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SKOLEM DIFFERENCE MEAN LABELING IN DUPLICATE GRAPHS OF SOME PATH RELATED GRAPHS

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Abstract: In this paper, we prove that the extended duplicate graphs of path graph, comb graph and twig graph admit skolem difference mean labeling, skolem odd difference mean labeling and skolem even difference mean labeling.

Keywords and Phrases: Graph labeling, duplicate graph, skolem mean labeling, skolem difference mean labeling, skolem odd difference mean labeling, skolem even difference mean labeling.

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1. Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 2000 papers [1]. The concept of duplicate graph was introduced by Sampath kumar and he proved many results on it [5]. Somasundaram and Ponraj introduced the concept of Mean labeling of graphs and proved the existence of the same in some standard graphs

[6]. Murugan and Subramanian introduced the concept of skolem difference mean labeling and proved some standard results on skolem difference mean labeling [3]. Lau et.al., introduced the concept of skolem odd and skolem even difference mean labeling and proved the existence of the same in some standard graphs [2]. Thirusangu, Ulaganathan and Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [9]. Thirusangu, Ulaganathan and Vijaya kumar have proved that the duplicate graph of Ladder graph L_m, $m \geq 2$, is cordial , total cordial and prime cordial [8]. Vijaya kumar, Ulaganathan and Thirusangu, proved the existence of 3- Equitable and 3 - Cordial Labeling in Duplicate Graph of Some Graphs [9].

2. Skolem Difference Mean Labeling

Definition 2.1. *A graph G(p,q) is said to have skolem difference mean labeling if there exists a function $f : V \to \{1, 2, 3, , p+q\}$ such that the edge labels are induced by the function $f^* : E \to \{1, 2, 3, , q\}$ defined by $f^*\langle uv\rangle = \lceil \frac{f(u)-f(v)}{2} \rceil$, such that the edge labels are distinct [3].*

2.1. Algorithm - SDMLP

V$\leftarrow \{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\}$.
E$\leftarrow \{\{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\}$.
Case (i): when m is odd
Fix $v_1 \leftarrow 3$, $v_2 \leftarrow 1$, $v_1^{'} \leftarrow 2$, $v_3^{'} \leftarrow 2m + 1$;
For $1 \leq k \leq \frac{m-1}{2}$
$v_{2k-1} \leftarrow 2k + 3$, $v_{2k+2} \leftarrow 2k + 2$;
For $1 \leq k \leq \frac{m-3}{2}$
$v_{2k+3}^{'} \leftarrow 2m - 2k + 3$;
For $1 \leq k \leq \frac{m+1}{2}$
$v_{2k}^{'} \leftarrow 4m - 2k + 5$;
Case (ii): when m is even
Fix $v_1 \leftarrow 3$, $v_2 \leftarrow 1$, $v_1^{'} \leftarrow 2$, $v_3^{'} \leftarrow 2m + 1$:
For $1 \leq k \leq \frac{m}{2}$
$v_{2k+1} \leftarrow 2k + 3$, $v_{2k}^{'} \leftarrow 4m - 2k + 5$;
For $1 \leq k \leq \frac{m-2}{2}$
$v_{2k+2} \leftarrow 2k + 2$, $v_{2k+3}^{'} \leftarrow 2m - 2k + 2$:

Theorem 2.2. *The extended duplicate graph of the path graph $EDG(P_m)$, $m \geq 2$ admits skolem difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\}$ and $\{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\}$ be

the set of vertices and the edges of the $EDG\,(P_m)$.
Case (i): when m is odd
Using the algorithm SDMLP, $2m+2$ vertices are labeled using 1, 2, 3,..., $4m+3(=p+q)$. Using the induced function $f^*(uv) = \lceil \frac{f(u)-f(v)}{2})\rceil$, the m edges namely $e_2^{'}$, e_3, $e_4^{'}$, e_5, $e_6^{'}$, e_7,...,$e_{m-3}^{'}$, e_{m-2}, $e_{m-1}^{'}$, e_m receive labels $2m$, $2m-1$, $2m-2$, $2m-3$, $2m-4$, $2m-5$,..., $m+4$, $m+3$, $m+2$, $m+1$ respectively, the $m-1$ edges namely, e_2, $e_3^{'}$, e_4, $e_5^{'}$, e_6, $e_7^{'}$,..., e_{m-3}, $e_{m-2}^{'}$, e_{m-1}, $e_m^{'}$ receive labels m, $m-1$, $m-2$, $m-3$, $m-4$, $m-5$,..., 5, 4, 3, 2 respectively and the 2 edges namely e_1 and $e_1^{'}$ receive labels $2m$ and 1 respectively. Thus the $2m+1(=q)$ edges are labeled with 1, 2, 3,..., $2m+1(=q)$.
Case (ii): when m is even
Using the algorithm SDMLP, $2m+2$ vertices are labeled using 1, 2, 3,..., $4m+3(=p+q)$. Using the induced function f^* defined in case(i), the m edges namely e_1, $e_2^{'}$, e_3, $e_4^{'}$, e_5, $e_6^{'}$,..., e_{m-3}, $e_{m-2}^{'}$, e_{m-1}, $e_m^{'}$ receive labels $2m$, $2m-1$, $2m-2$, $2m-3$, $2m-4$, $2m-5$,..., $m+4$, $m+3$, $m+2$, $m+1$ respectively, the $m-1$ edges namely e_2, $e_3^{'}$, e_4, $e_5^{'}$, e_6, $e_7^{'}$,..., $e_{m-3}^{'}$, e_{m-2}, $e_{m-1}^{'}$, e_m receive labels m, $m-1$, $m-2$, $m-3$, $m-4$, $m-5$,..., 5, 4, 3, 2 respectively and the 2 edges namely e_1 and $e_1^{'}$ receive labels 2m and 1 respectively. Thus the $2m+1(=q)$ edges are labeled with 1, 2, 3,..., $2m+1(=q)$. Hence, the extended duplicate graph of the path graph $EDG(P_m)$, $m \geq 2$ admits skolem difference mean labeling.

2.2. Algorithm - SDMLC

V$\leftarrow \{v_1, v_2, ..., v_{2m}, v_1^{'}, v_2^{'}, ..., v_{2m}^{'}\}$
E$\leftarrow \{e_1, e_2, ..., e_{2m}, e_1^{'}, e_2^{'}, ..., e_{2m-1}^{'}\}$
Case (i): when m is odd
Fix $v_1 \leftarrow 1$, $v_1^{'} \leftarrow 8m-1$;
For $1 \leq k \leq \frac{m+1}{2}$
$v_{4k-2} \leftarrow 4k-1$, $v_{4k-2}^{'} \leftarrow 4m-4k+3$;
For $1 \leq k \leq \frac{m-1}{2}$
$v_{4k-1} \leftarrow 4k+1$, $v_{4k} \leftarrow 4k$, $v_{4k+1} \leftarrow 4k+2$; $v_{4k-1}^{'} \leftarrow 4m-4k+1$, $v_{4k}^{'} \leftarrow 8m-4k+1$, $v_{4k-2}^{'} \leftarrow 8m-4k-1$;
Case (ii): when m is even
Fix $v_1 \leftarrow 1$, $v_1^{'} \leftarrow 8m-1$;
For $1 \leq k \leq \frac{m}{2}$
$v_{4k-2} \leftarrow 4k-1$, $v_{4k-1} \leftarrow 4k+1$, $v_{4k} \leftarrow 4k$;
For $1 \leq k \leq \frac{m-1}{2}$
$v_{4k-2} \leftarrow 4m-4k+3$, $v_{4k-1}^{'} \leftarrow 4m-4k+1$, $v_{4k}^{'} \leftarrow 8m-4k+1$;
For $1 \leq k \leq \frac{m-2}{2}$

$v_{4k+1} \leftarrow 4k+2,\ v'_{4k+1} \leftarrow 8m-4k-1;$

Theorem 2.3. *The extended duplicate graph of the comb graph $EDG(CB_m)$, $m \geq 2$ admits skolem difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m}, v'_1, v'_2, ..., v'_{2m}\}$ and $\{e_1, e_2, ..., e_{2m}, e'_1, e'_2, ..., e'_{2m-1}\}$ be the set of vertices and the edges of $EDG(CB_m)$.
Case (i): when m is odd
Using the algorithm SDMLC, $4m$ vertices are labeled using 1, 2, 3,..., $4m-1(= p+q)$. Using the induced function f^* defined in theorem 2.2, the edge e_{2m} receive label $4m-1$, the $2m-1$ edges namely e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, e_8,..., e'_{2m-5}, e'_{2m-4}, e_{2m-31}, e_{2m-2}, e'_{2m-1} receive labels $4m-2$, $4m-3$, $4m-4$, $4m-5$, $4m-6$, $4m-7$, $4m-8$, $4m-9$,..., $2m+4$, $2m+3$, $2m+2$, $2m+1$, $2m$ respectively and the $2m-1$ edges namely e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8,..., e_{2m-5}, e_{2m-4}, e'_{2m-3}, e'_{2m-2}, e_{2m-1} receive labels $2m-1$, $2m-2$, $2m-3$, $2m-4$, $2m-5$, $2m-6$, $2m-7$, $2m-8$,..., 5, 4, 3, 2, 1 respectively. Thus the $4m-1$ edges are labeled with 1, 2, 3,..., $4m-1$.
Case (ii): when m is even
Using the algorithm SDMLC, $4m$ vertices are labeled using 1, 2, 3,..., $4m-1(= p+q)$. Using the induced function f^* defined in theorem 2.2, the edge e_{2m} receive label $4m-1$, the $2m-1$ edges namely e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, e_8,..., e_{2m-5}, e_{2m-4}, e'_{2m-3}, e'_{2m-2}, e_{2m-1} receive labels $4m-2$, $4m-3$, $4m-4$, $4m-5$, $4m-6$, $4m-7$, $4m-8$, $4m-9$,..., $2m+4$, $2m+3$, $2m+2$, $2m+1$, $2m$ respectively and the $2m-1$ edges namely e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8,..., e'_{2m-5}, e'_{2m-4}, e_{2m-31}, e_{2m-2}, e'_{2m-1} receive labels $2m-1$, $2m-2$, $2m-3$, $2m-4$, $2m-5$, $2m-6$, $2m-7$, $2m-8$,..., 5, 4, 3, 2, 1 respectively. Thus the $4m-1$ edges are labeled with 1, 2, 3,..., $4m-1$. Hence the extended duplicate graph of the comb graph $EDG(CB_m)$, $m \geq 2$ admits skolem difference mean labeling.

2.3. Algorithm - SDMLT

V$\leftarrow \{v_1, v_2, ..., v_{3m+2}, v'_1, v'_2, ..., v'_{3m+2}\}$
E$\leftarrow \{e_1, e_2, ..., e_{3m+2}, e'_1, e'_2, ..., e'_{3m+1}\}$
Case (i): when m is odd
Fix $v_1 \leftarrow 12m+4$, $v_2 \leftarrow 1$, $v \leftarrow 6m+3$, $v_4 \leftarrow 6m+5$, $v_5 \leftarrow 6m+7$, $v'_1 \leftarrow 2$, $v'_2 \leftarrow 12m+7$.
For $1 \leq k \leq \frac{m+1}{2}$
$v'_{6k+3} \leftarrow 12m-6k+11$, $v'_{6k-2} \leftarrow 12m-6k+9$, $v'_{6k-1} \leftarrow 12m-6k+7$;
For $1 \leq k \leq \frac{m-1}{2}$
$v'_{6k} \leftarrow 6k+4$, $v'_{6k+1} \leftarrow 6k+6$, $v'_{6k+2} \leftarrow 6k+8$, $v_{6k} \leftarrow 6k-3$, $v_{6k+1} \leftarrow 6k-1$,

$v_{6k+2} \leftarrow 6k+1$, $v_{6k+3} \leftarrow 6m-6k+12$, $v_{6k+4} \leftarrow 6m-6k+10$, $v_{6k+5} \leftarrow 6m-6k+8$;
Case (ii): when m is even
Fix $v_1 \leftarrow 12m+4$, $v_2 \leftarrow 1$, $v_3 \leftarrow 6m+3$, $v_4 \leftarrow 6m+5$, $v_5 \leftarrow 6m+7$, $v_1^{'} \leftarrow 2$, $v_2^{'} \leftarrow 12m+7$.
For $1 \leq k \leq \frac{m}{2}$
$v_{6k-3}^{'} \leftarrow 12m-6k+11$, $v_{6k-2}^{'} \leftarrow 12m-6k+9$, $v_{6k-1}^{'} \leftarrow 12m-6k+7$, $v_{6k}^{'} \leftarrow 6k+4$, $v_{6k+1}^{'} \leftarrow 6k+6$, $v_{6k+2}^{'} \leftarrow 6k+8$, $v_{6k} \leftarrow 6k-3$, $v_{6k+1} \leftarrow 6k-1$, $v_{6k+2} \leftarrow 6k+1$;
For $1 \leq k \leq \frac{m-2}{2}$
$v_{6k+3} \leftarrow 6m-6k+12$, $v_{6k+4} \leftarrow 6m-6k+10$, $v_{6k+5} \leftarrow 6m-6k+8$;

Theorem 2.4. *The extended duplicate graph of the twig graph $EDG(T_m)$, $m \geq 2$ admits skolem difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{3m+2}, v_1^{'}, v_2^{'}, ..., v_{3m+2}^{'}\}$ and $\{e_1, e_2, ..., e_{3m+2}, e_1^{'}, e_2^{'}, ..., e_{3m+1}^{'}\}$ be the set of vertices and the edges of $EDG(T_m)$.
Case (i): when m is odd
Using the algorithm SDMLT, the $12m+7$ vertices are labeled using 1, 2, 3,..., $12m+7(=p+q)$. Using the induced function f^* defined in theorem 2.2, the two edges e_1, $e_1^{'}$ receive labels 2, 1 respectively, the $3m$ edges namely e_2, e_3, e_4, $e_5^{'}$, $e_6^{'}$, $e_7^{'}$, e_8, e_9, e_{10}, $e_{11}^{'}$, $e_{12}^{'}$, $e_{13}^{'}$,..., $e_{3m-4}^{'}$, $e_{3m-3}^{'}$, $e_{3m-2}^{'}$, e_{3m-1}, e_{3m}, e_{3m+1} receive labels $6m+2$, $6m+1$, $6m$, $6m-1$, $6m-2$, $6m-3$, $6m-4$, $6m-5$, $6m-6$, $6m-7$, $6m-8$, $6m-9$,..., $3m+8$, $3m+7$, $3m+6$, $3m+5$, $3m+4$, $3m+3$ respectively, the $3m$ edges namely $e_2^{'}$, $e_3^{'}$, $e_4^{'}$, e_5, e_6, e_7, $e_8^{'}$, $e_9^{'}$, $e_{10}^{'}$, e_{11}, e_{12}, e_{13},..., e_{3m-4}, e_{3m-3}, e_{3m-2}, $e_{3m-1}^{'}$, $e_{3m}^{'}$, $e_{3m+1}^{'}$ receive labels $3m+2$, $3m+1$, $3m$, $3m-1$, $3m-2$, $3m-3$, $3m-4$, $3m-5$, $3m-6$,..., 8, 7, 6, 5, 4, 3 respectively and the edge e_{3m+2} receive label $6m+3$. Thus the $6m+3$ edges are labeled with 1, 2, 3,..., $6m+3$.
Case (ii): when m is even
Using the algorithm SDMLT, the $12m+7$ vertices are labeled using 1, 2, 3,..., $12m+7(=p+q)$. Using the induced function f^* defined in theorem 2.2, the two edges e_1, $e_1^{'}$ receive labels 2, 1 respectively, the 3m edges namely e_2, e_3, e_4, $e_5^{'}$, $e_6^{'}$, $e_7^{'}$, e_8, e_9, e_{10}, $e_{11}^{'}$, $e_{12}^{'}$, $e_{13}^{'}$,..., e_{3m-4}, e_{3m-3}, e_{3m-2}, $e_{3m-1}^{'}$, $e_{3m}^{'}$, $e_{3m+1}^{'}$ receive labels $6m+2$, $6m+1$, $6m$, $6m-1$, $6m-2$, $6m-3$, $6m-4$, $6m-5$, $6m-6$, $6m-7$, $6m-8$, $6m-9$,..., $3m+8$, $3m+7$, $3m+6$, $3m+5$, $3m+4$, $3m+3$ respectively, the $3m$ edges namely $e_2^{'}$, $e_3^{'}$, $e_4^{'}$, e_5, e_6, e_7, $e_8^{'}$, $e_9^{'}$, $e_{10}^{'}$, e_{11}, e_{12}, e_{13},..., $e_{3m-4}^{'}$, $e_{3m-3}^{'}$, $e_{3m-2}^{'}$, e_{3m-1}, e_{3m}, e_{3m+1}, receive labels $3m+2$, $3m+1$, $3m$, $3m-1$, $3m-2$, $3m-4$, $3m-5$, $3m-6$, $3m-7$,..., 8, 7, 6, 5, 4, 3 respectively and the edge e_{3m+2} receive label $6m+3$. Thus the $6m+3$ edges are labeled with 1, 2, 3,..., $6m+3$. Hence the extended duplicate graph of the twig graph $EDG(T_m)$, $m \geq 2$ admits skolem difference mean labeling.

Illustration:

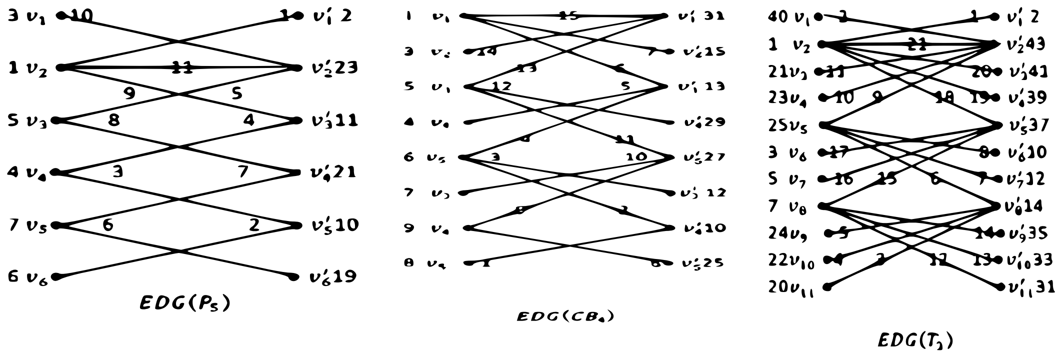

Figure 1. Skolem difference mean labeling in
$EDG\left(P_5\right), EDG\left(CB_4\right)$ *and* $EDG\left(T_3\right)$

3. Skolem Odd Difference Mean Labeling

Definition 3.1. *A graph G(p,q) is skolem odd difference mean graph if there exists an injective function* $f : V \rightarrow \{0, 1, 2, 3, ..., p + 3q - 3\}$ *and an induced function* $f^* : E \rightarrow \{1, 3, 5, , 2q - 1\}$ *such that each edge uv is labeled with* $\lceil \frac{f(u) - f(v)}{2} \rceil$ *[2], [4].*

3.1. Algorithm - SODMLP

V$\leftarrow \{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\}$
E$\leftarrow \{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\}$
Case (i): when m is odd
Fix $v_1 \leftarrow 8m$, $v_3 \leftarrow 8m - 4$, $v_1^{'} \leftarrow 8m - 2$, $v_2^{'} \leftarrow 8m + 2$;
For $1 \leq k \leq \frac{m+1}{2}$
$v_{2k} \leftarrow 4k - 4$;
For $1 \leq k \leq \frac{m-1}{2}$
$v_{2k+1}^{'} \leftarrow 8m - 4k - 2$, $v_{2k+2}^{'} \leftarrow 8m + 4k - 17$;
For $1 \leq k \leq \frac{m-3}{2}$
$v_{2k+3} \leftarrow 8m - 4k - 23$;
Case (ii): when m is even
Fix $v_1 \leftarrow 8m$, $v_1^{'} \leftarrow 8m - 2$, $v_2^{'} \leftarrow 8m + 2$;
For $1 \leq k \leq \frac{m}{2}$
$v_{2k} \leftarrow 4k - 4$, $v_{2k+1}^{'} \leftarrow 8m - 4k - 2$;

For $1 \leq k \leq \frac{m-2}{2}$
$v_{2k+3} \leftarrow 8m-4k-6$, $v'_{2k+2} \leftarrow 8m+4k-17$;

Theorem 3.2. *The extended duplicate graph of the path graph $EDG(P_m)$, $m \geq 2$ admits skolem odd difference mean labeling.*

Proof. Let V$\leftarrow \{v_1, v_2, ..., v_{m+1}, v'_1, v'_2, ..., v'_{m+1}\}$ and $\{e_1, e_2, ..., e_{m+1}, e'_1, e'_2, ..., e'_m\}$ be the set of vertices and the edges of the $EDG(P_m)$.
Case (i): when m is odd
Using the algorithm SODMLP, the $2m+2$ vertices are labeled using 0, 1, 2, 3,..., $8m+2(=p+3q-3)$. Using the induced function $f^*(uv) = \lceil \frac{f(u)-f(v)}{2} \rceil$, the m edges namely, e'_1, e_2, e'_3, e_4, e'_5, e_6, e'_7,..., e_{m-3}, e'_{m-2}, e_{m-1}, e'_m receive labels $4m-1$, $4m-3$, $4m-5$, $4m-7$, $4m-9$, $4m-11$,..., $2m+7$, $2m+5$, $2m+3$, $2m+1$ respectively and the edge e_{2m+1} receives the label $4m+1$. Thus the $2m+1$ edges are labeled with 1, 3, 5,..., $4m+1(=2q-1)$.
Case (ii): when m is even
Using the algorithm SODMLP, the $2m+2$ vertices are labeled using 0, 1, 2, 3,..., $8m+2(=p+3q-3)$. Using the induced function f^* defined in case (i), the m edges namely e_1, e'_2, e_3, e'_4, e_5, e'_6,..., e_{m-3}, e'_{m-2}, e_{m-1}, e'_m receive labels 1, 3, 5, 7, 9, 11,..., $2m-7$, $2m-5$, $2m-3$, $2m-1$ respectively and the m edges namely e'_1, e_2, e'_3, e_4, e'_5, e_6, e'_7,..., e'_{m-3}, e_{m-2}, e'_{m-1}, e_m receive labels $4m-1$, $4m-3$, $4m-5$, $4m-7$, $4m-9$, $4m-11$,..., $2m+7$, $2m+5$, $2m+3$, $2m+1$ respectively and the edge e_{2m+1} receives the label $4m+1$. Thus the $2m+1$ edges are labeled with 1, 3, 5,..., $4m+1(=2q-1)$. Hence the extended duplicate graph of the path graph $EDG(P_m)$, $m \geq 2$ admits skolem odd difference mean labeling.

3.2. Algorithm - SODMLC

V$\leftarrow \{v_1, v_2, ..., v_{2m}, v'_1, v'_2, ..., v'_{2m}\}$
E$\leftarrow \{e_1, e_2, ..., e_{2m}, e'_1, e'_2, ..., e'_{2m-1}\}$
Case (i): when m is odd
Fix $v_1 \leftarrow 0$, $v'_1 \leftarrow 8m-2$
For $1 \leq k \leq \frac{m+1}{2}$
$v'_{4k-2} \leftarrow 16m-8k+2$, $v_{4k-2} \leftarrow 16m-8k$;
For $1 \leq k \leq \frac{m-1}{2}$ $v_{4k-1} \leftarrow 16m-8k-4$, $v_{4k} \leftarrow 8k-4$, $v_{4k+1} \leftarrow 8k$, $v'_{4k-1} \leftarrow 16m-8k-2$, $v'_{4k} \leftarrow 8m+8k-6$, $v'_{4k+1} \leftarrow 8m+8k-2$;
Case (ii): when m is even
Fix $v_1 \leftarrow 0$, $v'_1 \leftarrow 8m-2$

For $1 \leq k \leq \frac{m}{2}$
$v_{4k-2} \leftarrow 16m - 8k$, $v_{4k-1} \leftarrow 16m - 8k - 4$, $v_{4k} \leftarrow 4k - 4$, $v'_{4k-2} \leftarrow 16m - 8k + 2$, $v'_{4k-1} \leftarrow 16m - 8k - 2$, $v'_{4k} \leftarrow 8m + 8k - 6$;
For $1 \leq k \leq \frac{m-2}{2}$
$v_{4k+1} \leftarrow 8k$, $v'_{4k+1} \leftarrow 8m + 8k - 2$;

Theorem 3.3. *The extended duplicate graph of the comb graph $EDG\,(CB_m)$, $m \geq 2$ admits skolem odd difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m}, v'_1, v'_2, ..., v'_{2m}\}$ and $\{e_1, e_2, ..., e_{2m}, e'_1, e'_2, ..., e'_{2m-1}\}$ be the set of vertices and the edges of the $EDG\,(CB_m)$.
Case (i): when m is odd
Using the algorithm SODMLC, the $4m$ vertices are labeled using 0, 1, 2,..., $16m - 6(= p + 3q - 3)$. Using the induced function f^* defined in theorem 3.2, the $2m - 2$ edges namely e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8,..., e_{2m-5}, e_{2m-4}, e'_{2m-3}, e'_{2m-2} receive labels $8m - 3$, $8m - 5$, $8m - 7$, $8m - 9$, $8m - 11$, $8m - 13$, $8m - 15$, $8m - 17$,..., $4m + 9$, $4m + 7$, $4m + 5$, $4m + 3$ respectively, the edges e_{2m-1}, e_{2m} receive labels $4m + 1$, $4m - 1$ and the $2m - 1$ edges namely e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, e_8,..., e'_{2m-5}, e'_{2m-4}, e_{2m-3}, e_{2m-2}, e'_{2m-1} receive labels $4m - 3$, $4m - 5$, $4m - 7$, $4m - 9$, $4m - 11$, $4m - 13$, $4m - 15$, $4m - 17$,..., 9, 7, 5, 3, 1 respectively. Thus the $4m - 1$ edges are labeled with 1, 3, 5, 7,..., $8m - 5$, $8m - 3(= 2q - 1)$.
Case (ii): when m is even
Using the algorithm SODMLC, the $4m$ vertices are labeled using 0, 1, 2,..., $16m - 6(= p + 3q - 3)$. Using the induced function f^* defined in theorem 3.2, the $2m - 2$ edges namely e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8,..., e'_{2m-5}, e'_{2m-4}, e_{2m-3}, e_{2m-2} receive labels $8m - 3$, $8m - 5$, $8m - 7$, $8m - 9$, $8m - 11$, $8m - 13$, $8m - 15$, $8m - 17$,..., $4m + 9$, $4m + 7$, $4m + 5$, $4m + 3$ respectively, the two edges e'_{2m-1}, e_{2m} receive labels $4m + 1$, $4m - 1$ respectively and the $2m - 1$ edges namely namely e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, e_8,..., e_{2m-5}, e_{2m-4}, e'_{2m-3}, e'_{2m-2}, e_{2m-1} receive labels $4m - 3$, $4m - 5$, $4m - 7$, $4m - 9$, $4m - 11$, $4m - 13$, $4m - 15$, $4m - 17$,..., 9, 7, 5, 3, 1 respectively. Thus the $4m - 1$ edges are labeled with 1, 3, 5, 7,..., $8m - 5$, $8m - 3(= 2q - 1)$. Hence, the extended duplicate graph of the comb graph $EDG\,(CB_m)$, $m \geq 2$ admits skolem odd difference mean labeling.

3.3. Algorithm - SODMLT

V$\leftarrow \{v_1, v_2, ..., v_{3m+2}, v'_1, v'_2, ..., v'_{3m+2}\}$
E$\leftarrow \{e_1, e_2, ..., e_{3m+2}, e'_1, e'_2, ..., e'_{3m+1}\}$
Case (i): when m is odd
Fix $v_1 \leftarrow 0$, $v'_1 \leftarrow 14$, $v_1 \leftarrow 24m + 9$, $v'_2 \leftarrow 24m + 10$;

For $1 \leq k \leq \frac{m+1}{2}$
$v_{6k-3} \leftarrow 36k - 16$, $v_{6k-2} \leftarrow 36k - 12$, $v_{6k-1} \leftarrow 36m - 8$, $v'_{6k-3} \leftarrow 12k - 9$, $v'_{6k-2} \leftarrow 12k - 5$, $v'_{6k-1} \leftarrow 12k - 1$;
For $1 \leq k \leq \frac{m-1}{2}$
$v_{6k} \leftarrow 24m - 36k + 13$, $v_{6k+1} \leftarrow 24m - 36k + 9$, $v_{6k+2} \leftarrow 24m - 36k + 5$, $v'_{6k} \leftarrow 24m - 12k + 18$, $v'_{6k+1} \leftarrow 24m - 12k + 14$, $v'_{6k+2} \leftarrow 24m - 12k + 10$;
Case (ii): when m is even
Fix $v_1 \leftarrow 0$, $v'_1 \leftarrow 14$, $v_2 \leftarrow 24m + 9$, $v'_2 \leftarrow 24m + 10$;
For $1 \leq k \leq \frac{m}{2}$
$v_{6k-3} \leftarrow 36k - 1$, $v_{6k-2} \leftarrow 36k - 12$, $v_{6k-1} \leftarrow 36k - 8$, $v_{6k} \leftarrow 24m - 36k + 13$, $v_{6k+1} \leftarrow 24m - 36k + 9$, $v_{6k+2} \leftarrow 24m - 36k + 5$; $v'_{6k-3} \leftarrow 12k - 9$, $v'_{6k-2} \leftarrow 12k - 5$, $v'_{6k-1} \leftarrow 12k - 1$, $v'_{6k} \leftarrow 24m - 12k + 18$, $v'_{6k+1} \leftarrow 24m - 12k + 14$, $v'_{6k+2} \leftarrow 24m - 12k + 10$;

Theorem 3.4. *The extended duplicate graph of the twig graph $EDG(T_m)$, $m \geq 2$ admits skolem odd difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{3m+2}, v'_1, v'_2, ..., v'_{3m+2}\}$ and $\{e_1, e_2, ..., e_{3m+2}, e'_1, e'_2, ..., e'_{3m+1}\}$ be the set of vertices and the edges of the $EDG(T_m)$.
Case (i): when m is odd
Using the algorithm SODMLT, the $6m + 4$ vertices are labeled using 0, 1, 2, 3,..., $24m + 10 (= p + 3q - 3)$. Using the induced function f^* defined in theorem 3.2, the 8 edges namely e_1, e_2, e_3, e_4, e'_1, e'_2, e'_3, e'_4 receive labels $12m + 5$, $12m + 3$, $12m + 1$, $12m - 1$, $12m - 3$, $12m - 5$, $12m - 7$, $12m - 9$ respectively, the $6m - 6$ edges namely e_5, e_6, e_7, e'_5, e'_6, e'_7, e_8, e_9, e_{10}, e'_8, e'_9, e'_{10},..., e_{3m-1}, e_{3m}, e_{3m+1}, e'_{3m-1}, e'_{3m}, e'_{3m+1} receive labels $12m - 11$, $12m - 13$, $12m - 15$, $12m - 17$, $12m - 19$, $12m - 21$, $12m - 23$, $12m - 25$, $12m - 27$, $12m - 29$, $12m - 31$, $12m - 33$,..., 13, 11, 9, 7, 5, 3, respectively and the edge e_{3m+2} receives the label 1. Thus the $6m + 3$ edges are labeled with 1, 3, 5,..., $12m + 5 (= 2q - 1)$.
Case (ii): when m is even
Using the algorithm SODMLT, the $6m + 4$ vertices are labeled using 0, 1, 2, 3,..., $24m + 10 (= p + 3q - 3)$. Using the induced function f^* defined in theorem 3.2, the 8 edges namely e_1, e_2, e_3, e_4, e'_1, e'_2, e'_3, e'_4 receive labels $12m + 5$, $12m + 3$, $12m + 1$, $12m - 1$, $12m - 3$, $12m - 5$, $12m - 7$, $12m - 9$ respectively, the $6m - 6$ edges namely e_5, e_6, e_7, e'_5, e'_6, e'_7, e_8, e_9, e_{10}, e'_8, e'_9, e'_{10},..., e_{3m-1}, e_{3m}, e_{3m+1}, e'_{3m-1}, e'_{3m}, e'_{3m+1} receive labels $12m - 11$, $12m - 13$, $12m - 15$, $12m - 17$, $12m - 19$, $12m - 21$, $12m - 23$, $12m - 25$, $12m - 27$, $12m - 29$, $12m - 31$, $12m - 33$,..., 13, 11, 9, 7, 5, 3, respectively and the edge e_{3m+2} receives the label 1. Thus the $6m + 3$ edges are

labeled with 1, 3, 5,..., $12m + 5(= 2q - 1)$. Hence the extended duplicate graph of the twig graph $EDG(T_m)$, $m \geq 2$ admits skolem odd difference mean labeling.

Illustration:

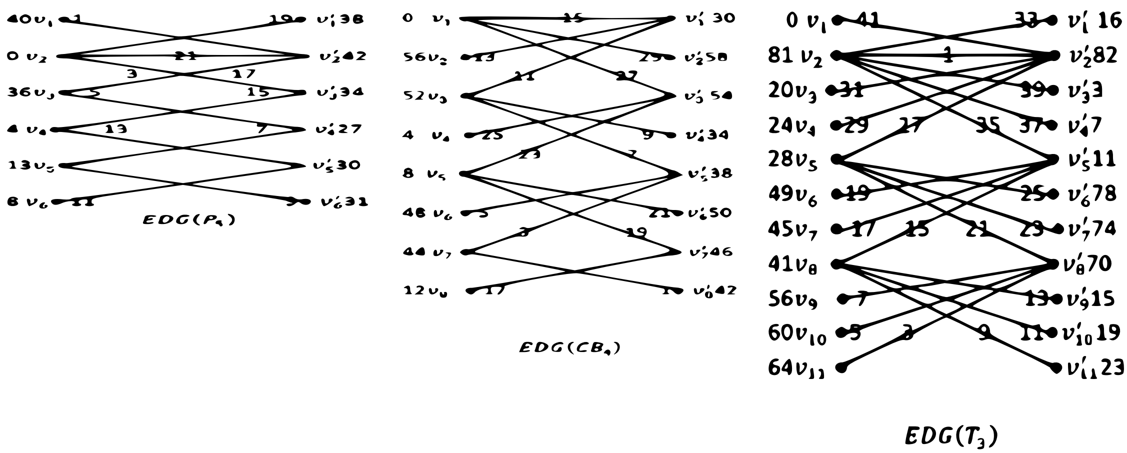

Figure 2. Skolem odd difference mean labeling in $EDG(P_5), EDG(CB_4)$ and $EDG(T_3)$

4. Skolem Even Difference Mean Labeling

Definition 4.1. *A graph G(p,q) is a skolem even difference mean graph if there exists an injective function* $f : V \longrightarrow \{0, 1, 2, 3, ..., p+3q-1\}$ *and an induced function* $f^* : E \longrightarrow \{2, 4, 6, ..., 2q\}$ *such that each edge uv is labeled with* $\lceil \frac{f(u) - f(v)}{2} \rceil$ *[2].*

4.1. Algorithm - SEDMLP

V$\leftarrow \{v_1, v_2, ..., v_{m+1}, v_1', v_2', ..., v_{m+1}'\}$

E$\{e_1, e_2, ..., e_{m+1}, e_1', e_2', ..., e_m'\}$

Case (i): when m is odd

Fix $v_1 \leftarrow 4$, $v_1' \leftarrow 8m + 4$, $v_2 \leftarrow 0$, $v_2' \leftarrow 8m$;

For $1 \leq k \leq \frac{m+1}{2}$

$v_{2k-1} \leftarrow 4k$, $v_{2k}' \leftarrow 8m - 4k - 1$; For $1 \leq k \leq \frac{m-1}{2}$

$v_{2k+2} \leftarrow 4k - 1$, $v_{2k+1}' \leftarrow 4m - 4k - 1$;

Case (ii): when m is even

Fix $v_1 \leftarrow 4$, $v_1' \leftarrow 8m + 4$, $v_2 \leftarrow 0$, $v_2' \leftarrow 8m$;

For $1 \leq k \leq \frac{m}{2}$

$v_{2k+1} \leftarrow 4k+4$, $v'_{2k+1} \leftarrow 4m-4k-1$;
For $1 \leq k \leq \frac{m-2}{2}$
$v_{2k+2} \leftarrow 4k-1$, $v'_{2k+2} \leftarrow 8m-4k$

Theorem 4.2. *The extended duplicate graph of the path graph $EDG\,(P_m)$, $m \geq 2$ admits skolem even difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{m+1}, v'_1, v'_2, ..., v'_{m+1}\}$ and $\{e_1, e_2, ..., e_{m+1}, e'_1, e'_2, ..., e'_m\}$ be the set of vertices and the edges of the $EDG\,(P_m)$.
Case (i): when m is odd
Using the algorithm SEMLP, the $2m+2$ vertices are labeled using 0, 1, 2, 3,..., $8m+4(=p+3q-1)$. Using the induced function $f^*(uv) = \lceil \frac{f(u)-f(v)}{2} \rceil$, the 3 edges namely, e'_1, e_{m+1}, e_1 receive labels $4m+2$, $4m$, $4m-2$ respectively, the $m-1$ edges namely e'_2, e_3, e'_4, e_5, e'_6, e_7,..., e'_{m-3}, e_{m-2}, e'_{m-1}, e_m receive labels $4m-4$, $4m-6$, $4m-8$, $4m-10$, $4m-12$, $4m-14$,..., $2m+6$, $2m+4$, $2m+2$, $2m$ respectively and the $m-1$ edges namely e_2, e'_3, e_4, e'_5, e_6, e'_7,..., e_{m-3}, e'_{m-2}, e_{m-1}, e'_m receive labels $2m-2$, $2m-4$, $2m-6$, $2m-8$, $2m-10$, $2m-12$,..., 8, 6, 4, 2 respectively. Thus the $2m+1$ edges are labeled with 2, 4, 6, 8,... ,$4m+2$.
Case (ii): when m is even
Using the algorithm SEMLP, the $2m+2$ vertices are labeled using 0, 1, 2, 3,..., $8m+4(=p+3q-1)$. Using the induced function f^* defined in case (i), the 3 edges namely e'_1, e_{m+1}, e_1 receive labels $4m+2$, $4m$, $4m-2$ respectively, the $m-1$ edges namely e'_2, e_3, e'_4, e_5, e'_6, e_7,..., e'_{m-2}, e_{m-1}, e'_m receive labels $4m-4$, $4m-6$, $4m-8$, $4m-10$, $4m-12$, $4m-14$,..., $2m+4$, $2m+2$, $2m$ respectively and the $m-1$ edges namely e_2, e'_3, e_4, e'_5, e_6, e'_7,..., e_{m-2}, e'_{m-1}, e_m receive labels $2m-2$, $2m-4$, $2m-6$, $2m-8$, $2m-10$, $2m-12$,..., 6, 4, 2 respectively. Thus the $2m+1$ edges are labeled with 2, 4, 6, 8,..., $4m+2$. Hence, the extended duplicate graph of the path graph $EDG\,(P_m)$, $m \geq 2$ admits skolem even difference mean labeling.

4.2. Algorithm - SEDMLC

V$\leftarrow \{v_1, v_2, ..., v_{2m}, v'_1, v'_2, ..., v'_{2m}\}$
E$\leftarrow \{e_1, e_2, ..., e_{2m}, e'_1, e'_2, ..., e'_{2m-1}\}$
Case(i): when m is odd
Fix $v_1 \leftarrow 0, v'_1 \leftarrow 16m-4$
For $1 \leq k \leq \frac{m+1}{2}$
$v'_{4k-2} \leftarrow 16m-8k, v_{4k-2} \leftarrow 8m+8k-9$;
For $1 \leq k \leq \frac{m-1}{2}$

$v_{4k-1} \leftarrow 8m+8k-5, v_{4k} \leftarrow 4k+4, v_{4k+1} \leftarrow 8k, v'_{4k-1} \leftarrow 16m-8k-4, v'_{4k} \leftarrow 16m+8k-15, v'_{4k+1} \leftarrow 16m+8k-19;$

Case(ii): when m is even

Fix $v_1 \leftarrow 0, v'_1 \leftarrow 16m-4$

For $1 \leq k \leq \frac{m}{2}$

$v'_{4k-2} \leftarrow 16m-8k+4, v'_{4k-1} \leftarrow 16m-8k, v'_{4k} \leftarrow 16m-8k+3, v_{4k-2} \leftarrow 8m+8k-9, v_{4k-1} \leftarrow 8m+8k-5, v_{4k} \leftarrow 8k-4;$

For $1 \leq k \leq \frac{m-2}{2}$

$v_{4k+1} \leftarrow 8k, v'_{4k-2} \leftarrow 16m-8k-1;$

Theorem 4.3. *The extended duplicate graph of the comb graph $EDG(CB_m), m \geq 2$ admits skolem even difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m}, v'_1, v'_2, ..., v'_{2m}\}$ and $\{e_1, e_2, ..., e_{2m}, e'_1, e'_2, ..., e'_{2m-1}\}$ be the set of vertices and the edges of the $EDG(CB_m)$.

Case(i): when m is odd

Using the algorithm SEDMLC, the $4m$ vertices are labeled using 0, 1, 2,..., $16m-4(=p+3q-1)$. Using the induced function f^* defined in theorem 4.2, the $2m-2$ edges namely $e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8, ..., e_{2m-5}, e_{2m-4}, e'_{2m-3}, e'_{2m-2}$ receive labels $8m-4, 8m-6, 8m-8, 8m-10, 8m-12, 8m-14, 8m-16, 8m-18, ..., 4m+8, 4m+6, 4m+4, 4m+2$ respectively, the $2m-2$ edges namely $e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, e_8, ..., e'_{2m-5}, e'_{2m-4}, e_{2m-3}, e_{2m-2}$ receive labels $4m-2, 4m-4, 4m-6, 4m-8, 4m-10, 4m-12, 4m-14, 4m-16, ..., 10, 8, 6, 4$ respectively and the 3 edges namely $e_{2m-1}, e'_{2m-1}, e_{2m}$ receive labels $4m, 2, 8m-2(=2q)$ respectively. Thus the $4m-1(=q)$ edges are labeled with 2, 4 ,6, 8,..., $8m-2$.

Case(ii): when m is even

Using the algorithm SEDMLC, the $4m$ vertices are labeled using 0, 1, 2,..., $16m-4(=p+3q-1)$. Using the induced function f^* defined in theorem 4.2, the $2m-2$ edges namely $e_1, e_2, e'_3, e'_4, e_5, e_6, e'_7, e'_8, ..., e'_{2m-5}, e'_{2m-4}, e_{2m-3}, e_{2m-2}$ receive labels $8m-4, 8m-6, 8m-8, 8m-10, 8m-12, 8m-14, 8m-16, 8m-18, ..., 4m+8, 4m+6, 4m+4, 4m+2$ respectively, the $2m-2$ edges namely e'_1, e'_2, e_3, e_4, e'_5, e'_6, e_7, $e_8, ..., e_{2m-5}$, $e_{2m-4}, e'_1 e'_{2m-3}$, e'_{2m-2}, receive labels $4m-2, 4m-4, 4m-6, 4m-8, 4m-10, 4m-12, 4m-14, 4m-16, ..., 10, 8, 6, 4$ respectively and the 3 edges namely $e_{2m-1}, e'_{2m-1}, e_{2m}$ receive labels $2, 4m, 8m-2(=2q)$ respectively. Thus the $4m-1(=q)$ edges are labeled with 2, 4 ,6, 8,..., $8m-2$. Hence, the extended duplicate graph of the comb graph $EDG(CB_m), m \geq 2$ admits skolem even difference mean labeling.

4.3. Algorithm - SEDMLT

V$\leftarrow \{v_1, v_2, ..., v_{3m+2}, v_1^{'}, v_2^{'}, ..., v_{3m+2}^{'}\}$
E$\leftarrow \{e_1, e_2, ..., e_{3m+2}, e_1^{'}, e_2^{'}, ..., e_{3m+1}^{'}\}$
Case(i): when m is odd
Fix $v_1 \leftarrow 0, v_2 \leftarrow 4, v_1^{'} \leftarrow 12m+8, v_2^{'} \leftarrow 24m+12;$
For $1 \le k \le \dfrac{m+1}{2}$
$v_{6k-3} \leftarrow 12k-4, v_{6k-2} \leftarrow 12k, v_{6k-1} \leftarrow 12k+4, v_{6k-3}^{'} \leftarrow 12m-12k+15, v_{6k-2}^{'} \leftarrow 12m-12k+11, v_{6k-1}^{'} \leftarrow 12m-12k+7;$
For $1 \le k \le \dfrac{m-1}{2}$
$v_{6k} \leftarrow 12k-5, v_{6k+1} \leftarrow 12k-1, v_{6k+2} \leftarrow 12k+3, v_{6k}^{'} \leftarrow 24m-12k+20, v_{6k+1}^{'} \leftarrow 24m-12k+16, v_{6k+2}^{'} \leftarrow 24m-12k+12;$
Case(ii): when$m$ is even
Fix $v_1 \leftarrow 0, v_2 \leftarrow 4, v_1^{'} \leftarrow 12m+8, v_2^{'} \leftarrow 24m+12;$
For $1 \le k \le \dfrac{m}{2}$
$v_{6k-3} \leftarrow 12k-5, v_{6k-2} \leftarrow 12k-1, v_{6k-1} \leftarrow 12k+3, v_{6k} \leftarrow 12k-4, v_{6k+1} \leftarrow 12k, v_{6k+2} \leftarrow 12k+4, v_{6k-3}^{'} \leftarrow 12m-12k+16, v_{6k-2}^{'} \leftarrow 12m-12k+12, v_{6k-1}^{'} \leftarrow 12m-12k+8, v_{6k}^{'} \leftarrow 24m-12k+19, v_{6k+1}^{'} \leftarrow 24m-12k+15, v_{6k+2}^{'} \leftarrow 24m-12k+11;$

Theorem 4.4. *The extended duplicate graph of the twig graph* $EDG(T_m), m \ge 2$ *admits skolem even difference mean labeling.*

Proof. Let $\{v_1, v_2, ..., v_{3m+2}, v_1^{'}, v_2^{'}, ..., v_{3m+2}^{'}\}$ and $\{e_1, e_2, ..., e_{3m+2}, e_1^{'}, e_2^{'}, ..., e_{3m+1}^{'}\}$ be the set of vertices and the edges of the $EDG(T_m)$.
Case(i): when m is odd
Using the Algorithm SEDMLT the $6m+4$ vertices are labeled using 0, 1, 2, 3,..., $24m+10(=p+3q-1)$. Using the induced function f^* defined in theorem 4.2, the $3m$ edges namely e_2, e_3, e_4, $e_5^{'}$, $e_6^{'}$, $e_7^{'}$, e_8, e_9, e_{10}, $e_{11}^{'}$, $e_{12}^{'}$, $e_{13}^{'}$,..., $e_{3m-4}^{'}$, $e_{3m-3}^{'}$, $e_{3m-2}^{'}$, e_{3m-1}, e_{3m}, e_{3m+1} receive labels $6m, 6m-2, 6m-4, 6m-6, 6m-8, 6m-10, 6m-12, 6m-4, 6m-16, 6m-18, 6m-20, 6m-22, ..., 12, 10, 8, 6, 4, 2$ respectively, the $3m$ edges namely $e_2^{'}, e_3^{'}, e_4^{'}, e_5, e_6$, $e_7, e_8^{'}, e_9^{'}$, $e_{10}^{'}, e_{11}, e_{12}$, $e_{13}, ..., e_{3m-4}$, $e_{3m-3}, e_{3m-2}, e_{3m-1}^{'}$, $e_{3m}^{'}$, $e_{3m+1}^{'}$ receive labels $12m+2, 12m, 12m-2, 12m-4, 12m-6, 12m-8, 12m-10, 12m-12, 12m-14, 12m-16, 12m-18, 12m-20, ..., 6m+14, 6m+12, 6m+10, 6m+8, 6m+6, 6m+4$ respectively and the 3 edges namely $e_1, e_1^{'}, e_{3m+2}$ receive labels $12m+6, 6m+2, 12m+4$ respectively. Thus the $2m+1$ edges are labeled with 2, 4, 6,..., $12m+6(=2q)$.
Case(ii): when m is even
Using the Algorithm SEDMLT the $6m+4$ vertices are labeled using 0, 1, 2, 3,..., $24m+10(=p+3q-1)$. Using the induced function f^* defined in theorem 4.2, the

$3m$ edges namely $e_2, e_3, e_4, e'_5, e'_6, e'_7, e_8, e_9, e_{10}, e'_{11}, e'_{12}, e'_{13}, ..., e'_{3m-4}, e'_{3m-3}, e'_{3m-2}, e_{3m-1}, e_{3m}, e_{3m+1}$ receive labels $6m, 6m-2, 6m-4, 6m-6, 6m-8, 6m-12, 6m-14, 6m-16, 6m-18, 6m-20, 6m-22, ..., 12, 10, 8, 6, 4, 2$ respectively, the $3m$ edges namely $6m-16, 6m-18, 6m-20, 6m-22, ..., 12, 10, 8, 6, 4, 2$ respectively, the $3m$ edges namely $e'_2, e'_3, e'_4, e_5, e_6, e_7, e'_8, e'_9, e'_{10}, e_{11}, e_{12}, e_{13}, ..., e_{3m-4}, e_{3m-3}, e_{3m-2}, e'_{3m-1}, e'_{3m}, e'_{3m+1}$ receive labels $12m+2, 12m, 12m-2, 12m-4, 12m-6, 12m-8, 12m-10, 12m-12, 12m-14, 12m-16, 12m-18, 12m-20, ..., 6m+14, 6m+12, 6m+10, 6m+8, 6m+6, 6m+4$ respectively and the 3 edges namely e_1, e'_1, e_{3m+2} receive labels $12m+6, 6m+2, 12m+4$ respectively. Thus the $2m+1$ edges are labeled with 2,4,6,..., $12m+6(=2q)$. Hence, the extended duplicate graph of the twig graph $EDG(T_m), m \geq 2$ admits skolem even difference mean labeling.

Illustration:

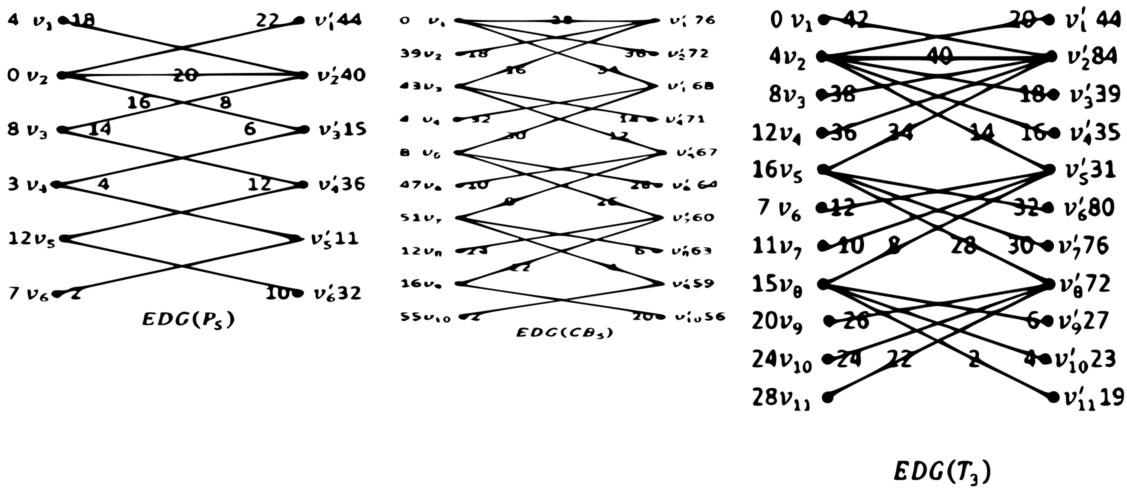

Figure 3. Skolem even difference mean labeling in $EDG(P_5), EDG(CB_4)\ and EDG(T_3)$

5. Conclusion We have proved that extended duplicate graph of path graph $EDG(P_m), m \geq 2$, extended duplicate graph of comb graph $EDG(CB_m), m \geq 2$ and extended duplicate graph of twig graph $EDG(T_m), m \geq 2$ admit skolem difference mean labeling, skolem odd difference mean labeling and skolem even difference mean labeling.

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