*J. of Ramanujan Society of Math. and Math. Sc.* **ISSN : 2319-1023**
*Vol.2, No.1 (2013), pp. 11-16*

# Certain Transformation Formulae for Bilateral Basic Hypergeometric Series

*(Received April 03, 2013)*

Renu Chugh, Prakriti Rai* and Smita Sharma**
Department of Mathematics, Maharishi Dayanand University Rohtak
email:chugh.r1@gmail.com
*Department of Mathematics, Amity University Noida
email:prai@amity.edu
**Department of Mathematics, KIIT Gurgaon, Gurgaon
email:smitasood10@gmail.com

**Abstract:** Bilateral expansions of the results of Satya Prakash Singh, Amit Kumar Singh[1] and Soni Singh[2] have been carried out.
**Keywords:** Bilateral basic hypergeometric series, Basic hypergeometric series.

## 1. Introduction, Notation and Definition:

Throughout this paper we shall adopt the following notations and definitions; For any numbers a and q real or complex and $|q| < 1$, let

$$(a)_n := \begin{cases} 1, & if\, n = 0 \\ (1-a)(1-aq)\left(1-aq^2\right)\ldots\left(1-aq^{n-1}\right), & if\, n \geq 1 \end{cases} \tag{1.1}$$

and

$$(a)_\infty := \prod_{n=0}^{\infty}(1-aq^n) \tag{1.2}$$

The basic hypergeometric series ${}_{r+1}\varphi_r$ and the bilateral basic hypergeometric series ${}_r\psi_r$ are given by

$${}_{r+1}\varphi_r\left[\begin{matrix} a_1, a_2, \ldots, a_{r+1} & ; z \\ b_1, b_2, \ldots, b_r & \end{matrix}\right] := \sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n\ldots(a_{r+1})_n}{(q)_n(b_1)_n(b_2)_n\ldots(b_r)_n}z^n \tag{1.3}$$

and

$${}_{r+1}\psi_r\left[\begin{matrix} a_1, a_2, \ldots, a_r & ; z \\ b_1, b_2, \ldots, b_r & \end{matrix}\right] := \sum_{n=-\infty}^{\infty}\frac{(a_1)_n(a_2)_n\ldots(a_r)_n}{(b_1)_n(b_2)_n\ldots(b_r)_n}z^n \tag{1.4}$$

respectively. The advantage of writing down the bilateral series is to introduce one more parameter and thus to obtain an entire infinite family.

**2. Transformation formulae for Bilateral Basic Hypergeometric Series:**

In this section we have proven the following result which is a bilateral extension of Satya Prakash Singh and Amit Kumar Singh result[1]

$$
{}_{12}\Psi_{12}\left(\frac{\alpha q^M, q^{M+1}\sqrt{\alpha}, -q^{M+1}\sqrt{\alpha}, \beta q^M, \gamma q^M, \frac{\alpha q^M}{\beta\gamma}}{q^{1+M}, \sqrt{\alpha}q^M, -\sqrt{\alpha}q^M, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}, q^{1+M},}\right)
$$

$$
\left(\frac{, aq^M, q^{M+1}\sqrt{a}, -q^{M+1}\sqrt{a}, cq^M, \frac{aq^{N+2M}}{c}, q^{-N}; q; q^2}{\sqrt{a}q^M, -\sqrt{a}q^M, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1}}\right)
$$

$$
={}_{10}\Psi_{10}\left(\frac{aq^M, q^{1+M}\sqrt{a}, -q^{1+M}\sqrt{a}, cq^M, \frac{aq^{N+2M}}{c}, q^{-N}, \alpha q^{1+M}, \beta q^{1+M}, \gamma q^{1+M}, \frac{\alpha q^{1+M}}{\beta\gamma}; q; q}{q^{1+M}, \sqrt{a}q^M, -\sqrt{a}q^M, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1}, q^{1+M}, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}}\right)
$$

$$
\times\frac{(\alpha q;q)_M(\beta q;q)_M(\gamma q;q)_M\left(\frac{\alpha q}{\beta\gamma};q\right)_M(\alpha;q^2)_M}{(\alpha;q)_M(q^2\alpha;q^2)_M\left(\frac{\alpha}{\beta\gamma};q\right)_M(\beta;q)_M(\gamma;q)_M q^M}
$$

$$
+{}_{10}\Psi_{10}\left(\frac{\alpha q^M, q^{1+M}\sqrt{\alpha}, -q^{1+M}\sqrt{\alpha}, \beta q^M, \frac{aq^{N+2M+1}}{c}, q^{-N+1}, \gamma q^M, aq^{1+M}, cq^{1+M}, \frac{\alpha q^M}{\beta\gamma}; q; q}{q^{1+M}, \sqrt{\alpha}q^M, -\sqrt{\alpha}q^M, \frac{\alpha q^{1+M}}{\beta}, cq^{-N+1}, aq^{N+2M+1}, \frac{aq^{1+M}}{c}, q^{1+M}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}}\right)
$$

$$
\times\frac{(cq;q)_M(aq;q)_M(a;q^2)_M\left(\frac{aq^{N+M+1}}{c};q\right)_M\left(q^{-(N+M)+1};q\right)_M}{(a;q)_M(c;q)_M(aq^2;q^2)_M\left(\frac{aq^{N+M}}{c};q\right)_M\left(q^{-(N+M)};q\right)_M q^M} \tag{2.1}
$$

**Proof:**

Satya Prakash Singh and Amit Kumar Singh transformation[1] is given by

$$
{}_{12}\varphi_{11}\left(\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \frac{\alpha}{\beta\gamma}, a, q\sqrt{a}, -q\sqrt{a}, c, \frac{aq^N}{c}, q^{-N}; q; q^2}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, q, \sqrt{a}, -\sqrt{a}, \frac{aq}{c}, cq^{-N+1}, aq^{N+1}}\right)
$$

$$
={}_{10}\varphi_9\left(\frac{a, q\sqrt{a}, -q\sqrt{a}, c, \alpha q, \beta q, \gamma q, \frac{\alpha q}{\beta\gamma}, \frac{aq^N}{c}, q^{-N}; q; q}{\sqrt{a}, -\sqrt{a}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, \frac{aq}{c}, cq^{-N+1}, aq^{N+1}, q}\right)
$$

$$
+{}_{10}\varphi_9\left(\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{a}, \beta, \gamma, \frac{\alpha}{\beta\gamma}, aq, cq, \frac{aq^{N+1}}{c}, q^{-N+1}; q; q}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, \frac{aq}{c}, q, cq^{-N+1}, aq^{N+1}}\right) \tag{2.2}
$$

Consider the above transformation (where $N$ has to be replaced by $N+M$)

$$
{}_{12}\varphi_{11}\left(\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \frac{\alpha}{\beta\gamma}, a, q\sqrt{a}, -q\sqrt{a}, c, \frac{aq^{(N+M)}}{c}, q^{-(N+M)}; q; q^2}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, q, \sqrt{a}, -\sqrt{a}, \frac{aq}{c}, cq^{-(N+M)+1}, aq^{(N+M+1)}}\right)
$$

$$
={}_{10}\varphi_9\left(\frac{a, q\sqrt{a}, -q\sqrt{a}, c, \alpha q, \beta q, \gamma q, \frac{\alpha q}{\beta\gamma}, \frac{aq^{(N+M)}}{c}, q^{-(N+M)}; q; q}{\sqrt{a}, -\sqrt{a}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, \frac{aq}{c}, cq^{-(N+M)+1}, aq^{(N+M)+1}, q}\right)
$$

$$
+{}_{10}\varphi_9\left(\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{a}, \beta, \gamma, \frac{\alpha}{\beta\gamma}, aq, cq, \frac{aq^{(N+M)+1}}{c}, q^{-(N+M)+1}; q; q}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \beta\gamma q, \frac{aq}{c}, q, cq^{-(N+M)+1}, aq^{(N+M)+1}}\right) \tag{2.3}
$$

Putting $r = S + M$ in the above result, then on simplification we get the required result.

$$
{}_{12}\Psi_{12}\left(\frac{\alpha q^M, q^{M+1}\sqrt{\alpha}, -q^{M+1}\sqrt{\alpha}, \beta q^M, \gamma q^M, \frac{\alpha q^M}{\beta\gamma}, aq^M,}{q^{1+M}, \sqrt{\alpha}q^M, -\sqrt{\alpha}q^M, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}, q^{1+M},}\right.
$$

$$
\left.\frac{q^{M+1}\sqrt{a}, -q^{M+1}\sqrt{a}, cq^M, \frac{aq^{N+2M}}{c}, q^{-N}; q; q^2}{\sqrt{a}q^M, -\sqrt{a}q^M, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1}}\right)
$$

$$
={}_{10}\Psi_{10}\left(\frac{aq^M, q^{1+M}\sqrt{a}, -q^{1+M}\sqrt{a}, cq^M, \frac{aq^{N+2M}}{c}, q^{-N}, \alpha q^{1+M}, \beta q^{1+M}, \gamma q^{1+M}, \frac{\alpha q^{1+M}}{\beta\gamma}; q; q}{q^{1+M}, \sqrt{a}q^M, -\sqrt{a}q^M, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1}, q^{1+M}, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}}\right)
$$

$$
\times\frac{(\alpha q;q)_M(\beta q;q)_M(\gamma q;q)_M\left(\frac{\alpha q}{\beta\gamma};q\right)_M(\alpha;q^2)_M}{(\alpha;q)_M(q^2\alpha;q^2)_M\left(\frac{\alpha}{\beta\gamma};q\right)_M(\beta;q)_M(\gamma;q)_M q^M}
$$

$$
+{}_{10}\Psi_{10}\left(\frac{\alpha q^M, q^{1+M}\sqrt{\alpha}, -q^{1+M}\sqrt{\alpha}, \beta q^M, \frac{aq^{N+2M+1}}{c}, q^{-N+1}, \gamma q^M, aq^{1+M}, cq^{1+M}, \frac{\alpha q^M}{\beta\gamma}; q; q}{q^{1+M}, \sqrt{\alpha}q^M, -\sqrt{\alpha}q^M, \frac{\alpha q^{1+M}}{\beta}, cq^{-N+1}, aq^{N+2M+1}, \frac{aq^{1+M}}{c}, q^{1+M}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}}\right)
$$

$$
\times\frac{(cq;q)_M(aq;q)_M(a;q^2)_M\left(\frac{aq^{N+M+1}}{c};q\right)_M\left(q^{-(N+M)+1};q\right)_M}{(a;q)_M(c;q)_M(aq^2;q^2)_M\left(\frac{aq^{N+M}}{c};q\right)_M\left(q^{-(N+M)};q\right)_M q^M} \tag{2.4}
$$

## 3. Application

**Case 1:** When we take $M = 0$ in the above equation we get the transformation (2.2)

**Case 2:** As $c = q$ we get

$$
{}_{11}\Psi_{11}\left(\frac{\alpha q^M, q^{M+1}\sqrt{\alpha}, -q^{M+1}\sqrt{\alpha}, \beta q^M, \gamma q^M, \frac{\alpha q^M}{\beta\gamma}, aq^M, q^{M+1}\sqrt{a}, -q^{M+1}\sqrt{a}, aq^{N+2M-1}, q^{-N}; q; q^2}{\sqrt{\alpha}q^M, -\sqrt{\alpha}q^M, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M}, q^{1+M}, \sqrt{a}q^M, -\sqrt{a}q^M, aq^M, q^{-N+2}, aq^{N+2M+1}}\right)
$$

$$={}_9\Psi_9\left(\begin{matrix} q^{1+M}\sqrt{a}, -q^{1+M}\sqrt{a}, q^{M+1}, aq^{N+2M-1}, q^{-N}, \alpha q^{1+M}, \beta q^{1+M}, \gamma q^{1+M}, \frac{\alpha q^{1+M}}{\beta\gamma}; q; q \\ q^{1+M}, \sqrt{a}q^{M}, -\sqrt{a}q^{M}, q^{-N+2}, aq^{N+2M+1}, q^{1+M}, \frac{\alpha q^{1+M}}{\beta}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M} \end{matrix}\right)$$

$$\times \frac{(\alpha q;q)_M (\beta q;q)_M (\gamma q;q)_M \left(\frac{\alpha q}{\beta\gamma};q\right)_M (\alpha;q^2)_M}{(\alpha;q)_M (q^2\alpha;q^2)_M \left(\frac{\alpha}{\beta\gamma};q\right)_M (\beta;q)_M (\gamma;q)_M q^M}$$

$$+{}_{10}\Psi_{10}\left(\begin{matrix} \alpha q^{M}, q^{1+M}\sqrt{\alpha}, -q^{1+M}\sqrt{\alpha}, \beta q^{M}, aq^{N+2M}, q^{-N+1}, \gamma q^{M}, aq^{1+M}, q^{2+M}, \frac{\alpha q^{M}}{\beta\gamma}; q; q \\ q^{1+M}, \sqrt{\alpha}q^{M}, -\sqrt{\alpha}q^{M}, \frac{\alpha q^{1+M}}{\beta}, q^{-N+2}, aq^{N+2M+1}, aq^{M}, q^{1+M}, \frac{\alpha q^{1+M}}{\gamma}, \beta\gamma q^{1+M} \end{matrix}\right)$$

$$\times \frac{(q^2;q)_M (aq;q)_M (a;q^2)_M (aq^{N+M};q)_M (q^{-(N+M)+1};q)_M}{(a;q)_M (q;q)_M (aq^2;q^2)_M (aq^{N+M-1};q)_M (q^{-(N+M)};q)_M q^M} \tag{3.1}$$

**Case 3:** When $\beta = q$ we get

$${}_{10}\Psi_{10}\left(\begin{matrix} q^{M+1}\sqrt{\alpha}, -q^{M+1}\sqrt{\alpha}, \gamma q^{M}, \frac{\alpha q^{M-1}}{\gamma}, aq^{M}, q^{M+1}\sqrt{a}, -q^{M+1}\sqrt{a}, cq^{M}, \frac{aq^{N+2M}}{c}, q^{-N}; q; q^2 \\ q^{1+M}, \sqrt{\alpha}q^{M}, -\sqrt{\alpha}q^{M}, \frac{\alpha q^{1+M}}{\gamma}, \gamma q^{2+M}, \sqrt{a}q^{M}, -\sqrt{a}q^{M}, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1} \end{matrix}\right)$$

$$={}_{10}\Psi_{10}\left(\begin{matrix} aq^{M}, q^{1+M}\sqrt{a}, -q^{1+M}\sqrt{a}, cq^{M}, \frac{aq^{N+2M}}{c}, q^{-N}, \alpha q^{1+M}, q^{2+M}, \gamma q^{1+M}, \frac{\alpha q^{M}}{\gamma}; q; q \\ q^{1+M}, \sqrt{a}q^{M}, -\sqrt{a}q^{M}, \frac{aq^{1+M}}{c}, cq^{-N+1}, aq^{N+2M+1}, q^{1+M}, \alpha q^{M}, \frac{\alpha q^{1+M}}{\gamma}, \gamma q^{2+M} \end{matrix}\right)$$

$$\times \frac{(\alpha q;q)_M (q^2;q)_M (\gamma q;q)_M \left(\frac{\alpha}{\gamma};q\right)_M (\alpha;q^2)_M}{(\alpha;q)_M (q^2\alpha;q^2)_M \left(\frac{\alpha}{q\gamma};q\right)_M (q;q)_M (\gamma;q)_M q^M}$$

$$+{}_8\Psi_8\left(\begin{matrix} q^{1+M}\sqrt{\alpha}, -q^{1+M}\sqrt{\alpha}, \frac{aq^{N+2M+1}}{c}, q^{-N+1}, \gamma q^{M}, aq^{1+M}, cq^{1+M}, \frac{\alpha q^{M-1}}{\gamma}; q; q \\ q^{1+M}, \sqrt{\alpha}q^{M}, -\sqrt{\alpha}q^{M}, cq^{-N+1}, aq^{N+2M+1}, \frac{aq^{1+M}}{c}, \frac{\alpha q^{1+M}}{\gamma}, \gamma q^{2+M} \end{matrix}\right)$$

$$\times \frac{(cq;q)_M (aq;q)_M (a;q^2)_M \left(\frac{aq^{N+M+1}}{c};q\right)_M (q^{-(N+M)+1};q)_M}{(a;q)_M (c;q)_M (aq^2;q^2)_M \left(\frac{aq^{N+M}}{c};q\right)_M (q^{-(N+M)};q)_M q^M} \tag{3.2}$$

**4. Another transformation formula for Bilateral Basic Hypergeometric Series:** In this section we have proven another result which is a bilateral generalization of Soni Singh result[2] which is as follows:

$${}_8\psi_8\left(\begin{matrix} aq^{M}, q^{M+1}\sqrt{a}, -q^{M+1}\sqrt{a}, bq^{M}, cq^{M}, dq^{M}, eq^{M}, q^{-N}, \\ q^{1+M}, \sqrt{a}q^{M}, -\sqrt{a}q^{M}, \frac{aq^{1+M}}{b}, \frac{aq^{1+M}}{c}, \frac{aq^{1+M}}{d}, \frac{aq^{1+M}}{e}, aq^{N+2M+1} \end{matrix}; q; \frac{a^2q^{N+2+M}}{bcde}\right)$$

$$\times\frac{(a;q)_M\left(q^2a;q^2\right)_M(b;q)_M(c;q)_M\left(\frac{a^2q^{2+N+M}}{bcde}\right)^M}{\left(a;q^2\right)_M\left(aq^{(N+M+1)};q\right)_M}$$

$$=\frac{(aq;q)_M\left(aq^{1+M};q\right)_N\left(\frac{aq}{de};q\right)_M\left(\frac{aq^{1+M}}{de};q\right)_N}{\left(\frac{aq^{1+M}}{d};q\right)_N\left(\frac{aq^{1+M}}{e};q\right)_N}$$

$$\times{}_4\psi_4\left(\begin{matrix}\frac{aq^{1+M}}{bc},dq^M,eq^M,q^{-N};q;q\\ \frac{aq^{1+M}}{b},\frac{aq^{1+M}}{c},\frac{deq^{-N}}{a},q^{1+M}\end{matrix}\right)\tag{4.1}$$

**Proof:** Soni Singh transformation is given by

$$\begin{aligned}&{}_8\varphi_7\left(\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,c,d,e,q^{-n}\\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq/d,aq/e,aq^{n+1}\end{matrix};q;a^2q^{2+n}/bcde\right)\\ &=\frac{(aq,aq/de;q)_n}{(aq/d,aq/e;q)_n}{}_4\varphi_3\left(\begin{matrix}aq/bc,d,e,q^{-n}\\ aq/b,aq/c,deq^{-n}/a\end{matrix};q;q\right)\end{aligned}\tag{4.2}$$

Consider the above transformation (where $N$ has to be replaced by $N+M$)

$$\begin{aligned}&{}_8\varphi_7\left(\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,c,d,e,q^{-(N+M)}\\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq/d,aq/e,aq^{n+1}\end{matrix};q;a^2q^{2+N+M}/bcde\right)\\ &=\frac{(aq,aq/de;q)_{N+M}}{(aq/d,aq/e;q)_{N+M}}{}_4\varphi_3\left(\begin{matrix}aq/bc,d,e,q^{-(N+M)}\\ aq/b,aq/c,deq^{-(N+M)}/a\end{matrix};q;q\right)\end{aligned}\tag{4.3}$$

Putting $r=S+M$ in (4.3), then on simplification we get the required result.

$${}_8\psi_8\left(\frac{aq^M,q^{M+1}\sqrt{a},-q^{M+1}\sqrt{a},bq^M,cq^M,dq^M,eq^M,q^{-N},;q;\frac{a^2q^{N+2+M}}{bcde}}{q^{1+M},\sqrt{a}q^M,-\sqrt{a}q^M,\frac{aq^{1+M}}{b},\frac{aq^{1+M}}{c},\frac{aq^{1+M}}{d},\frac{aq^{1+M}}{e},aq^{N+2M+1}}\right)$$

$$\times\frac{(a;q)_M\left(q^2a;q^2\right)_M(b;q)_M(c;q)_M\left(\frac{a^2q^{2+N+M}}{bcde}\right)^M}{\left(a;q^2\right)_M\left(aq^{(N+M+1)};q\right)\ M}$$

$$=\frac{(aq;q)_M\left(aq^{1+M};q\right)_N\left(\frac{aq}{de};q\right)_M\left(\frac{aq^{1+M}}{de};q\right)_N}{\left(\frac{aq^{1+M}}{d};q\right)_N\left(\frac{aq^{1+M}}{e};q\right)_N}{}_4\psi_4\left(\begin{matrix}\frac{aq^{1+M}}{bc},dq^M,eq^M,q^{-N};q;q\\ \frac{aq^{1+M}}{b},\frac{aq^{1+M}}{c},\frac{deq^{-N}}{a},q^{1+M}\end{matrix}\right)\tag{4.4}$$

## 5. Application

**Case 1:** When $a=q$, transformation (4.4) reduces to

$${}_7\psi_7\left(\frac{q^{M+\frac{3}{2}},-q^{M+\frac{3}{2}},bq^M,cq^M,dq^M,eq^M,q^{-N},}{q^{M+\frac{1}{2}},-q^{M+\frac{1}{2}},\frac{q^{2+M}}{b},\frac{q^{2+M}}{c},\frac{q^{2+M}}{d},\frac{q^{2+M}}{e},q^{N+2M+2}};q;\frac{q^{N+4+M}}{bcde}\right)$$

$$\times \frac{(q;q)_M\left(q^3;q^2\right)_M(b;q)_M(c;q)_M\left(\frac{q^{4+N+M}}{bcde}\right)^M}{(q;q^2)_M\left(q^{(N+M+2)};q\right)_M}$$

$$= \frac{\left(q^2;q\right)_M\left(q^{2+M};q\right)_N\left(\frac{q^2}{de};q\right)_M\left(\frac{q^{2+M}}{de};q\right)_N}{(\frac{q^{2+M}}{d};q)_N(\frac{q^{2+M}}{e};q)_N} {}_4\psi_4\left(\begin{matrix}\frac{q^{2+M}}{bc}, dq^M, eq^M, q^{-N}; q; q\\ \frac{q^{2+M}}{b}, \frac{q^{2+M}}{c}, \frac{deq^{-(N+1)}}{}, q^{1+M}\end{matrix}\right) \quad (5.1)$$

**Case 2:** When $a = de$, (4.4) yields

$${}_8\psi_8\left(\begin{matrix}deq^M, q^{M+1}\sqrt{de}, -q^{M+1}\sqrt{de}, bq^M, cq^M, dq^M, eq^M, q^{-N},\\ q^{1+M}, \sqrt{de}q^M, -\sqrt{de}q^M, \frac{deq^{1+M}}{b}, \frac{deq^{1+M}}{c}, \frac{eq^{1+M}}{d}, \frac{dq^{1+M}}{e}, deq^{N+2M+1}\end{matrix}; q; \frac{deq^{N+2+M}}{bc}\right)$$

$$\times \frac{(de;q)_M\left(q^2de;q^2\right)_M(b;q)_M(c;q)_M\left(\frac{deq^{2+N+M}}{bc}\right)^M}{(de;q^2)_M\left(deq^{(N+M+1)};q\right)_M}$$

$$= \frac{(deq;q)_M\left(deq^{1+M};q\right)_N(q;q)_M\left(q^{1+M};q\right)_N}{(eq^{1+M};q)_N(dq^{1+M};q)_N} {}_3\psi_3\left(\begin{matrix}\frac{deq^{1+M}}{bc}, dq^M, eq^M; q; q\\ \frac{deq^{1+M}}{b}, \frac{ade}{c}, q^{1+M}\end{matrix}\right) \quad (5.2)$$

## References

[1] Satya Prakash Singh and Amit Kumar Singh, On a Transformation formula for elliptic Hypergeometric series, South East Asian J.Math.Sc.,10(2011),79-87.

[2] Soni Singh, A Summation Formulae for Bilateral Basic Hypergeometric Series and Extended Bilateral Bailey Transform, South East Asian J.Math.Sc.,10 (2011),41-51.

[3] Gasper, G and Rahman, M., Basic Hypergeometric Series,(second edition),Cambridge University Press 2004.