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CORDIALITY IN THE PATH UNION OF VERTEX SWITCHING OF CYCLES IN INCREASING ORDER

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Abstract: The *Cordial labeling* of a graph G is a function $f : V(G) \rightarrow \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$, where $v_f(i)$ for $i = 0,1$ denote the number of vertices with label i and $e_{f*}(i)$ for $i = 0,1$ denote the number of edges with label i. The graph which admits cordial labeling is called the *Cordial graph*. In this paper, we prove that the path union of vertex switching of cycles in increasing order is cordial.

Keywords and Phrases: Cordial labeling, Path union, Vertex switching.

2010 Mathematics Subject Classification: 05C78.

1. Introduction

Graph labeling methods trace their origin to the graceful labeling introduced by Rosa [7] in 1967. For the past five decades variations in labeling methods have evolved. One such labeling method is the cordial labeling introduced by Cahit [4] in 1987. The *Cordial labeling* of a graph G is a function $f : V(G) \rightarrow \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property that $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$, where $v_f(i)$ for $i = 0,1$ denote the number of vertices with label i and $e_{f*}(i)$ for $i = 0,1$ denote the number of edges with label i. The graph which admits cordial labeling is called the *Cordial graph*. Various graphs are shown to be cordial. Andar et al. [1,2,3] have proved that the helms, closed helms, flowers, gears and sunflower graphs and multiple shells are cordial. Again in [1,2,3] the one point union of helms, flowers, gears, sunflower graphs are shown to be cordial. Cahit [5] has proved that every tree is cordial. In [5] Cahit has shown that all fans are cordial, the wheel W_n when $n \not\equiv 3(mod\ 4)$ is cordial, the complete graph K_n is cordial if and only if $n \leq 3$, the bipartite graph $K_{m,n}$ is cordial for all m and n, the friendship graph $C_3^{(t)}$ is cordial if and only if $t \not\equiv 2(mod\ 4)$. An extensive survey of cordial labeling methods is available in [6] by Gallian.

In this paper, we prove that the path union of vertex switching of cycles in increasing order is cordial.

2. Main Results

In this section we first recall the definition for cycle graph, vertex switching of a graph, path union of graphs. Later we prove that the path union of vertex switching of cycles in increasing order is cordial.

Definition 2.1. *A sequence of vertices* $[v_0, v_1, v_2, ..., v_n, v_0]$ *is a cycle of length* $n+1$ *if* $v_{i+1}v_i \epsilon E$, $i = 0, 1, 2, 3, ..., n$ *and* $v_n v_0 \epsilon E$. *A cycle of length* n *is denoted by* C_n.

Definition 2.2. [8] *A vertex switching* G_v *of a graph* G *is the graph obtained by taking a vertex* v *of* G, *removing all the edges incident to* v *and adding edges joining* v *to every other vertex which are not adjacent to* v *in* G.

Definition 2.3. *The path union of a graph* G *is the graph obtained by adding an edge from* n *copies* $G_1, G_2, ..., G_n, (n \geq 2)$ *of* G *from* G_i *to* G_{i+1} *for* $i = 1, 2, ..., n-1$. *We denote this graph by* $P(n \circ G)$.

Theorem 2.4. *The path union of vertex switching of cycles in increasing order is cordial.*

Proof. Let G be a cycle C_n with n vertices that are denoted as $v_1, v_2, ..., v_n$ in the anticlockwise direction. Let H be the vertex switching graph of the graph G with $v_1 \epsilon G$ as the switching vertex. The graphs G and H are shown in Figure 1.

Case 1: Even cycles ($n_i \equiv 0(mod\ \ 2)$ and $n_i \geq 6$).

Let $H_1, H_2, ..., H_m$ be the copies of H in an increasing order as shown in Figure 2. The first copy H_1 of H is described as follows. Denote the switching vertex of H_1 as v_1^1. Denote the remaining vertices in H_1 as $v_2^1, v_3^1, ..., v_6^1$ in the anticlockwise direction. The second copy H_2 of H is described as follows. Denote the switching vertex of H_2 as v_1^2. Denote the remaining vertices in H_2 as $v_2^2, v_3^2, ..., v_8^2$ in the anticlockwise direction. Finally the last copy H_m of H is described by denoting the switching vertex as v_1^m. The remaining vertices of H_m are denoted as $v_2^m, v_3^m, ..., v_k^m$ where $k = n_i$ for $1 \leq i \leq m$ in the anticlockwise direction.

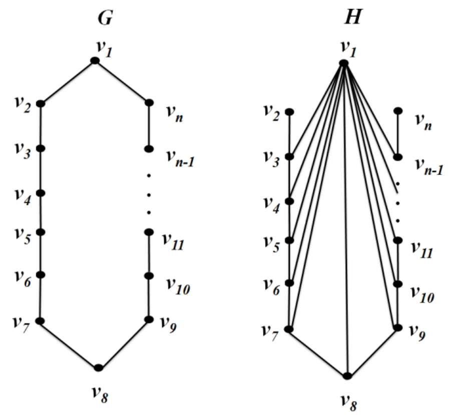

Figure 1: The graphs G and H

Let $H^{'}$ be the graph obtained by adding an edge e_i between the switching vertices v_1^i and v_1^{i+1} of the copies H_i and H_{i+1} ,$1 \leq i \leq (m-1)$. The graph $H^{'}$ so obtained is called the path union of vertex switching of even cycles as shown in the Figure 2.

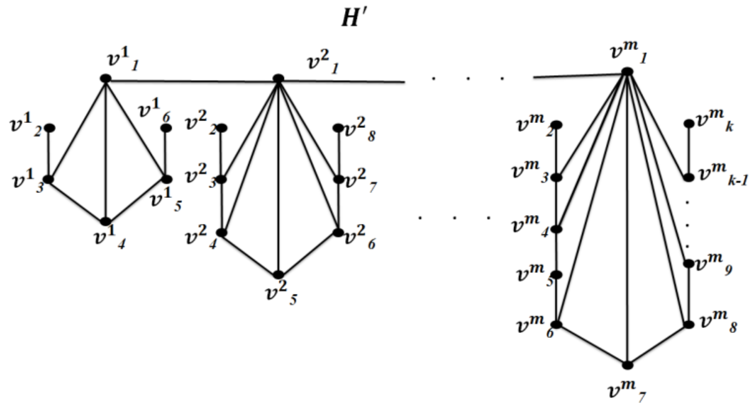

Figure 2: The graph $H^{'}$ which is the path union of the vertex switching of even cycles in increasing order

Note that in $H^{'}$ the switching vertices are v_1^i for $1 \leq i \leq m$ and the remaining vertices are v_j^i for $(1 \leq i \leq m)$,$(2 \leq j \leq n_i)$. If p denotes number of vertices in $H^{'}$ then $p = \sum_{i=1}^{m} n_i$ and if q denotes the number of edges in $H^{'}$ then $q = \sum_{i=1}^{m} n_i + m^2 + (m-1)$.

The vertices of $H^{'}$ are labeled as follows

For $(1 \leq i \leq m)$ where $i \equiv 0, 1(mod\ 4)$

$$f(v_j^i) = \begin{cases} 0, & 1 \leq j \leq n_i \quad j \equiv 0, 1(mod\ 4) \\ 1, & 1 \leq j \leq n_i \quad j \equiv 2, 3(mod\ 4) \end{cases}$$

For $(1 \leq i \leq m)$ where $i \equiv 2, 3(mod\ 4)$

$$f(v_j^i) = \begin{cases} 1, & 1 \leq j \leq n_i \quad j \equiv 0, 1(mod\ 4) \\ 0, & 1 \leq j \leq n_i \quad j \equiv 2, 3(mod\ 4) \end{cases}$$

Case i**:** m is odd
$v_f(0) = \frac{p}{2}$, $v_f(1) = \frac{p}{2}$ and $e_{f*}(0) = \lceil \frac{q}{2} \rceil$, $e_{f*}(1) = \lfloor \frac{q}{2} \rfloor$
$\Longrightarrow |v_f(0) - v_f(1)| = |\frac{p}{2} - \frac{p}{2}| = 0$
$\Longrightarrow |e_{f*}(0) - e_{f*}(1)| = |\lceil \frac{q}{2} \rceil - \lfloor \frac{q}{2} \rfloor| = 1$
Case ii**:** m is even
$v_f(0) = \frac{p}{2}$, $v_f(1) = \frac{p}{2}$ and $e_{f*}(0) = \lfloor \frac{q}{2} \rfloor$,$e_{f*}(1) = \lceil \frac{q}{2} \rceil$
$\Longrightarrow |v_f(0) - v_f(1)| = |\frac{p}{2} - \frac{p}{2}| = 0$
$\Longrightarrow |e_{f*}(0) - e_{f*}(1)| = |\lfloor \frac{q}{2} \rfloor - \lceil \frac{q}{2} \rceil| = 1$
From the above definition it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$
Therefore the graph H' is cordial. We illustrate the above case in Figure 3.
Illustration:
$m = 5, p = 50, q = 79$ $v_f(0) = 25, v_f(1) = 25$
$e_{f*}(0) = 40, e_{f*}(1) = 39$
$\Longrightarrow |v_f(0) - v_f(1)| = |25 - 25| \leq 1$
$\Longrightarrow |e_{f*}(0) - e_{f*}(1)| = |40 - 39| \leq 1$

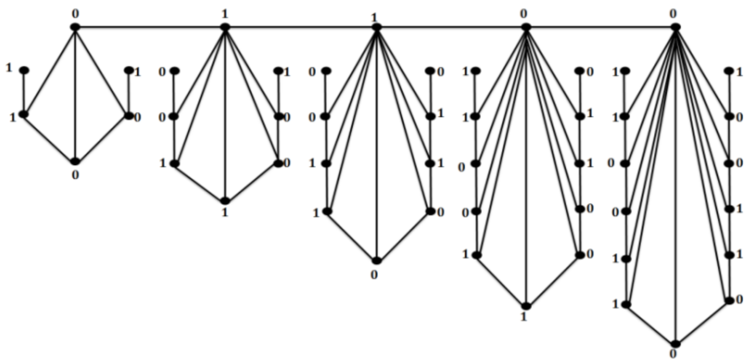

Figure 3: Path union of the vertex switching of even cycles in increasing order is cordial when m=5

Case 2: Odd cycles ($n_i \equiv 1(mod\ \ 2)$ and $n_i \geq 5$)

Let $H_1, H_2, ..., H_m$ be the copies of H in an increasing order as shown in Figure 4. The first copy H_1 of H is described as follows. Denote the switching vertex of H_1 as v_1^1. Denote the remaining vertices in H_1 as $v_2^1, v_3^1, ..., v_5^1$ in the anticlockwise direction. The second copy H_2 of H is described as follows. Denote the switching vertex of H_2 as v_1^2. Denote the remaining vertices in H_2 as $v_2^2, v_3^2, ..., v_7^2$ in the anticlockwise direction. Finally the last copy H_m of H is described by denoting the switching vertex as v_1^m. The remaining vertices of H_m are denoted as $v_2^m, v_3^m, ..., v_k^m$ where $k = n_i$ for $1 \leq i \leq m$ in the anticlockwise direction.

Let H' be the graph obtained by adding an edge e_i between the switching vertices v_1^i and v_1^{i+1} of the copies H_i and H_{i+1}, $1 \leq i \leq (m-1)$. The graph H' so obtained is called the path union of vertex switching of odd cycles as shown in the Figure 4.

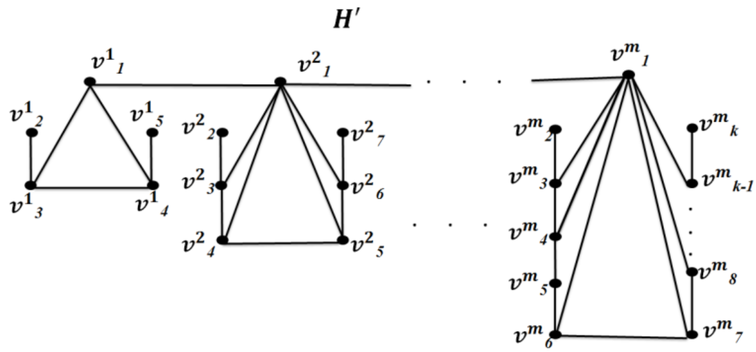

Figure 4: The graph H' which is the path union of the vertex switching of odd cycles in increasing order

Note that in H' the switching vertices are v_1^i for $1 \leq i \leq m$ and the remaining vertices are v_j^i for $(1 \leq i \leq m)$,$(2 \leq j \leq n_i)$. If p denotes number of vertices in H' then $p = \sum_{i=1}^{m} n_i$ and if q denotes the number of edges in H' then $q = \sum_{i=1}^{m} n_i + m^2 - 1$.

The vertices of H' are labeled as follows

$$f(v_1^i) = \begin{cases} 0, & 1 \leq i \leq m \quad i \equiv 0, 1 (mod\ 4) \\ 1, & 1 \leq i \leq m \quad i \equiv 2, 3 (mod\ 4) \end{cases}$$

For $(1 \leq i \leq m)$ where $i \equiv 1 (mod\ 2)$

$$f(v_j^i) = \begin{cases} 1, & 2 \leq j \leq n_i \quad j \equiv 2, 3 (mod\ 4) \\ 0, & 2 \leq j \leq n_i \quad j \equiv 0, 1 (mod\ 4) \end{cases}$$

For $(1 \leq i \leq m)$ where $i \equiv 0 (mod\ 2)$

$$f(v_j^i) = \begin{cases} 0, & 2 \leq j \leq n_i \quad j \equiv 2, 3 (mod\ 4) \\ 1, & 2 \leq j \leq n_i \quad j \equiv 0, 1 (mod\ 4) \end{cases}$$

$$f(v_j^i) = \{\ 1, \quad j = n_i \quad j \equiv 2, 3 (mod\ 4)$$

Case i**:** m is odd
$v_f(0) = \lfloor \frac{p}{2} \rfloor$, $v_f(1) = \lceil \frac{p}{2} \rceil$ and $e_{f*}(0) = \lceil \frac{q}{2} \rceil$, $e_{f*}(1) = \lfloor \frac{q}{2} \rfloor$
$\implies |v_f(0) - v_f(1)| = ||\lfloor \frac{p}{2} \rfloor - \lceil \frac{p}{2} \rceil|| = 1$
$\implies |e_{f*}(0) - e_{f*}(1)| = ||\lceil \frac{q}{2} \rceil - \lfloor \frac{q}{2} \rfloor|| = 1$
Case ii**:** m is even
$v_f(0) = \frac{p}{2}$, $v_f(1) = \frac{p}{2}$ and $e_{f*}(0) = \lfloor \frac{q}{2} \rfloor$,$e_{f*}(1) = \lceil \frac{q}{2} \rceil$
$\implies |v_f(0) - v_f(1)| = |\frac{p}{2} - \frac{p}{2}| = 0$
$\implies |e_{f*}(0) - e_{f*}(1)| = ||\lfloor \frac{q}{2} \rfloor - \lceil \frac{q}{2} \rceil|| = 1$
From the above definition it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$
Therefore the graph H' is cordial. We illustrate the above case in Figure 5.
Illustration:
$m = 6, p = 60, q = 95$, $v_f(0) = 30, v_f(1) = 30$,
$e_{f*}(0) = 48, e_{f*}(1) = 47$
$\implies |v_f(0) - v_f(1)| = |30 - 30| \leq 1$
$\implies |e_{f*}(0) - e_{f*}(1)| = |48 - 47| \leq 1$

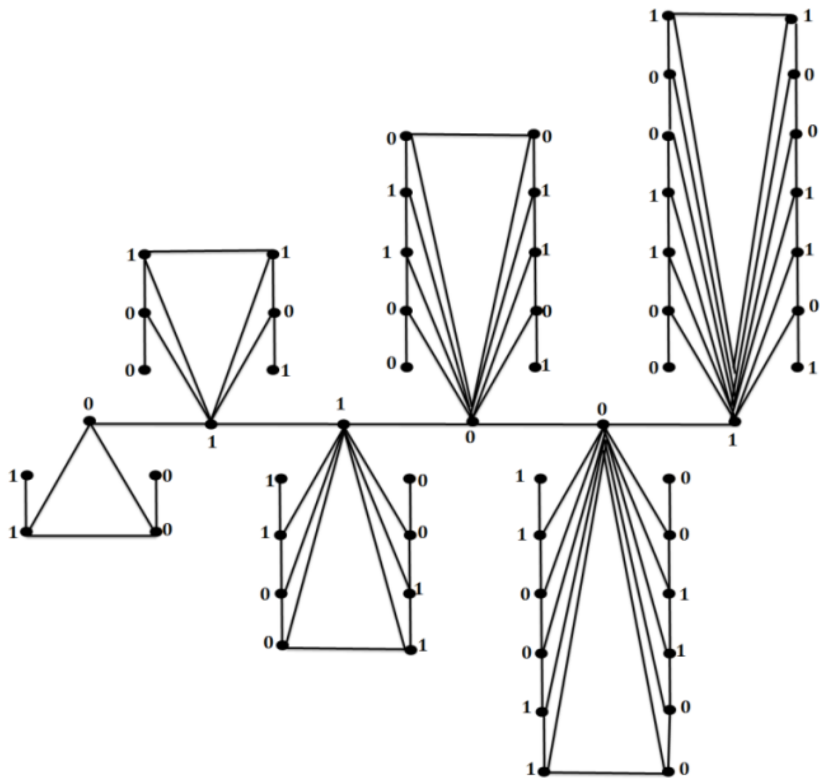

Figure 5: Path union of the vertex switching of odd cycles in increasing order is cordial when m=6

3. Conclusion

In this paper we proved that the path union of vertex switching of odd and even cycles in increasing order is cordial. Further we intend to prove the path union of some other cycle graphs is cordial.

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