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ODD GRACEFUL LABELING ON EXTENDED SUNFLOWER GRAPH

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Abstract: In 1991, Gnanajothi [3] introduced a labeling method called *odd graceful labeling.* A graph G with q edges is said to be odd graceful if there is an injection f from $V(G) \to \{0, 1, 2, ..., (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are $1, 3, 5, ..., (2q-1)$. In this paper,we prove the odd gracefulness on extended sunflower graph.

Keywords and Phrases: Odd graceful labeling, Cycle, Dutch Windmill graph, Revised sunflower graph, extended sunflower graph.

2010 Mathematics Subject Classification: 05C78.

1. Introduction and Definition

The *graph labeling* is one of the important area in graph theory. Graph labeling methods trace their origin to the graceful labeling introduced by Rosa [5] in 1967. A graceful labeling of a graph G with q edges and vertex set V is an injection $f : V(G) \to \{0, 1, 2, ..., q\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label $|f(u) - f(v)|$.

In 1991, Gananajothi [3] introduced *odd graceful labeling.* A graph G with q edges is said to be odd graceful if there is an injection f from $V(G) \to \{0, 1, 2, ..., (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $1, 3, 5, ..., (2q-1)$. Gnanajothi [3] proved that every cycle graph is odd graceful if and only if n is even.She also proved that the graph obtained from $P_n \times P_2$ by deleting an edge that joins to end points of the paths is odd graceful. Govindarajan and Srividya [4] proved odd graceful labeling of every odd cycle $C_n, n \geq 7$ with parallel P_k chords for $k = 2, 4$ after the removal of two edges from the cycle C_n. Badr [1] proved that the revised friendship graphs $F(kC_8), F(kC_{12}), F(kC_{16})$ *and* $F(kC_{20})$ are odd graceful, where k is any positive integer. Joseph Gallian [2] has given a broad and a dynamic survey on various graph labeling methods.

Graph theory plays a vital role in many fields. Labeled graphs serves as useful mathematical models for a broad range of applications such as the design of good radar type codes, synch-set codes, missile guidance codes and radio astronomy problems.

In this paper, we define a new graph called extended Sunflower graph and we prove that the extended Sunflower graph is odd graceful.

Definition 1.1. *The **Dutch windmill graph** is the graph obtained by taking n copies of the cycle C_a with a vertex in common. Dutch windmill graph is also called as Friendship graph and is denoted by* $[D_a^{(n)}]$

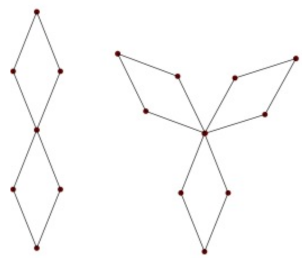

Figure 1 : The Dutch windmill graphs $D_4^{(2)}$ and $D_4^{(3)}$

Definition 1.2. *The **extended sunflower graph** is the graph which is obtained by attaching n isomorphic copies of Dutch Windmill Graphs $(D_4^{(n)})$ at each vertex of a cycle.*

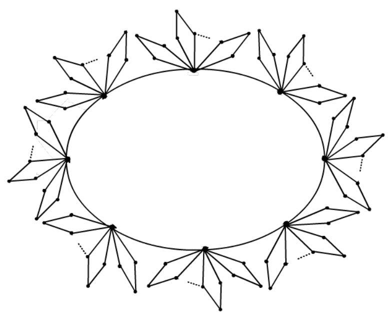

Figure 2 : The Extended sunflower graph

2. Main Result

In this section, we prove that extended sunflower graph which is obtained by attaching n isomorphic copies of Dutch Windmill Graphs $(D_4^{(n)})$ at each vertex of a cycle is odd graceful.

Theorem 2.1. *The extended sunflower graph is odd graceful when* $m \equiv 0$ $(mod 4)$.

Proof. Let G be the graph obtained by attaching m isomorphic copies of Dutch windmill graph $(D_4^{(n)})$ at every vertex of cycle C_m where $m \equiv 0$ $(mod 4)$. Let $|V(G)| = p$ and $|E(G)| = q$. We describe the graph G as follows. The vertices in the cycle C_m in G are denoted as $u_1, u_2, u_3, ..., u_m$ in the clockwise direction. The middle vertices of the first copy of Dutch Windmill graph attached at u_1 are denoted by $v_{11}, v_{12}, v_{13}, ..., v_{1j}$ where $1 \leq j \leq 2n$.The middle vertices of the second copy of Dutch Windmill graph attached at u_2 are denoted by $v_{21}, v_{22}, v_{23}, ..., v_{2j}$ where $1 \leq j \leq 2n$.In general, The middle vertices of the i^{th} copy of Dutch windmill graph attached at vertex u_i will denoted by v_{ij} where $1 \leq i \leq m$ and $1 \leq j \leq 2n$. The topmost vertices of first copy of Dutch Windmill graph are denoted by $w_1^1, w_2^1, w_3^1, ..., w_n^1$.Similarly,The topmost vertices of second copy of Dutch Windmill graph are denoted by $w_1^2, w_2^2, w_3^2, ..., w_n^2$.In general, The topmost vertices of the i^{th} copy of Dutch windmill graph will denoted by w_i^k where $1 \leq i \leq n$ and $1 \leq k \leq m$. The Graph G has $p = (3n + 1)m$ vertices and $q = (4n + 1)m$ edges.

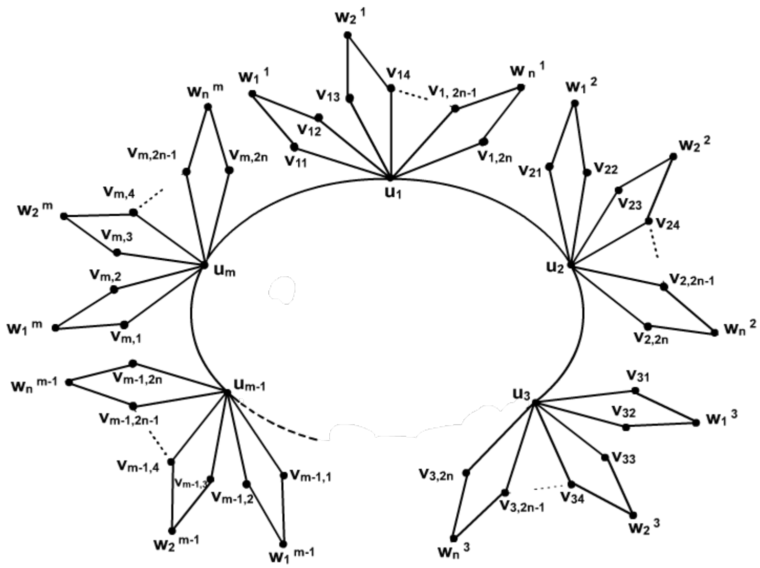

Figure 3 : $m-$ isomorphic copies of Dutch Windmill Graphs attached at each vertex of cycle C_m

The vertices of extended sunflower graph Graph is labeled by first labeling the vertices of a cycle C_m and then labeling the other vertices of Dutch Windmill Graph.

The **vertex labels** for the cycle C_m given below.

$$\begin{aligned} \mathrm{f}(u_{2i-1}) &= (2i-2)(2n+1) && for \quad 1 \le i \le \tfrac{m}{4} \\ \mathrm{f}(u_{2i-1}) &= (2i-2)(2n+1)+2 && for \quad \tfrac{m}{4}+1 \le i \le \tfrac{m}{2} \\ \mathrm{f}(u_{2i}) &= (2q-1-4n)-(i-1)(4n+2) && for \quad 1 \le i \le \tfrac{m}{2} \end{aligned} \tag{2.1}$$

The vertex labels for the Dutch windmill graph is as follows:

$$\begin{aligned} f(v_{2i-1,j}) &= (2q-1)-(4n+2)(i-1)-(2j-2) \\ &\qquad for \quad 1 \le i \le (\tfrac{m}{2}), \quad 1 \le j \le 2n \\ f(v_{2i,j}) &= 2+(4n+2)(i-1)+(2j-2) \\ &\qquad for \quad 1 \le i \le (\tfrac{m}{4}), \quad 1 \le j \le 2n \\ f(v_{2i,j}) &= 4+(4n+2)(i-1)+(2j-2) \\ &\qquad for \quad \tfrac{m}{4}+1 \le i \le (\tfrac{m}{2}), \quad 1 \le j \le 2n \end{aligned} \tag{2.2}$$

$$\begin{aligned} f(w_l^{(2k-1)}) &= (2q-4)-(k-1)(12n+2)-8(l-1) \\ &\qquad for \quad 1 \le l \le n, \quad 1 \le k \le (\tfrac{m}{2}) \\ f(w_l^{(2k)}) &= (4n+5)+(k-1)(12n+2)+8(l-1) \\ &\qquad for \quad 1 \le l \le n, \quad 1 \le k \le (\tfrac{m}{4}) \\ f(w_l^{(2k)}) &= (4n+7)+(k-1)(12n+2)+8(l-1) \\ &\qquad for \quad 1 \le l \le n, \quad (\tfrac{m}{4}+1) \le k \le (\tfrac{m}{2}) \end{aligned} \tag{2.3}$$

From the equations (2.1) to (2.3) we see that the vertex labels for Extended Sunflower Graph are distinct.

Now we compute the edge labels for the Extended Sunflower Graph by first computing the edge labels for cycle C_m for $m \equiv 0\ (mod4)$ as follows:

$$\begin{aligned}
|f(u_{2i-1}) - f(u_{2i})| &= |4i + 4n(2i-1) - 2q - 3|, && for\ 1 \leq i \leq (\tfrac{m}{4}) \\
|f(u_{2i-1}) - f(u_{2i})| &= |4i + 4n(2i-1) - 2q - 1|, && for (\tfrac{m}{4}+1) \leq i \leq (\tfrac{m}{2}) \\
|f(u_{2i+1}) - f(u_{2i})| &= |2q - 4i(2n+1) + 1|, && for\ 1 \leq i \leq (\tfrac{m}{4} - 1) \\
|f(u_{2i+1}) - f(u_{2i})| &= |2q - 4i(2n+1) - 1|, && for (\tfrac{m}{4}) \leq i \leq (\tfrac{m}{2} - 1) \\
|f(u_1) - f(u_m)| &= |2p - m + 1|, &&
\end{aligned} \tag{2.4}$$

The edge labels for the remaining edges of the Dutch Windmill Graph is computed as follows

$$\begin{aligned}
&|f(u_{2i-1}) - f(v_{2i-1,j})| = 4i + 8n(i-1) + 2j - (2q+5), \\
&\qquad for \quad 1 \leq i \leq (\tfrac{m}{4}), \quad 1 \leq j \leq 2n \\
&|f(u_{2i-1}) - f(v_{2i-1,j})| = 4i + 8n(i-1) + 2j - (2q+3), \\
&\qquad for \quad (\tfrac{m}{4}+1) \leq i \leq (\tfrac{m}{2}), \quad 1 \leq j \leq 2n \\
&|f(u_{2i}) - f(v_{2i,j})| = (2q+3) + 4n - 2j - 4i(2n+1), \\
&\qquad for \quad 1 \leq i \leq (\tfrac{m}{4}), \quad 1 \leq j \leq 2n \\
&|f(u_{2i}) - f(v_{2i,j})| = (2q+1) + 4n - 2j - 4i(2n+1), \\
&\qquad for \quad (\tfrac{m}{4}+1) \leq i \leq (\tfrac{m}{2}), \quad 1 \leq j \leq 2n
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
&|f(v_{2i-1,j}) - f(w_l^{(2k-1)})| = 4n(3k - i - 2) + 2(k - i - j) + (8l - 3) \\
&\qquad for \quad 1 \leq i \leq (\tfrac{m}{2}), \quad 1 \leq j \leq 2n, 1 \leq k \leq (\tfrac{m}{2}), 1 \leq l \leq n \\
&|f(v_{2i,j}) - f(w_l^{(2k)})| = 4n(i - 3k + 1) + 2(i + j - k) - (8l - 3), \\
&\qquad for \quad 1 \leq i \leq (\tfrac{m}{2}), \quad 1 \leq j \leq 2n, 1 \leq k \leq (\tfrac{m}{2}), 1 \leq l \leq n
\end{aligned} \tag{2.6}$$

From the above computed edge labels we see that the edge labels for the Extended Sunflower Graph are the distinct odd numbers from the set $1, 3, 5, ..., (2q-1)$.

Hence the Extended Sunflower Graph is odd graceful.

Corollary 2.2. *The revised sunflower graph which is obtained by attaching Dutch Windmill graphs* $[D_4^{(2)}]$ *at every vertex of the cycle* C_m *where* $m = 4l$, $l = 1, 2, ...$ *is odd graceful.*

Proof. From the above theorem, when $n = 2$, we get the odd graceful labeling for Revised Sunflower Graph.

We illustrate the above theorem as follows

3. Illustration When $m = 4, n = 4, p = 52, q = 68$

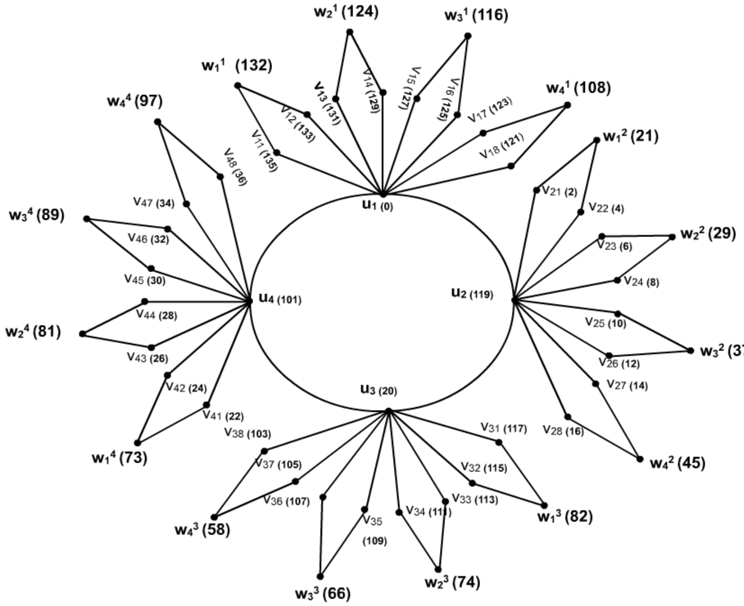

Figure 4 : 4− **isomorphic copies of Dutch Windmill Graphs attached at each vertex of cycle** C_4

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