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EDGE-ODD GRACEFUL LABELING OF UNIFORM n-WHEEL SPLIT GRAPH

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Abstract: In 2009, Solairaju and Chithra introduced edge - odd graceful labeling. A simple graph G with q edges is called an edge odd graceful graph, EOGG, if there is a bijection f from the edge set of the graph to the set $\{1, 3, 5, ...(2q - 1)\}$ such that when each vertex is assigned the sum of all values of the edges incident to it modulo $2q$, the resulting vertex labels are distinct. The graphs related to Paths and cycles, the wheel and complete graphs, the tree and related graphs are edge-odd graceful. The prism of cycle c_n for $(n \geq 3)$,the prism of star graph and prism of wheel graph are edge - odd graceful. In this paper, we define an uniform n - wheel split graph and prove that uniform 3- wheel split graph admit edge - odd graceful labeling.

Keywords and Phrases: Edge-odd graceful graph, uniform n-wheel split graph.

2010 Mathematics Subject Classification: 05C78.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Let G be a simple graph with q edges. Let $V(G)$ and $E(G)$ denote the vertex set and the edge set of G respectively. In 1967, Rosa defined graceful labeling as a function f of a graph G with q edges if f is an injection from the vertices of G to the set {0,1,....q} such that when each edge xy is assigned the label $|f(x) - f(y)|$,the resulting edge labels are distinct. In 1991, Gnanojothi introduced odd graceful labeling such as if there is an injection f from

$V(G)$ to the set {0,1,2,...,(2q-1)} such that when each edge xy is assigned the label $|f(x) - f(y)|$,the resulting edge labels are in the set {1,3,5,....(2q-1)}.

In 2009, Solairaju and Chithra defined edge-odd graceful labeling as a bijection f from $E(G)$ to the set {1,3,5,...(2q-1)} so that the induced mapping f^+ from $V(G)$ to the set {0,1,2,...(2q-1)} given by $f^+(x) = \sum f(xy)(mod2q)$ where the vertex x is adjacent to the vertex y. The edge labels and vertex labels are distinct. A graph that admitted an edge - odd graceful labeling is called an edge - odd graceful graph denoted by EOGG. There are many usages of graph labeling in several areas like data mining, communication networks, image processing, cryptosystems, computer science applications. The main applications of graph labeling are the areas of coding theory, radar, astronomy, circuit design , missile guidance, X-ray crystallography.

2. Main Result

In this section we prove the uniform n - wheel split graph when $n = 3$ and r is odd, admits edge - odd graceful labeling.

Definition 2.1. *The uniform n - wheel split graph is defined as follows: Let u_i, $1 \leq i \leq n$ be the vertices of the complete graph K_n. For $1 \leq i \leq n$, let $W^i_{r+1} = C^i_r + K_1$, where r is a positive integer, be wheels with hubs w_i and let u_i be adjacent to w_i.The graph thus constructed is called an uniform n - wheel split graph and is denoted by K_n W_r. The number of vertices in K_n W_r is $n(2 + r)$, $r \geq 3$, $n \geq 3$.An uniform n - wheel split graph is shown in Figure 2(a).*

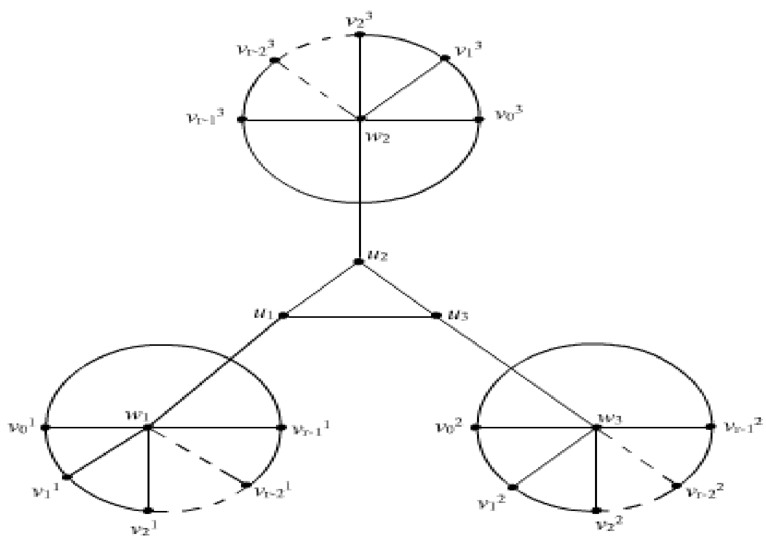

Figure 1: An uniform n - wheel split graph, K_n W_r

Theorem 2.2. *The uniform n - wheel split graph admits edge - odd graceful labeling, when n = 3 and r is odd.*

Proof. Let G denote the uniform n - wheel split graph for $n = 3$. Let $|V(G)| = p$ and $|E(G)| = q$. First we have to prove that the edge labels of G are odd and distinct.
Define $f : E(G) \to \{1, 3, 5, ...(2q-1)\}$ by

$$
\begin{aligned}
f(u_i u_{i+1}) &= 2i - 1 \text{ for } 1 \leq i \leq 2 \\
f(u_n u_1) &= n + 2 \\
f(u_i w_i) &= n - 2i + 10 \text{ for } 1 \leq i \leq 3 \\
f(w_1 v_j^1) &= 13 + 2j \text{ for } 0 \leq j \leq r - 1 \\
f(w_3 v_j^2) &= 2r + 13 + 2j \text{ for } 0 \leq j \leq r - 1 \\
f(w_2 v_j^3) &= 4r + 13 + 2j \text{ for } 0 \leq j \leq r - 1 \\
f(v_0^1 v_{j+2}^1) &= 11r - j + 10 \text{ for } j = 0, 2, \ldots r - 3 \\
f(v_0^2 v_{j+2}^2) &= 9r - j + 10 \text{ for } j = 0, 2, \ldots r - 3 \\
f(v_0^3 v_{j+2}^3) &= 7r - j + 10 \text{ for } j = 0, 2, \ldots r - 3 \\
f(v_j^1 v_{j+1}^1) &= 12r + 11 - 2j \text{ for } 0 \leq j \leq r - 2 \\
f(v_j^2 v_{j+1}^2) &= 10r + 11 - 2j \text{ for } 0 \leq j \leq r - 2 \\
f(v_j^3 v_{j+1}^3) &= 8r + 11 - 2j \text{ for } 0 \leq j \leq r - 2 \qquad (1)
\end{aligned}
$$

From equation (1) it is clear that all the edge labels are odd and distinct.

We now define the vertex labels of G as follows:

$$
\begin{aligned}
f^+(u_1) &= 17(mod2q) \\
f^+(u_2) &= 13(mod2q) \\
f^+(u_3) &= 15(mod2q) \\
f^+(w_i) &= [f(w_i u_i) + f(w_i v_j^i)](mod2q) \\
&\quad 0 \leq j \leq r - 1, 1 \leq i \leq 3 \\
f^+(v_0^i) &= [f(v_0^i v_{j+2}^i) + f(v_0^i v_1^i) + f(w_i v_0^i)](mod2q) \\
&\quad j = 0, 2, \ldots, r - 3, 1 \leq i \leq 3 \\
f^+(v_j^i) &= [f(w_i v_j^i) + f(v_j^i v_{j+1}^i) + f(v_j^i v_{j-1}^i)](mod2q) \\
&\quad j = 1, 2, \ldots r - 2, 1 \leq i \leq 3 \\
f^+(v_j^i) &= [f(v_j^i v_{j-1}^i) + f(v_j^i v_0^i) + f(w_i v_j^i)](mod2q) \\
&\quad j = r - 1, 1 \leq i \leq 3 \qquad (2)
\end{aligned}
$$

It is clear from equation (2) that all the vertex labels $f^+(u_1)$, $f^+(u_2)$, $f(u_3)$, $f^+(w_i)$, $f^+(v_j^i)$, $f^+(v_0^i)$ are distinct. Hence the graph admits edge - odd graceful labeling.

An edge - odd graceful labeling of uniform n - wheel split graph is shown in figure 2(b)

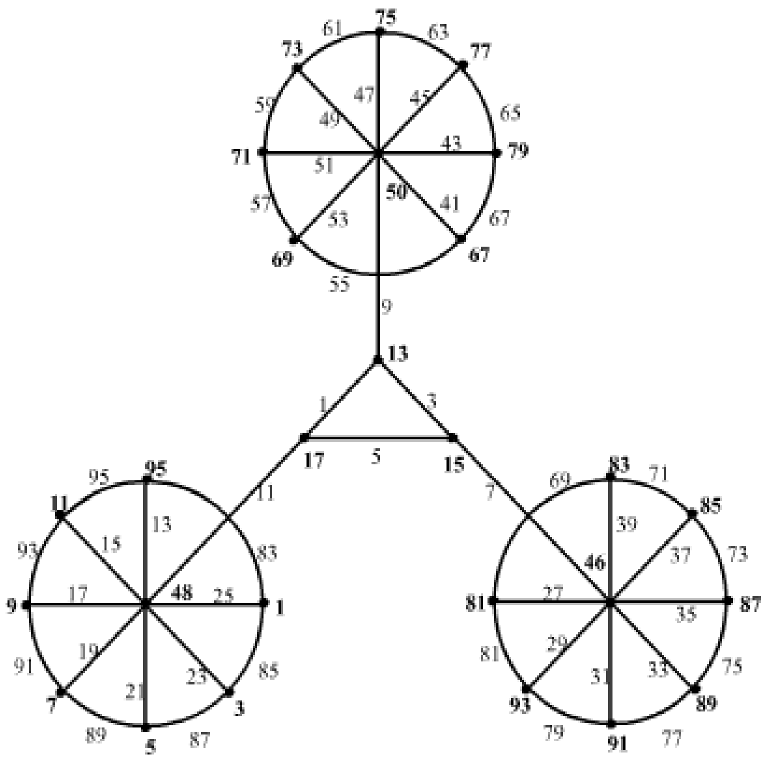

Figure 2: An uniform 3-wheel split graph, $K_3\ W_7$

3. Conclusion

In this paper we have proved Uniform n - wheel split graph, for $n = 3$ admits edge - odd graceful labeling. We further intend to prove cyclic split graph,uniform r - fan split graph for edge - odd gracefulness.

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