South East Asian J. of Math. & Math. Sci. **ISSN : 0972-7752 (Print)**
Vol. 14, No. 3 (2018), pp. 21-30

INVERTIBLE SUBHYPERGROUP NANO TOPOLOGY INDUCED BY CHEMICAL REACTION

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(Received: February 12, 2018)

Abstract: This paper deals with an approximation space and a hypergroup theory. Also we introduce invertible subhypergroup nano topology. Moreover, the notion of a normal and closed subhypergroup gives the characterization of invertible subhypergroup nano topology. Finally, we have shown that copper has same crystalline form but different chemical composition using nano homeomorphic in invertible subhypergroup nano topology.

Keywords and Phrases: Nano topology, Hypergroup, invertible subhypergroup, normal subhypergroup, semihypergroup.

2010 Mathematics Subject Classification: 20N20, 37J05, 54B05.

1. Introduction

Pawlak introduced "rough set theory"[11], a mathematical tool for dealing with vagueness or uncertainity. Since 1982, the theory and applications of rough sets have impressively developed. The algebraic approach to rough sets was studied by some authors, for instance by Bonikowaski[1], Iwinski[6]. Kuroki[7] considered the rough ideal in a semigroup, Kuroki and Wang[8] studied the lower and upper approximations with respect to normal subgroups, Davvaz[5] introduced rough subrings and rough ideals, with respect to an ideal of a ring.
On the other hand, algebraic hyperstructures, particularly hypergroups, were introduced by Marty[10] in 1934. Since then, algebraic hyperstructures have been intensively studied, both from the theoretical point of view and especially for their applications in other fields such as nonEuclidean geometries, graphs and hypergraphs, fuzzy sets, automata, cryptography, artificial intelligence, codes, probabilities,lattices and so on. An interesting book dedicated to the applications of

hyperstructures is [3], written by Corsini-Leoreanu. Nano topology is studied by Lellis Thivagar and Carmel Richard[9].
In this paper, we present a general framework for the study of approximations in invertible subhypergroups nano topology. We also consider rough approximations of a closed and normal subhypergroup to get nano topology. Finally, we apply the above approach, by introducing redox reaction and analying indiscrete nanotopology.

2. Preliminaries

Now we would like to present some basic notations and results about hypergroups which will be necessary in the following paragraphs.

Definition 2.1. *[2]: Let H be a nonempty set. A hyperoperation on H is a map $\circ : H \times H \longrightarrow P^*(H)$, where $P^*(H)$ is the set of all nonempty subsets of H. For any nonnempty subsets A,B of H we denote by $AB = \cup_{a\in A, b\in B} ab$. The couple $(H, \circ)$ is called a hypergroupoid.*

Definition 2.2. *[2]: $(H, \circ)$ is said to be hypergroup if for all x,y,z of H, we have $(xy)z = x(yz)$ and $Hx = xH = H$. Let $K \subset H$, $K \neq \emptyset$.*

Definition 2.3. *[2]: $(K, \circ)$ is said to be subhypergroup of $(H, \circ)$ if for any $a \in K$, we have $Ka = aK = K$. A subhypergroup K of H is*

(i) closed if for all a,b of K and x,y of H such that $a \in bx$ and $a \in yb$, it follows that x,y belong to K.

(ii) left invertible if for all x,y of H such that $x \in Ky$,it follows that $y \in Kx$. We say that K is invertible if it is left and right invertible.

(iii) normal if for any $x \in H$, we have $xK = Kx$.

Proposition 2.4. *[2]: (i) K is right invertible in H if and only if the following implication is valid: $b \in Ka \implies a \in Kb$ for all a,$b \in H$.*
(ii) K is left invertible in H if and only if the following implication is valid: $b \in aK \implies a \in bK$ for all a,$b \in H$.

Proposition 2.5. *[2]: (i) K is right invertible in H if and only if the following implication is valid : $Ka \neq Kb \implies Ka \cap Kb = \emptyset$ for all a,$b \in H$.*
(ii) K is left invertible in H if and only if the following implication is valid: $aK \neq bK \implies aK \cap bK = \emptyset$ for all a,$b \in H$.

Definition 2.6. *[2]: A nonempty subset A of H is called a complete part of H if for any $n \in N^*$ and any $x_1, x_2, ...x_n$ of H such that $A \cap \Pi_{i=1}^n x_i \neq \emptyset$, it follows that $\Pi_{i=1}^n x_i \subseteq A$.*

Definition 2.7. *[2]: If A is a nonempty subset of H, then the complete closure $\mathcal{C}(A)$ of A is the intersection of all complete parts of H, which contains A.*

Remark 2.8. *[2]: If K is an invertible subhypergroup of H, then the following relation: $x\ R_K y$ if and only if $x \in yK$ is an equivalence relation. The symmetry and transitivity are immediate. Moreover, for any $x \in H$ and $k \in K$, there exist $y \in H$, such that $x \in yk \subset yK$. Since $y \in xK$, we obtain $x \in xK$, that is if the reflexivity holds too. Hence, if K is an invertible subhypergroup, then $\{xK\}_{x\in H}$ is a partition of H such that for any $x \in H$, we have $x \in xK$.*

Definition 2.9. *[9]: Let $\mathcal{U}$ be a nonempty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair ($\mathcal{U}$,R) is said to be the approximation space. Let $X \subseteq \mathcal{U}$.*

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in\mathcal{U}}\{x : R(x) \subseteq X\}$,where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \bigcup_{x\in\mathcal{U}}\{x : R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects,which can be classified neither as X nor as X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X)$- $L_R(X)$.

Definition 2.10. *[9]: Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U},\emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:*

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X.We call ($\mathcal{U}$,$\tau_R(X)$) as the nano topological space.The elements of $\tau_R(X)$ are called nano-open sets.

Proposition 2.11. *[9]: Let $\mathcal{U}$ be a nonempty finite universe and $X \subset \mathcal{U}$, $\mathcal{U}/R$ be an indiscernibility relation on $\mathcal{U}$ then*

(i) ***Nano Type-1***$(\mathcal{NT}_1)$*: If $L_R(X) = U_R(X) = X$, then the nano topology, $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X)\}$.*

(ii) ***Nano Type-2***$(\mathcal{NT}_2)$*: If $L_R(X) = \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, U_R(X)\}$.*

(iii) ***Nano Type-3***$(\mathcal{NT}_3)$*: If $L_R(X) \neq \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), B_R(X)\}$.*

(iv) ***Nano Type-4***$(\mathcal{NT}_4)$*: If $L_R(X) = \emptyset$ and $U_R(X) = \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset\}$, is the indiscrete nano topology on U.*

(v) ***Nano Type-5***$(\mathcal{NT}_5)$*: If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$.*

3. Invertible subhypergroup Nano Topology

In this section we introduce the invertible subhypergroup nano topology and also the characterizations are given.

Definition 3.1. *: Let $(H, \circ)$ be a hypergroup, S be an invertible subhypergroup of H. For all $x \in H$, xS induces an equivalence relation on H and $A \subseteq H$, where $L_S(A)$, $U_S(A)$, $B_S(A)$ are follows.*

(i) The lower approximation of A with respect to S is the set of all objects, which can be for certain classified as A with respect to S and it is denoted by $L_S(A)$. That is, $L_S(A) = \{x \in H | xS \subseteq A\}$.

(ii) The upper approximation of A with respect to S,is the set of all objects, which can be possibly classified as A with respect to S and it is denoted by $L_S(A)$. That is, $U_S(A) = \{x \in H | xS \cap A \neq \emptyset\}$.

(iii) The boundary region of A with respect to S, is the set of all objects, which can be classified as A with respect to S and it is denoted by $B_S(A)$. That is, $B_S(A) = U_S(A) - L_S(A)$.

Definition 3.2. *: Let H be a hypergroup, S is an invertible subhypergroup on H and $\tau_S(A) = \{H, \emptyset, L_S(A), U_S(A), B_S(A)\}$ where $A \subseteq H$, $\tau_S(A)$ satisfies the following axioms:*

(i) H and $\emptyset \in \tau_S(A)$.

(ii) The union of the elements of any subcollection of $\tau_S(A)$ is in $\tau_S(A)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_S(A)$ is in $\tau_S(A)$.

That is $\tau_S(A)$ forms a invertible subhypergroup nanotopology on H with respect to A. We call $(H,\tau_S(A))$ as the invertible subhypergroup nano topological space. The elements are called invertible subhypergroup nano open sets.

Remark 3.3. *: We have $L_S(X) = L_{R_S}(X)$ and $U_S(X) = U_{R_S}(X)$ and $B_S(X) = B_{R_S}(X)$. Indeed for any $x \in H$, the equivalence class of x with respect to R_S is xS, where R_S is the equivalence relation with respect to invertible subhypergroup.*

Example 3.4. *: Let $H = \{e,a,b,c\}$. We consider the following hyperoperation on H*

$\circ$	e	a	b	c
e	e	$\{a,b\}$	$\{a,b\}$	$\{c\}$
a	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$
b	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$
c	$\{c\}$	$\{e\}$	$\{e\}$	$\{a,b\}$

Then $(H,\circ)$ is a hypergroup. $S = \{e\}$ be an invertible subhypergroup and $A = \{a,b\}$. Now $L_S(A) = \{a,b\}$ and $U_S(A) = \{a,b\}$, $B_S(A) = \emptyset$. Then $\tau_S(A) = \{H, \emptyset, \{a,b\}\}$.

Theorem 3.5. *: The following theorem hold for a hypergroup nano topological space.*

(i) $L_S(A) \subseteq A \subseteq U_S(A)$.

(ii) $L_S(A) = \emptyset = U_S(A)$.

(iii) $L_S(A) = H = U_S(A)$.

(iv) if $A \subseteq B$, then $L_S(A) \subseteq L_S(B)$ and $U_S(A) \subseteq U_S(B)$.

(v) $L_S(L_S(A)) = L_S(A)$.

(vi) $U_S(U_S(A)) = U_S(A)$.

(vii) $U_S(L_S(A)) = L_S(A)$.

(viii) $L_S(U_S(A)) = U_S(A)$.

(ix) $L_S(A) = (U_S(A^c))^c$ *where* $B^c = H\text{-}B$ *for any* $B \subseteq H$.

(x) $U_S(A) = (L_S(A^c))^c$.

(xi) $U_S(A \cap B) \subseteq U_S(A) \cap U_S(B)$.

(xii) $L_S(A \cap B) = L_S(A) \cap L_S(B)$.

(xiii) $L_S(xS) = U_S(xS)$, *for all* $x \in H$.

Proof.

(i) If x $\in L_S(A)$ that is x $\in$ xS $\subseteq$ A this implies x $\in$ A and hence x $\in U_S(A)$.

(ii) and (iii) obvious.

(iv) If A $\subseteq$ B and x $\in L_S(A)$ then x $\in U_S(A)$, so u $\in L_S(A)$. We have uS = xS and uS $\subseteq$ A, so x $\in L_S(A)$ and so x $\in L_S(B)$. Therefore, $L_S(A) \subseteq L_S(B)$ and similarly $U_S(A) \subseteq U_S(B)$.

(v) If x $\in L_S(L_S(A))$ then there exists u $\in$ xS and u $\in L_S(A)$. We have uS = xS and uS $\subseteq$ A so x $\in L_S(A)$. Conversely, if x $\in L_S(A)$ then we show that there exist u $\in$ xS $\subseteq L_S(A)$. Take u = x.

(vi) If x $\in U_S(U_S(A))$ then there exists u $\in$ xS and u $\in U_S(A)$. We have uS = xS and uS $\cap$ A $\neq \emptyset$, so x $\in U_S(A)$. Conversely, if x $\in U_S(A)$ then we show that there exists u $\in$ xS $\cap\ U_S(A)$. Take u = x.

(vii) and (viii) proof is similar.

(ix) Replace A^c = A then $U_S(A)^c$ we know that $U_S(A)^c = L_S(A^c)$. Now $L_S(A) = U_S(A)^c$. Hence $L_S(A) = (U_S(A^c))^c$.

(x) proof is similar.

(xi) Let x $\in U_S(A \cup B)$. Then x $\in$ xS $\cap$ (A $\cup$ B). It follows that x $\in$ xS $\cap$ A and x $\in$ xS $\cap$ B and hence x $\in U_S(A)$ or x $\in U_S(B)$ that is x $\in U_S(A) \cup U_S(B)$.

(xii) obvious.

(xiii) If x $\in L_S(xS)$ since xS = xS and similarly it cannot be empty then $L_S(xS) = U_S(xS)$.

Theorem 3.6. : *If H is a hypergroup, A is a complete part of H and S is an invertible subhypergroup of H, then invertible subhypergroup nano topology is* $\tau_S(A) = \{H, \emptyset,\ L_S(A)\}$.

Proof. : Indeed,if x ∈ $U_S(A)$, then xS ∩ A ≠ ∅ and by hypothesis, it follows that xS ⊆ A, hence x∈ A. On the other hand,if a ∈ A then a ∈ aS ∩ A,hence we obtain aS ⊆ A. Hence a ∈ $L_S(A)$. Therefore, we obtain the nano topology $\tau_S(A) = \{H, \emptyset, L_S(A)\}$.

Theorem 3.7. : *Let (H,∘) be a hypergroup, A be a complete part of H and S is an invertible subhypergroup of H, then the nano topology is* $\tau_S(A) = \{H, \emptyset,\ L_S(A)\}$.

Proof. : We already have $L_S(A)$ ⊆ A ⊆ $U_S(A)$. Let x ∈ $U_S(A)$ that is a ∈ xS ∩ A. We obtain xS = aS such that x, hence x ∈ $\mathcal{C}(a) \subseteq \mathcal{C}(A)$ = A. On the other hand, if a ∈ A, then $aS_A = \mathcal{C}(a)$ ⊆ A. Hence a ∈ $L_S(A)$. Therefore the invertible subhypergroup nano topology is $\tau_S(A) = \{H, \emptyset,\ L_S(A)\}$.

Theorem 3.8. : *Let (H,∘) be a hypergroup, S be an invertible subhypergroup of H,* S^* *be a closed subhypergroup of H and* $L_S(S^*) \neq \emptyset$ *if and only if* $S \subseteq\ S^*$. *Then* $\tau_S(S^*) = \{H, \emptyset,\ L_S(S^*),\ B_S(S^*)\}$

Proof. : (i) If S^* ⊆ S, S^* ≠ S, then $L_S(S^*)$. We have x ∈ xS ⊆ S^* . Let s ∈ S and u ∈ xS ⊆ S^*. Since S^* is closed, it follows that s ∈ S^*. Hence S ⊆ S^* and since S^* ⊆ S, we obtain S = S^*, which is a contradiction. Hence we get $L_S(S^*)$ = S^* which is of the type $\tau_S(S^*) = \{\mathcal{U}, \emptyset,\ L_S(S^*),\ B_S(S^*)\}$.
(ii) If S ⊆ S^*, then for any x ∈ S^*, we have xS ⊆ S^*, so S^* ⊆ $L_S(S^*)$. Moreover, we have $L_S(S^*)$ ⊆ S^* and hence $L_S(S^*)$ = S^*.
(iii) If S ⊈ S^* and S^* ⊈ S, then $L_S(S^*)$ ⊆ S^* and we consider the following cases:
(a) If x ∈ S then xS = S ⊈ S^*,hence x ∉ $L_S(S^*)$.
(b) If x ∈ S^* -S and if we suppose xS ⊆ S^*, then we consider y ∈ S - S^* and we have xy ⊆ S^*. Since x ∈ S^* and S^* is closed, it follows that y ∈ S^*, which is a contradiction.
Hence, xS ⊈ S^*, which means that x ∉ $L_S(S^*)$. We obtain $L_S(S^*) = \emptyset$.
Therefore, $L_S(S^*) \neq \emptyset$ if and only if S ⊆ S^* and in this case we have $L_S(S^*)$ = S^*. Here we get the invertible subhypergroup nano topology as $\tau_S(S^*) = \{H, \emptyset,\ L_S(S^*),\ B_S(S^*)\}$.

Theorem 3.9. : *Let (H,∘) be a hypergroup,* S_1 *be a invertible subhypergroup of H,* S_2 *be a normal subhypergroup of H and* $L_{S_1}(S_2) = U_{S_1}(S_2)$, *then invertible subhypergroup nano topology is* {H,∅, $L_{S_1}(S_2)$}.

Proof. : If S_1 is invertible subhypergroup and for x,y ∈ H we get x ∈ $S_1 y \Longrightarrow$ y ∈ $S_1 x$ and S_2 is normal subhypergroup. Therefore $xS_2 = S_2 x$. Assume $xS_1 \in$

$L_{S_1}(S_2) = \{x \in H/xS_1 \subset S_2\}$ if and only if x $\in$ H and $xS_1 \subset S_2$ if and only if x $\in$ H and $xS_1 \cap S_2 \neq \emptyset$ if and only if $xS_1 \in U_{S_1}(S_2)$.Hence $L_{S_1}(S_2) = U_{S_1}(S_2)$.So the invertible subhypergroup nano topology is {H,$\emptyset$, $L_{S_1}(S_2)$}.

4. Application

In this section we get invertible subhypergroup Nano topology through Redox reaction in terms of spontaneous reaction.

Definition 4.1. *[13] : Reduction is defined as the gain of electrons or a decrease in oxidation state by a molecule, atom, or ion.*

Definition 4.2. *[13] : Oxidation is defined as the loss of electrons or an increase in oxidation state by a molecule, atom, or ion.*

Definition 4.3. *[13] : Redox is defined as a contraction of the name for chemical reduction-oxidation reaction. A reduction reaction always occurs with an oxidation reaction.Redox reactions include all chemical reactions in which atoms have their oxidation state changed.*

Example 4.4. *[13] : Simple redox process, such as the carbon to yield carbondioxide or the reduction of carbon by hydrogen to yield methane or a complex process such as the oxidation of glucose in the human body through a series of complex electron transfer processes.*

Definition 4.5. *[13] : The oxidation alone and the reduction alone are each called a half-reaction, because two half- reactions always occur together to form a whole reaction.*

Definition 4.6. *[13] : Each half reaction has a standard reduction potential (E^0), which is equal to the potential difference at equilibrium under standard conditions of an electrochemical cell in which the cathode reaction is the half-reaction considered, and the anode is a Standard Hydrogen Electrode(SHE). For a redox reaction, the potential of the cell is defined by: $E^0_{cell} = E^0_{cathode}$ - E^0_{anode}. If the potential of a redox reaction (E^0_{cell}) is positive, this reaction will spontaneous.*

Definition 4.7. *: Homeomorphism is said to be similarity in crystalline form but not necessarily in chemical composition.*

Copper(Cu) is a ductile metal with very high thermal and electrical conductivity. It is used as a conductor of heat and electricity, a building material, and a constituent of various metal alloys. Cu can be in four oxidation state: Cu(0),Cu(I),Cu(II),Cu(III). In nature, copper mainly is as $CuFeS_2$, with oxidation state of II for Cu. Also, Cu can be as Cu_2S or Cu_2O with the oxidation state of I. Pure copper is obtained by electrolytic refining using sheets of pure copper as cathode and impure copper as anode. In this process different ions of

Cu, Cu(II) or Cu(I), reduced to Cu(0) at cathode. Cu(III) is generally uncommon, however some its complexes are known [12].

The standard reduction potential (E^0) for conversion of each oxidation state to other are $E^0(Cu^{3+}/Cu^{2+}) = 2.4$V, $E^0(Cu^{2+}/Cu^{+}) = 0.153$V, $E^0(Cu^{2+}/Cu) = 0.342$V, $E^0(Cu^{+}/Cu) = 0.521$V , where potential versus SHE. According to these standard potentials, the following reactions are spontaneous.

(i) $Cu^{3+} + Cu^{+} \longrightarrow Cu^{2+}$.

(ii) $Cu^{3+} +$ Cu $\longrightarrow Cu^{2+} + Cu^{+}$.

Characterisation of Copper via invertible subhypergroup Nano topology

Therefore, all possible products in reactions between oxidation states of Cu which can be produced spontaneously are listed in the following table:

$*$	Cu	Cu^{+}	Cu^{2+}	Cu^{3+}
Cu	Cu	Cu Cu^{+}	Cu^{2+}, Cu	$Cu^{2+}Cu^{+}$
Cu^{+}	Cu, Cu^{+}	Cu^{+}	Cu^{2+}, Cu^{+}	Cu^{2+}
Cu^{2+}	Cu, Cu^{2+}	Cu^{2+}, Cu^{+}	Cu^{2+}	Cu^{2+}, Cu^{3+}
Cu^{3+}	Cu^{+}, Cu^{2+}	Cu^{2+}	Cu^{2+}, Cu^{3+}	Cu^{3+}

Here we rename Cu, Cu^{+}, Cu^{2+}, Cu^{3+} as follows: Cu = a, Cu^{+} = b, Cu^{2+} = c, Cu^{3+} = d.

$*$	a	b	c	d
a	a	a,b	a,c	b,c
b	a,b	b	b,c	c
c	a,c	b,c	c	c,d
d	b,c	c	c,d	d

The hyperstructures ({a,b},*),({a,c},*),({b,c},*) and ({c,d},*) are hypergroups. If we consider H = {a,b} and S = H is invertible subhypergroup then {{a,b}} is a partition of H we have a $\in$ aS and b $\in$ bS. Now A = {a} where $L_S(A) = \emptyset$ and $U_S(A) = U$ the invertible subhypergroup nano topology is $\tau_S(A) = \{H, \emptyset\}$. Similarly if we consider
H = {b,c},H ={a,c},H = {c,d} gives an indiscrete nano topology.

Observation : From the above illustration if spontaneous reaction for Cu forms hypergroup then the invertible subhypergroup nano topology is an indiscrete nano topology. Two nano topological spaces carrying the trivial topology are homeomorphic iff they have the same cardinality and hence indiscrete space is homeomorphic and by Definition 4.7 copper has same crystalline form but different chemical composition.

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