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PATH UNION OF n NON ISOMORPHIC COPIES OF COMPLETE BIPARTITE GRAPH IS ODD GRACEFUL

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Abstract: In 1991, Gnanajothi [4] introduced a labeling method called *odd graceful labeling* to label the vertices of a graph. A graph G with q edges is said to be odd graceful if there is an injection f from $V(G) \to \{0, 1, 2, ..., (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $1, 3, 5, ..., (2q-1)$. In this paper,we prove that path union of n non isomorphic copies of complete bipartite graph is odd graceful, when m is even.

Keywords and Phrases: Odd graceful labeling, cycles, complete bipartite graph.

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1. Introduction

A Graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Labeled graphs serve as useful models for a broad range of applications such as: coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management, secret sharing schemes, and models for constraint programming over finite domains.

The study on graph labeling began with the introduction to β-valuation by Rosa[10] in 1967. Golomb [5] called this β-valuation as *graceful labeling* in 1972. A graceful labeling of a graph G with q edges and vertex set V is an injection $f : V(G) \to \{0, 1, 2, ..., q\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label

$|f(u)-f(v)|$. A graph which admits a graceful labeling is called a *graceful graph*. An extensive work has been carried out on graceful labeling for the past five decades. Morgan [9] has shown that all lobsters with perfect matching's are graceful. Ghosh [3] has proved that three classes of lobsters are graceful using adjacency matrices. Mishra and Panigrahi [8] found classes of graceful lobsters of diameter at least five. Sethuraman and Jeba Jesintha [11], [12] have proved that all banana trees and extended banana trees are graceful.

In 1991, Gananajothi [4] introduced another labeling called *odd graceful labeling*. A graph G with q edges is said to be odd graceful if there is an injection f from $V(G) \rightarrow \{0, 1, 2, ..., (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are 1, 3, 5, ..., (2q-1). Gnanajothi [4] proved that the class of odd graceful graphs lies between the class of graphs with α-labeling and the class of bipartite graphs by showing that every graph with an α-labeling has an odd graceful labeling and every graph with an odd cycle is not odd graceful. Gnanajothi [4] also proved that the following graphs are odd graceful: The paths P_n, the cycle C_n if and only if n is even. Combs $P_n \bigodot K_1$ (graphs obtained by joining a single pendant edge to each vertex of P_n), books, crowns $C_n \bigodot K_1$ (graphs obtained by joining a single pendent edge to each vertex of C_n) if and only if n is even, the disjoint union of copies of C_4, the one-point union of copies of C_4, caterpillars, rooted trees of height 2, the graphs obtained from $P_n (n \geq 3)$ by adding exactly two leaves at each vertex of degree 2 of P_n. Ibrahim Moussa [6] proved that the graph $C_m \bigcup P_n$ is odd graceful if m is even. Javid [7] proved that the disjoint union of cycle and H isomorphic copies of paths is odd graceful. Eldergill[1] proved that the one point union of any number of copies of C_6 is odd graceful. For an exhaustive survey on odd graceful labeling, refer to the dynamic survey by Gallian [2]

In this paper, we prove that the path union of n non isomorphic copies of complete bipartite graph is odd graceful, when m is even.

2. Main Result

In this section,we prove that the path union of n non isomorphic copies of complete bipartite graph is odd graceful, when m is even.

Theorem 2.1. *The path union of n non isomorphic copies of complete bipartite graph is odd graceful.*

Proof. Let G be the path union of n non isomorphic copies of complete bipartite graph, Let $|V(G)| = p$ and $|E(G)| = q$. We describe the graph G as follows. The vertices in the complete bipartite graph with a bipartition into two sets of r_a and s_b vertices is isomorphic to K_{r_a,s_b} for $1 \leq a \leq n$, $1 \leq b \leq n$.

Denote the first copy of complete bipartite graph as $K^1_{r_1,s_1}$ having two partite set as X^1 and Y^1 .The vertices in X^1 denoted as $x^1_1, x^1_2, x^1_3, ..., x^1_{r_1}$ and the vertices in Y^1 denoted as $y^1_1, y^1_2, y^1_3, ..., y^1_{s_1}$.

The second copy of complete bipartite graph as $K^2_{r_2,s_2}$ having two partite set as X^2 and Y^2 .The vertices in X^2 denoted as $x^2_1, x^2_2, x^2_3, ..., x^2_{r_2}$ and the vertices in Y^2 denoted as $y^2_1, y^2_2, y^2_3, ..., y^1_{s_2}$.

The third copy of complete bipartite graph as $K^3_{r_3,s_3}$ having two partite set as X^3 and Y^3 .The vertices in X^3 denoted as $x^3_1, x^3_2, x^3_3, ..., x^3_{r_3}$ and the vertices in Y^3 denoted as $y^3_1, y^3_2, y^3_3, ..., y^3_{s_3}$.

In generalk^{th}copy of complete bipartite graph as $K^k_{r_k,s_k}$ having two partite set as X^k and Y^k .The vertices in X^k denoted as $x^k_1, x^k_2, x^k_3, ..., x^k_{r_k}$ and the vertices in Y^k denoted as $y^k_1, y^k_2, y^k_3, ..., y^k_{s_2}$.

The graph G shown in Figure 1. Note that G has $p = \sum_{a=1}^{n}\sum_{b=1}^{n}(r_a + s_b)$ vertices and $q = \sum_{a=1}^{n}\sum_{b=1}^{n}(r_a s_b) + n - 1$ edges.

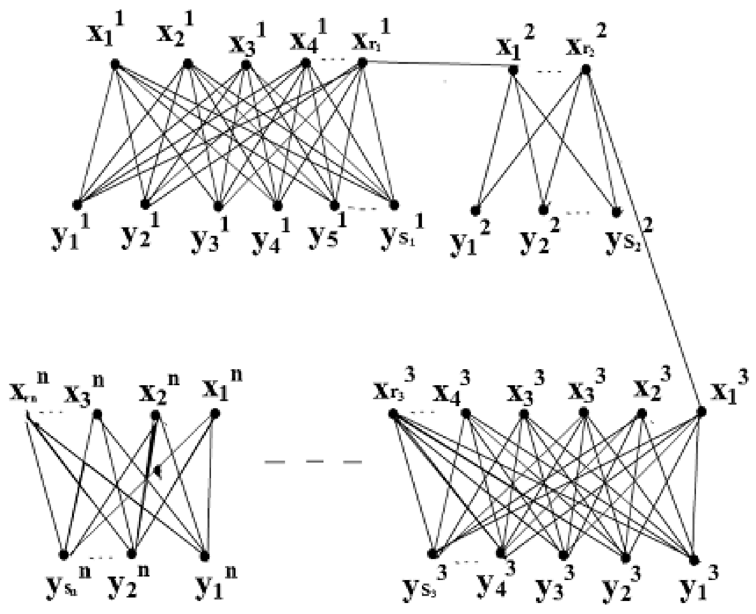

Figure : 1 Path union of n non isomorphic copies complete bipartite graph

The vertices of G are labeled based on the parameters 'r'' and 's'' as follows.

When n is even

The vertex label for n non isomorphic copies of the complete bipartite graph $K_{r,s}$ is computed as follows.

$$f(x_i^1) = 2i - 2 \qquad \text{for} \quad 1 \le i \le r_1$$

$$f(x_i^{2k+1}) \ = 2i - 2 + 2\sum_{\alpha=1}^{k}[s_{2\alpha} + r_{2\alpha-1}] \quad \text{for} \quad 1 \le i \le r_{2k+1}, 1 \le k \le \frac{n-2}{2} \tag{1}$$

$$f(x_i^2) \ = 2q - 1 - 2(r_1(s_1 - 1) + 1) - s_2(2i - 2) \ for 1 \le i \le r_2$$

$$f(x_i^{2k+2}) \ = 2q - 1 - 2(r_1(s_1 - 1) + 1) - s_{(2k+2)}(2i - 2)$$

$$-2\sum_{\alpha=1}^{k}[s_{2\alpha})(r_{2\alpha} - 1) + r_{2\alpha+1}(s_{2\alpha+1}) - 1) + 2]$$

$$\text{for} \ \ 1 \le i \le r_{2k+2}, 1 \le k \le \frac{n-2}{2} \tag{2}$$

$$f(y_j^1) = 2q - 1 + (2j - 2)r_1 \qquad for 1 \le j \le s_1$$

$$f(y_j^{2k+1}) \ = 2q - 1 - r_{2k+1}(2j - 2)$$

$$-2\sum_{\alpha=1}^{k}[r_{2\alpha-1}(s_{2\alpha-1} - 1) + s_{2\alpha}(r_{2\alpha}) - 1) + 2]$$

$$for \ 1 \le j \le s_{2k+1}, 1 \le k \le \frac{n-2}{2} \tag{3}$$

$$f(y_j^2) \ = 2r_1 + (2j - 2) \qquad for \ 1 \le j \le s_2$$

$$f(y_j^{2k+2}) \ = 2r_1 + (2j - 2) + 2\sum_{\alpha=1}^{k}(s_{2\alpha} + r_{2\alpha+1})$$

$$\text{for} \ 1 \le j \le s_{2k+2}, 1 \le k \le \frac{n-2}{2} \tag{4}$$

From equations $(1), (2), (3), (4)$ we observe that the vertices labels are distinct.

The edge label for n non isomorphic copies of the complete bipartite graph $K_{r,s}$ is computed as follows.

$$|f(x_i^1) - f(y_j^1)| = 2q - 1 + (2j-2)r_1 - (2i-2)$$

$$\text{for } \ 1 \leq i \leq r_1, 1 \leq i \leq s_1, 1 \leq j \leq s_2, \tag{5}$$

$$\begin{aligned} |f(x_i^2) - f(y_j^2)| \quad &= 2q - 1 - 2(r_1(s_1-1)+1) + s_2(2i-2) \\ &- [2r_1 + (2j-2)] \quad \text{for } \ 1 \leq i \leq r_1, 1 \leq i \leq s_1, 1 \leq j \leq s_2, \end{aligned} \tag{6}$$

$$\begin{aligned} |f(x_i^{2k+1}) - f(y_j^{2k+1})| \ &= 2i - 2 + 2\sum_{\alpha=1}^{k}(s_{2\alpha}) + r_{2\alpha-1} \\ &- [2q - 1 - r_{2k+1}(2j-2) \\ &- 2\sum_{\alpha=1}^{k}[r_{2\alpha-1}(s_{2\alpha-1}-1) + s_{2\alpha}(r_{2\alpha}-1) + 2] \\ &\text{for } \ 1 \leq i \leq r_{2k+1}, 1 \leq j \leq s_{2k+1}, 1 \leq k \leq \frac{n-2}{2} \end{aligned} \tag{7}$$

$$\begin{aligned} |f(x_i^{2k+2}) - f(y_j^{2k+2})| \quad &= 2q - 1 - 2(r_1(s_1-1)+1) + s_{(2k+2)}(2i-2) - 2 \\ &\sum_{\alpha=1}^{k}[s_{2\alpha}(r_{2\alpha}-1) + r_{2\alpha+1}(s_{2\alpha+1}) - 1) + 2] \\ &- [2r_1 + (2i-2) + 2\sum_{\alpha=1}^{k}(s_{2\alpha} + r_{2\alpha} + 1)] \\ &\text{for } \ 1 \leq i \leq r_{2k+2}, 1 \leq j \leq s_{2k+2}, 1 \leq k \leq \frac{n-2}{2} \end{aligned} \tag{8}$$

From equations $(5), (6), (7), (8)$ we observe that the vertices labels are odd and distinct.

An illustration for the above theorem is given as follows:

ILLUSTRATION : When $r_1 = 5, r_2 = 2, r_3 = 6, s_1 = 6, s_2 = 3, s_3 = 5$
$p = 27, q = 68$ $n = 3$

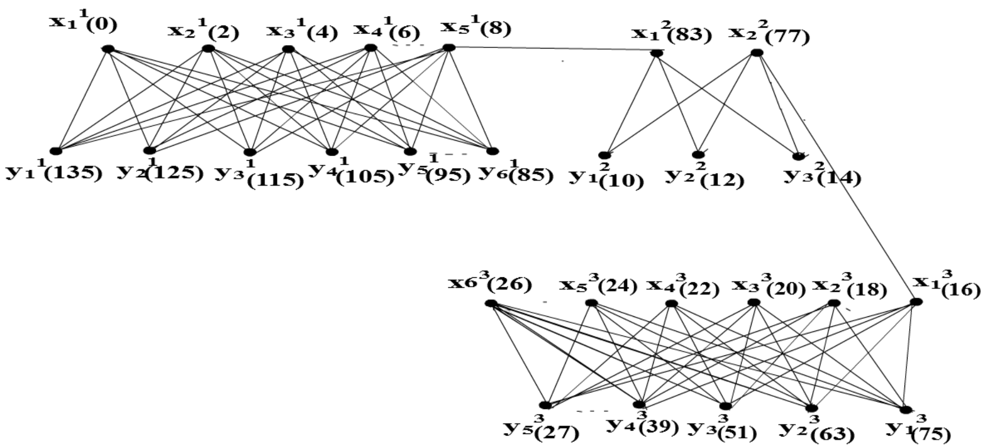

Fig : 2 Path union of 3 non isomorphic copies complete bipartite graph

3. Conclusion:

In this paper, we have proved that the path union of n non isomorphics copies of complete bipartite graph is odd graceful.

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