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# TOPOLOGY ON INVOLUTIVELY BORDERED WORDS

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**Abstract:** We introduce a topology on Involutively Bordered Words for Morphic and Antimorphic involution on the language $L \subset V^*$ and we have analysed certain topological concepts such as neighbourhood, closure, Hausdorff space, limit point.

**Keywords and Phrases:** Involutively Bordered Words, Topology, Prefix Topology.

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## 1. Introduction

The Main aim of this paper is to introduce Topology on Involutively Bordered Words for Morphic and Antimorphic involution inspired by the work of the authors [7]. DNA, Deoxyribonucleic Acid, is the molecule or group of molecules responsible for encoding all genetic information and instructions in living organisms. The structure of DNA was pieced together by Francis Crick and James Watson [8] based on X-ray images of Rosalind Franklin. DNA Single strand consists of four different types of units called nucleotides or bases strung together by an oriented backbone like beads on wire. The bases are Adenine (A), Guanine (G), Cytosine (C) and Thymine (T), A can chemically bind to an opposing T on another single strand, while C can bind to G. Bases that can bind are called Watson Crick (WK) complementary [4], and two DNA single strands with opposite orientation and with WK complementary bases at each position can bind to each other to form a DNA double strand in a process called base-pairing. DNA strands can be viewed as finite strings over the alphabet A,G,C,T and are used in DNA computing to encode information. These and other biochemical properties of DNA are all harnessed in biocomputing. The concept of DNA computation studied by by Adleman [1] opened a wide area of research. The main objective of the research by Lila Kari and Kalpana Mahalingam is to pursue theoretical properties of bio information by investigating formal language theoretic and combinatorics of words models of DNA-encoded information and DNA computations. In [5] Lila Kari and Kalpana Mahalingam investigates a DNA computing motivated generalization of a classical concept in Combinatorics namely Bordered Words. They extended the notion of bordered and unbordered words by replacing identity function with an arbitrary morphic or antimorphic involution. A Morphic involution function generalizes the identity function on V* and antimorphic involution models the DNA Watson Crick (WK) complementary. The WK complement of a DNA single strand is the reverse complement of original strand. Annal Deva Priya Darshini. C et.al have introduced a general notion of involutively factors of words and obtained several combinatorial properties of such words under morphic and antimorphic involution [2]. Rajkumar Dare. V and Annal Deva Priya Darshini defined $\theta-$ bordered factor of a word, the concept of $\theta-$ valence of a bordered factor and $\theta-$ subword complexity of a word [6].

The paper is organised as follows: Section 2 gives the definition of the topology on Involutively Bordered Words for Morphic and Antimorphic Involution and in Section 3 their Properties.

## 2. Preliminaries and Basic Definitions

A finite non empty set $V$ of symbols is called alphabet. A string is finite sequence of symbols from the alphabet. $V^*$ is the set of finite strings over $V$. Language is a set of strings, $L \subset V$. Concatenation of two strings $w$ and $v$ over the alphabet $V$ is the string obtained by appending the symbol of $v$ to the right end of $w$. Infinite word $u \in V^\infty$ is a mapping $u : \{1, 2, \ldots\} \to V$, $V^\omega$ denotes the set of all such mappings. $V^\infty = V^* \bigcup V^\omega$, where $V^*$ stands for the set of finite words over $V$. A finite word $u \in V^*$ is said to be a prefix of a $w \in V^\infty$ if there is a word $v \in V^\infty$ such that $w = uv$. If $u \neq w$, $u$ is a proper prefix of $w$. If $x \in V^*$ and $y \in V^\infty$, then $x$ is a subword of $y$ iff $y = uxv$ where $u \in V^*$ and $v \in V^\infty$. An involution is a function $\theta$ such that $\theta^2$ equals the identity. An antimorphism $f$ over an alphabet $V$ is a function such that $f(uv) = f(v)f(u)$ for all words $u, v \in V^*$. An morphism $f$ over an alphabet $V$ is a function such that $f(uv) = f(u)f(v)$ for all words $u, v \in V^*$.

We recall the definition of involutively bordered and unbordered words proposed in[5].

**Definition 2.1.** *[5] Let $\theta$ be a morphic or antimorphic involution on $V^*$. A word $u \in V^+$ is said to be $\theta$ - bordered if there exist a $v \in V^+$ such that $u = vx = y\theta(v)$ for some $x, y \in V^+$. A non empty word which is not $\theta$ - bordered is called $\theta$*

*unbordered.*

*We recall some basic definition for Topology. For this we refer to the reader George F. Simmons [3]*

*Let $X$ be a non - empty set. A class $T$ of subsets of $X$ is called a topology on $X$ if it satisfies two conditions*

1. *The union of every class of sets in $T$ is a set in $T$.*

2. *The intersection of every finite class of sets in $T$ is a set in $T$.*

*In other words, a topology on $X$ is a class of subsets of $X$ which is closed under the formation of arbitrary union and finite intersections. The sets in the class $T$ are called the open sets of the topological space $(X, T)$. A class $\{G_i\}$ of open subsets of $X$ is said to be an open cover of $X$, if each point in $X$ belongs to atleast one $\{G_i\}$. A compact space is a Topological space in which every open cover has a finite subcover. Neighbourhood of a point in a topological space is an open set which contains the point. Hausdorff space is a topological space in which each pair of distinct points can be separated by open sets in the sense they have distinct neighbourhoods. A point $x$ in $X$ is said to be a limit point of $A$ if each of its neighbourhood contains a point of $A$ different from $X$.*

*We recall some definition from the 'Subword Topology' by Rajkumar Dare and Rani Sironmoney [7]*

**Definition 2.2.** *[7] Let $u \in V^*$, $L \subset V^\infty$ and $S_u = \{x \in L \text{ suchthat } x = x'ux''$ where $x', x'' \in V^*\}$. In other words the set $S_u$ is the collection of words in the language with $u$ as a subword. The topology generated by $\{S_u : u \in V^*\}$ in $L$ is called Subword Topology.*

**Definition 2.3.** *[7] Let $u \in V^*$,$L \subset V^\infty$ and $P_u = \{x \in L \text{ suchthat } x = ux'$ where $x' \in V^*\}$. Set $P_u$ is the collection of words which has $u$ as its prefix. The topology generated by $\{P_u : u \in V^*\}$ in $L$ is called Prefix Topology.*

**Definition 2.4.** *[7] Let $u \in V^*$, $L \subset V^\infty$ and $SF_u = \{x \in L \text{ suchthat } x = x_1u_1x_2u_2 \ldots x_nu_nx_{n+1}$ where $u_i$ and $x_i \in V^*$ and $u = u_1u_2 \ldots u_n\}$. The topology generated by $\{SF_u : u \in V^*\}$ in $L$ is called Shuffle Topology.*

## 3. Topology on Involutively Bordered Words

In this section we introduce a topology arising from Involutively Bordered words. Since, involution are Morphic and Antimorphic for Involutively Bordered words, we define Topology on Involutively Bordered words for Morphic and Antimorphic involution and state some examples.

**Definition 3.1.** *Let $L \subset V^*$, $u \in V^*$ and $\theta$ be a morphic involution on $V^*$ then*

$$B_u = \{x \in L : u \text{ is a } \theta - \text{border of } x\}$$

*that is, $B_u = \{x \in L : x = ux' = y\theta(u)\}$ for involutively bordered words. The Topology generated by $\tau = \{B_u : u \in V^*\}$ in the language $L$ is called Involutively Bordered Topology for Morphic involution.*

**Example 3.2.** *For $V = a, b$, $u \in V^*$ , $L = \{a^n b^{2n} : n \geq 1\}, \theta : a \to b, b \to a$ . We define a topology generated by $B_u$ in $L$.*

*For $u = \{a^n : n \geq 1\}$, set of words in the language $L$ which have $u$ as a $\theta$- border are $B_u = L - \{a^m b^{2m} : 1 \leq m \leq n-1\}$ For $u \in V^* - \{a^n : n \geq 1\}$ for the language $L$*

$$B_u = \phi.$$

*Thus the collection $\{B_u : u \in V^*\}$ of the language $L$ having $u$ as a $\theta$ - border are*

$$B_u = \left\{L, L - \left\{a^m b^{2m} : 1 \leq m \leq n-1\right\}, \phi\right\}$$

*This collection is a topology. Union of every class of sets in $B_u$ is a set in $B_u$. Intersection of every finite class of sets in a set $B_u$ is a set in $B_u$. Since, arbitrary union and finite intersection are closed in $B_u$ for a language $L$, $\{B_u : u \in V^*\}$ generates a topology in the language $L = \{a^n b^{2n} : n \geq 1\}$. We call this topology as an involutively bordered topology for morphic involution.*

**Definition 3.3.** *Let $L \subset V^*$ and $u \in V^*$, let $\theta$ be a antimorphic involution on $V^*$ then $B_u = \{x \in L : u \; isa \; \theta - \; borderof \; x\}$ that is, $B_u = \{x \in L : x = uy\theta(u)\}$. The topology generated by $\tau = \{B_u : u \in V^*\}$ in Language $L$ is called Involutively Bordered Topology for antimorphic involution.*

**Example 3.4.** *For $V = \{a, b\}$, $L = \{(ba)^n : n \geq 1\}, \theta : a \to b, b \to a$. We define topology generated by $B_u$ in $L$.*

*For $u = (ba)^n$, set of words in the language $L$ which have $u$ as a $\theta$ - border are*

$$B_u = L - \{(ba)^n : 1 \leq m \leq n-1\}$$

*For $u = \{(ba)^n b : n \geq 1\}$, set of words in the language $L$ which have $u$ as a $\theta$ - border are*

$$B_u = L - \{(ba)^n : 1 \leq m \leq n\}$$

*For* $u \in V^* - \{\{b\} \bigcup \{(ba)^n\} \bigcup \{(ba)^n b\} : n \geq 1\}$ *for the language* $L$

$$B_u = \phi$$

*Thus, the collection* $\{B_u : u \in V^*\}$ *of the language* $L$ *having* $u$ *as a* $\theta$*- border are*

$$B_u = \{L, L - \{(ba)^m : m \geq 1\}, \phi\}$$

*This collection is a topology. Union of every class of sets in* $B_u$ *is a set in* $B_u$*. Intersection of every finite class of sets in a set* $B_u$ *is a set in* $B_u$*. Since, arbitrary union and finite intersection are closed in* $B_u$ *the language* $L$*,* $\{B_u : u \in V^*\}$ *generates a topology in language* $L = \{(ba)^n : n \geq 1\}$*. This topology is called as involutively bordered topology for antimorphic involution.*

**Proposition 3.5.** $B_u$ *are subset of* $P_u$ *but* $P_u$ *is not a subset of* $B_u$*.*

**Proof. Case 1**
Let

$$B_u = \{x \in L : u \text{ is a } \theta - \text{ border of } x\} \tag{1}$$

i.e.,

$$B_u = \{x \in L : x = ux' = y\theta(u)\} \tag{2}$$

for involutively bordered words for morphic involution

Involutively bordered words for antimorphic involution,

$$B_u = \{x \in L : x = uy\theta(u)\} \tag{3}$$

$$P_u = \{x \in L \text{ such that } x = ux' \text{ where } x' \in V^*\} \tag{4}$$

If $x \in B_u$ then we show that $x \in P_u$.

From ([2])

$$B_u = x = ux' = y\theta(u) \subseteq P_u = x = ux'$$

Hence,

$$B_u \subseteq P_u$$

**Case 2**
To prove that $P_u$ is not a subset of $B_u$ . We prove this case by contradiction. We assume that $P_u$ is a subset of $B_u$

From ([4]) $x = ux'$ but not all $x = y\theta(u)$

This contradicts our assumption that $x \in B_u$. Hence not all $P_u$ are subset of $B_u$.

**Proposition 3.6.** *Bordered Topology are contained in Prefix Topology but converse is not always true.*

**Proof.** Let, $\tau = \{B_u : u \in V^*\}$ be the topology generated in the language $L$ Since, $B_u$ is subset of $P_u$ from Proposition (3.1). Bordered Topology is contained in Prefix topology.
Converse From the Proposition(3.1) $P_u$ is not subset of $B_u$. Hence it is obvious that Prefix topologies are not contained in Bordered topology.

## 4. Properties of Involutively Bordered Words

**4.1. Neighbourhood.** Neighbourhood of $u \in V^*$ are set of words in the language for which $u$ is a $\theta$- border for morphic and antimorphic involution.

**Example 4.1.** *For $V = \{a, b\}$ and $L = \{a^n b^n : n \geq 1\}, \theta : a \to b, b \to a$ the neighbourhood of aa are $L - \{ab\}$ since all such words are $\theta$- border for morphic involution.*

**4.2. Closure.** In a topological space $X$, the closure of a set $L$ is defined as all neighbourhood of $x$ belonging to set $X$ that intersects with language $L$.

$$\overline{L} = \{x \in X : \text{ every neighbourhood of } x \text{ intersects } L\}$$

**Example 4.2.** *For $V = \{a, b\}$ and $L = \{(ab)^n : n \geq 1\}, \theta : a \to b, b \to a$ Neighbourhood for $v = aba$ is $L - \{ab\}$ for antimorphic involution. Intersecting points between language and neighbourhoods are $L - \{ab\}$.*

$$\text{Hence, Closure } \overline{L} = L - \{ab\}.$$

**4.3. Hausdorff Space.** A language $L \subset V^*$ is Hausdorff iff intersection of neighbourhood of distinct points in an open sets is empty. There are languages which are not Hausdorff.

**Example 4.3.** *For $V = \{a, b\}$ and $L = \{a^n b^n : n \geq 1\}, \theta : a \to b, b \to a$ is not a hausdorff space*

*For $u_1 = a$, neighbourhoods are language $L$ and for $u_2 = aa$, neighbourhood are $L - \{ab\}$ for morphic involution. Intersection of two distinct neighbourhoods is $L - \{ab\}$.*
*Hence, $L = \{a^n b^n : n \geq 1\}$ is not a Hausdorff space.*

**4.4. Limit Point.** A word $x$ is called a limit point of a language $L$ iff every neighbourhood of that point has a word in $L$ other than $x$. In other words Language $L$ has atleast one word for which $x$ is its $\theta$- border for morphic and antimorphic involution then $x$ is a limit point of $L$.

**Example 4.4.** *For* $V = \{a, b\}$ *and* $L = \{a^n b^n : n \geq 1\}, \theta : a \to b, b \to a$ *limit points are* $\{a\}$ *and* $\{aa\}$ *since they form a* $\theta$*- border for morphic involution.*

## 5. Conclusion

In this paper we have introduced a topology on Involutively Bordered Words for Morphic and Antimorphic involution on the language $L \subset V^*$ and we have analysed certain topological concepts such as neighbourhood, closure, hausdorff space and limit point.

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