau_R(X))$, if $\tau_R(X)$ =$\{\mathcal{U}, \emptyset, L_R(X), B_R(X)\}$ then $(\mathcal{U}, \tau_R(X))$ is not nano hyper connected because $L_R(X) \neq \emptyset$ and $B_R(X) \neq \emptyset$ but $L_R(X) \cap B_R(X) = \emptyset$

(ii) In a nano topological space $(\mathcal{U},\tau_R(X))$,if $\tau_R(X)=\{\mathcal{U},\emptyset,U_R(X),L_R(X),B_R(X)\}$ then
$(\mathcal{U},\ \tau_R(X))$is not nano hyper connected because $L_R(X)\neq\emptyset$ and $B_R(X)\neq\emptyset$ but
$L_R(X)\cap B_R(X)=\emptyset$

Theorem 3.7 : Every nano hyper connected space is extremally disconnected.
Proof : Let $(\mathcal{U},\tau_R(X))$is nano hyper connected. By the proposition 3.3 ,3.4 and 3.5

(i) If $\tau_R(X)=\{\mathcal{U},\emptyset\}$ By the property of nano closure Ncl($\mathcal{U}$)=$\mathcal{U}$ and Ncl($\emptyset$)= $\emptyset$ then $\tau_R(X)$ is extremally disconnected.

(ii) If $\tau_R(X)=\{\mathcal{U},\emptyset,U_R(X)\}$. Ncl($U_R(X)$)=$\mathcal{U}$ then $\tau_R(X)$ is extremally disconnected.

(iii) If $\tau_R(X)=\{\mathcal{U},\emptyset,L_R(X)\}$ this occous when $L_R(X)=U_R(X)=X$ and $B_R(X)=\emptyset$. By the property Ncl($L_R(X)$)=$[B_R(X)]^c$=$\emptyset^c$=$\mathcal{U}$. Therefore $\tau_R(X)$ is extremally disconnected.

Remark 3.8 : Converse of the above theorem is not true. Let $\mathcal{U}=\{x,y,z\}$, $\mathcal{U}/R=\{\{x\},\{y,z\}\}$. Let X=$\{x,y\}$ then $\tau_R(X)$=$\{\mathcal{U},\emptyset,\{x\},\{y,z\}\}$ then $(\mathcal{U},\tau_R(X))$ is extremally disconnected.But it is not hyper connected because $\{x\}\cap\{y,z\}=\emptyset$

Theorem 3.9 : In a nano hyper connected space$(\mathcal{U},\tau_R(X))$ if $U_R(X)$=$\mathcal{U}$ then $\tau_R(X)=$ NαO(X)=NSO(X).

Proof : By the theorem 3.7 every nano hyperconnected space is extremally disconnected. If $U_R(X)$=$\mathcal{U}$ then $\tau_R(X)=\{\mathcal{U},\emptyset\}$. The nano α -open sets and nano semi open sets are also $\mathcal{U},\emptyset$. Therfore we get the proof.

Theorem 3.10 : Every nano hyper connected space is nano connected.

Proof : Let $(\mathcal{U},\tau_R(X))$ is a nano hyper connected space.By proposition 3.3, 3.4, 3.5 any nano hyper connected space is of the form $\{\mathcal{U},\emptyset\}$ or $\{\mathcal{U},\emptyset,L_R(X)\}$ or $\{\mathcal{U},\emptyset,U_R(X)\}$. In all the three cases $\mathcal{U}$ cannot be written as a union of two nonempty disjoint nano open sets. Therefore every nano hyper connected space is nano connected.

Remark 3.11 Converse of the above theorem is not true. consider $\mathcal{U}=\{a,b,c,d\}$, $\mathcal{U}$/R $=\{\{a\},\{b,c\},\{d\}\}$. Let X $=\{a,b\}$ then $\tau_R(x)=\{\mathcal{U},\emptyset,\{a\},\{a,b,c\},\{b,c\}\}$. Here
$(\mathcal{U},\ \tau_R(X))$ is nano connected but not nano hyper connected.

4. Characterization of Nano Hyper Connected Spaces

Here the characterization of nano hyper connected spaces are completely clarified by the following theorems.Note that in a nano hyper connected space, a nonempty set is nano semi open if and only if it contains a nano open set.

Theorem 4.1 : In a nano hyper connected space ($\mathcal{U}$,$\tau_R(X)$)

(i) Every non empty nano open set is dense in $\mathcal{U}$

(ii) Nano interior of every proper closed set is empty.

Proof.: (i) Let ($\mathcal{U}$,$\tau_R(X)$)is nano hyper connected.By the proposition 3.3, 3.4 and 3.5 $\tau_R(X)$ =$\{\mathcal{U}, \emptyset\}$ or $\tau_R(X) = \{\mathcal{U}, \emptyset, U_R(X)\}$ or $\tau_R(X)$ =$\{\mathcal{U}, \emptyset, L_R(X)\}$. By the property of nano closure Ncl($\mathcal{U}$)=$\mathcal{U}$ and Ncl($U_R(X)$)=$\mathcal{U}$ and Ncl($L_R(X)$)= $[B_R(X)]^c = \emptyset^c = \mathcal{U}$. Therefore every non empty nano open set is dense in $\mathcal{U}$.
(ii) Let A be a nano open set then $\mathcal{U}$-A is closed. By the property $\mathcal{N}int$($\mathcal{U}$-A) = $\mathcal{U} - \mathcal{N}cl(A)$ and by the previous result $\mathcal{N}cl(A) = \mathcal{U}$. Therefore $\mathcal{N}int$($\mathcal{U}$-A) =$\emptyset$.

Theorem 4.2:In a nano hyper connected space ($\mathcal{U}$,$\tau_R(X)$), every non empty nano semi open set is dense in $\mathcal{U}$

Proof:Since ($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected, every non-empty set A$\in \mathcal{N}$ SO(X) contains a nano open set B. Since B$\subseteq$ A, $\mathcal{N}cl(B) \subseteq \mathcal{N}cl(A)$. By theorem 4.1 $\mathcal{N}cl(B)$=$\mathcal{U}$,Therefore $\mathcal{N}cl(A) = \mathcal{U}$.Henceevery nonempty nano semi open sset is dense in $\mathcal{U}$.

Theorem 4.3: In a nano hyper connected space ($\mathcal{U}$,$\tau_R(X)$), $\mathcal{N}$Pcl(A) = U for every non-empty set A$\in \mathcal{N}$ SO(X)
Proof: Let ($\mathcal{U}$,$\tau_R(X)$) is a nano hyperconnected space. Let A $\in \mathcal{N}SO(X)$ then A contains a nano open set V. Therefore V$\subset$ A implies $\mathcal{N}int(V) \subseteq \mathcal{N}int(A)$.
Hence $\mathcal{N}cl(\mathcal{N}int(V)) \subseteq \mathcal{N}cl(\mathcal{N}int(A))$. Since V is nano open $\mathcal{N}int(V)$= V and V is dense in $\mathcal{U}$, $\mathcal{N}cl(\mathcal{N}int(V)) = \mathcal{N}cl(V) = \mathcal{U}$. Therefore $\mathcal{U} \subseteq \mathcal{N}cl(\mathcal{N}int(A))$ which is possible only when $\mathcal{U} = \mathcal{N}cl(\mathcal{N}int(A))$ Therefore $\mathcal{N}Pcl(A) = \mathcal{U}$.

Theorem 4.4 The following conditions are equivalent for a nano topological space($\mathcal{U}$,$\tau_R(X)$)

(i) ($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected.

(ii) $\mathcal{N}$Pcl(A) = U for any non-empty set A$\in \mathcal{N}$ SO(X)

Proof: Proof of the theorem follows from theorems 4.1 and 4.3

Theorem 4.5 : A nano topological space ($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected if and only if $\mathcal{N}SO(\mathcal{U}, X)\diagdown\{\emptyset\}$ is a filter on $\mathcal{U}$.

Proof : Let ($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected. $\emptyset \notin \mathcal{N}SO(\mathcal{U}, X)\diagdown\{\emptyset\}$. Let A,B $\in \mathcal{N}SO(\mathcal{U}$,X) $\diagdown\{\emptyset\}$ then by the property of nano semi opensets, there exist two nano open sets C and D such that C$\subseteq$A and D$\subseteq$B.Since($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected $\emptyset \neq$ C$\cap D \subset$ A$\cap$B and hence A$\cap B \subset \mathcal{N}SO(\mathcal{U}$,X) $\diagdown\{\emptyset\}$. Suppose A $\subset\in \mathcal{N}SO(\mathcal{U}$,X) $\diagdown\{\emptyset\}$ then by everyset containing A is in $\mathcal{N}SO(\mathcal{U}, X)$. Therefore $\mathcal{N}SO(\mathcal{U}$,X) $\diagdown\{\emptyset\}$ is a filter on $\mathcal{U}$.
Converse part is obvious because $\tau_R(X) \subset \mathcal{N}SO(\mathcal{U}, X)$.

5. Nano Super Connected Spaces

We define nano super connected spaces,nano ultra conncted and derive the relation between them.

Definition 5.1 : A nano topological space is nano ultra connected if the intersection of any two nonempty nano closed sets is non empty.

Example 5.2 : Let U = $\{a, b, c, d\}$and $\mathcal{U}/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let X =$\{a, b\}$ then
$\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$. The closed sets are $\mathcal{U}, \emptyset, \{c, d\}$. Therefore($\mathcal{U}$,$\tau_R(X)$) is a nano Ultra connected space.

Definition 5.3 : A nano topological space is called nano s-space if every subset which contains a non empty nano open subset is nano open.

Definition 5.4 : A nano topological space is called nano super connected if it is nano connected and s-space.

Example 5.5 : Let U = $\{a, b, c, d\}$and $\mathcal{U}/R = \{\{a, b, c\}, \{d\}\}$. Let X =$\{a, b\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b, c\}\}$ Therefore($\mathcal{U}$,$\tau_R(X)$) is a nano super connected space.

Definition 5.6 : A nano topological space is called nano F-connected if it is both nano hyper connected and nano ultra connected.

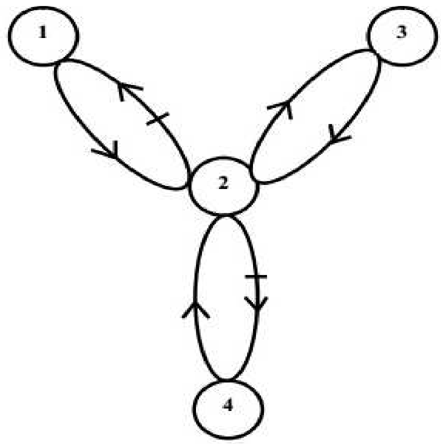
Example 5.7 : Let U = $\{a, b, c, d\}$and $\mathcal{U}/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let X =$\{a, b\}$ then
$\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$. The closed sets are $\mathcal{U}, \emptyset, \{c, d\}$. Therefore($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected a nano Ultra connected space.Therefore it is a F-space.

Proposition 5.8 : Every nano topological space is nano hyper connected if and only if it is nano ultra connected.
Proof : From the definition the proof is obvious.

Proposition 5.9:Every nano super connected space is nano hyper connected.
Proof : Since every nano super connected space is a nano connectd and s-space

and every nano connected space is nano hyper connected.Therfore every nano super connected space is nano hyper connected.

Remark 5.10 : Converse of the above result is not true.Every nano hyper connected space need not be nano super connected.Consider Let U = $\{a, b, c\}$and $\mathcal{U}/R = \{\{a\}, \{b, c\}\}$. Let X =$\{a\}$ then
$\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}\}$. Therefore($\mathcal{U}$,$\tau_R(X)$) is a nano hyper connected space but not a nano super connected space.
The relation between various connected spaces are represented in the following figure.



1. Nano Connected 2. Nano Hyper Connected
3. Nano Ultra Connected 4. Nano Super Connected

6. Applications

We introduce two nano functions nano feebly continuous and nano somewhat nearly continuous and show that nano hyper connectedness is preserved under nano feebly continuous function.

Definition 6.1 : A function f:($\mathcal{U}$,$\tau_R(X)$)$\longrightarrow$ ($\mathcal{V}$,$\tau_{R'}(Y)$) is said to be nano feebly continuous if for every nonempty nano open set V of $\mathcal{V}$, $f^{-1}(V) \neq \emptyset$ implies $\mathcal{N}int(f^{-1}(V)) \neq \emptyset$

Example 6.2 : Let $\mathcal{U} = \mathcal{V}$ =$\{a, b, c, d\}$ and $\mathcal{U}/R = \{\{a, b, c\}, \{d\}\}$. Let X = $\{a, b\}$ then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b, c\}\}$. Let $\mathcal{V}/R' = \{\{a\}, \{b, c\}, \{d\}\}$ and Y = $\{c\}$ then $\tau_{R'}(Y) = \{\mathcal{U}, \emptyset, \{b, c\}\}$. Let f : ($\mathcal{U}$,$\tau_R(X)$) $\longrightarrow$ ($\mathcal{V}$,$\tau_{R'}(Y)$) is a constant function. Let
V = $\{b, c\}$, $f^{-1}(V) \neq \emptyset$ implies $\mathcal{N}int(f^{-1}(V)) \neq \emptyset$

Proposition 6.3 Every nano semi continuous function is nano feebly continuous.
Proof : Let f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is nano semi continuous. Let V be a non empty nano open set in $\mathcal{V}$. Then $f^{-1}(V)$ is a non empty nano semi open set in $\mathcal{U}$ and $f^{-1}(V) \subset \mathcal{N}cl(\mathcal{N}int(f^{-1}(V)))$. Since $f^{-1}(V) \neq \emptyset$ then $\mathcal{N}int f^{-1}(V) \neq \emptyset$. Therefore f is nano feebly continuous.

Definition 6.4 : A function f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is said to be nano somewhat nearly continuous if for every nonempty nano open set V of $\mathcal{V}$, $f^{-1}(V) \neq \emptyset$ implies $\mathcal{N}int(\mathcal{N}cl(f^{-1}(V)) \neq \emptyset$

Example 6.5 : In example 6.2 the function f is nano somewhat nearly continuous.
Proposition 6.6 : Every nano feebly continuous function is nano somewhat nearly continuous.
Proof : Let f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is nano feebly continuous. Let V be a non empty nano open set in $\mathcal{V}$. Then $f^{-1}(V)$ is non empty and $\mathcal{N}int(f^{-1}(V)) \neq \emptyset$.Since $f^{-1}(V) \neq \emptyset$, $\mathcal{N}cl f^{-1}(V) \neq \emptyset$.Therefore $\mathcal{N}int(\mathcal{N}cl(f^{-1}(V))) \neq \emptyset$. Hence f is nano somewhat nearly continuous.

Remark 6.7 : But the converse is not true. Let $\mathcal{U} = \{a,b,c,d\}$ and $\mathcal{U}/R = \{\{a,b,c\},\{d\}\}$.Let X = $\{a,b\}$ then $\tau_R(X) = \{\mathcal{U},\emptyset,\{a,b,c\}\}$.Let $\mathcal{V}$=$\{1,2,3,4\}$,$\mathcal{V}/R'$ = $\{\{1\},\{2,3\},\{4\}\}$ and Y = $\{1,3\}$ then $\tau_{R'}(Y) = \{\mathcal{U},\emptyset,\{1\},\{1,2,3\},\{2,3\}\}$. Let f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is defined as f(a)=1, f(b)=2, f(c)=3,f(d)=4. V = $\{1\}$,$f^{-1}(V) \neq \emptyset$ implies $\mathcal{N}int(f^{-1}(V)) = \emptyset$. Therefore f is not nano feebly continuous.But f is nano somewhat nearly continuous.

Theorem 6.8 : Let $(\mathcal{U},\tau_R(X))$ is a nano hyper connected space. If f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is somewhat nearly continuous and G(f) is closed in U x V then f is constant.
Proof :Suppose f is not a constant function. Let x,y $\in$ U such that f(x)$\neq$ f(y) then (x, f(y))$\in$ XxY not in G(f). Since XxY is closed, there exist open sets U ,V such that x$\in$U and f(y)$\in$ V and (UxV)$\cap$ G(f)=$\emptyset$.Therefore f(U)$\cap$ V =$\emptyset$ implies U$\cap f^{-1}(V) = \emptyset$. Which is a contradiction to X is nano hyper connected. Therefore f is a constant function.

Corollary 6.9 : Let $(\mathcal{U},\tau_R(X))$ is a nano hyper connected space. If f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is nano continuous and G(f) is closed in U x V then f is constant.

Theorem 6.10 : Let $(\mathcal{U},\tau_R(X))$ is a nano hyper connected space. If f:$(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{V},\tau_{R'}(Y))$ is nano feebly continuous surjection then $(\mathcal{V},\tau_R(Y))$ is hyper connected.
Proof : Suppose $(\mathcal{V},\tau_R(Y))$ is not nano hyper connected. There exist two disjoint non empty open sets G and H of $\mathcal{V}$ such that G$\cap$H = $\emptyset$. Since f is nano feebly

continuous surjection, $f^{-1}(G) \neq \emptyset$ and $f^{-1}(H) \neq \emptyset$. Let U = $\mathcal{N}int(f^{-1}(H))$, V= $\mathcal{N}int(f^{-1}(G))$ then U $\neq \emptyset$ and V$\neq \emptyset$. U$\cap$ V = $\mathcal{N}int(f^{-1}(G)) \cap \mathcal{N}int(f^{-1}(H))$= $\mathcal{N}int(f^{-1}(G \cap H)) \neq \emptyset$.Which is a contradiction to ($\mathcal{U}$,$\tau_R(X)$) is nano hyper connected. Therefore ($\mathcal{V}$,$\tau_R(Y)$) is nano hyper connected.

Theorem 6.11 : Let ($\mathcal{V}$,$\tau_R(Y)$) is a nano hyper connected space. If f:($\mathcal{U}$,$\tau_R(X)$)$\longrightarrow$ ($\mathcal{V}$,$\tau_{R'}(Y)$) is nano open injection then ($\mathcal{U}$,$\tau_R(X)$) is hyper connected.
Proof : Let U and V are non empty nano open subset of $\mathcal{U}$. Since f is nano open, f(U) and f(V) are non empty nano open in $\mathcal{V}$.Since ($\mathcal{V}$,$\tau_{R'}(Y)$)is nano hyper connected, $f(U) \cap f(V) \neq \emptyset$. Since f is injective, $\emptyset \neq f(U) \cap f(V) = f(U \cap V)$. Therefore $U \cap V \neq \emptyset$. Hence ($\mathcal{V}$,$\tau_R(Y)$) is a nano hyper connected space.

Remark 6.12 : From the above discussion we have the following table.The symbol "1" in a cell means that a function on the corresponding row implies a function on the corresponding column. Finally the symbol "0" means that a function on the corresponding row does not implies the function on the corresponding column.

Nano Continuous function	A	B	C
A	1	1	1
B	0	1	1
C	0	0	1

A. Nano Semi Continuous B. Nano feebly continuous
C. Nano somewhat feebly continuous.

7. Conclusion

Here we defined nano hyper connected spaces and study its properties. We also introduce some nano functions and characterize the properties of hyper connectedness. Hyper connectedness is a strong property than connectedness, therefore all the results which are true in hyper connected space are automatically true in connected spaces.

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