South East Asian J. of Math. & Math. Sci. **ISSN : 0972-7752 (Print)**
Vol. 14, No. 3 (2018), pp. 01-14

A NEW VIEW TO NEAR-RING THEORY: SOFT NEAR-RINGS

Akın Osman Atagün and Aslıhan Sezgin

Department of Mathematics,
Amasya University, 05100 Amasya, Turkey.
E-mail: aslihan.sezgin@amasya.edu.tr

(Received: August 6, 2018)

Abstract: In this paper, we indicate the study of soft near-rings by using the definition of the soft sets. The notions of soft near-rings, soft subnear-rings, soft (left, right) ideals, (left, right) idealistic soft near-rings, soft homomorphisms and soft near-ring homomorphisms are introduced. Also we investigate the soft homomorphism and soft near-ring homomorphism with respect to the homomorphic image and we show that some structures of soft near-rings are preserved under soft near-ring isomorphism.

Keywords and Phrases: Soft sets, Soft near-rings, Idealistic soft near-rings, Soft near-ring homomorphisms.

2010 Mathematics Subject Classification: 03G25,20D05.

1. Introduction

Molodtsov [1] proposed a new approach for modeling vagueness and uncertainty, which is called soft set theory in 1999. Since its inception, Maji et al. [2] and Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagün [4] studied on soft set operations in more detail. Soft set theory has also wide-ranging applications in algebraic structures, for example Aktas and Çağman [5] studied soft groups and Sezgin and Atagün [6] studied on normalistic soft groups as well. Then, Feng et al. [7] introduced and investigated soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. In [8], Zhan and Jun introduced soft BL-algebras on fuzzy sets and in [9], Çağman and Enginoglu defined soft matrices and their operations and constructed a soft max-min decision making method. Acar et al. [10] introduced initial concepts of soft rings. Atagün and Sezgin [11] studied soft substructures of rings, fields and modules and Sezgin et al. [12] introduced the union soft substructures of near-rings and N-groups. Soft set has also studied in [22-24] as regards operations and algebraic structures.

Soft set theory emphasizes a balanced coverage of both theory and practice. Nowadays, it has promoted a breadth of the discipline of Informations Sciences with intelligent systems, approximate reasoning, expert and decision support systems, self-adaptation and self-organizational systems, information and knowledge, modeling and computing with words, especially soft decision making as in the following studies: [16-18] and some other fields as [19-27].
In this paper, we study the near-ring structure as regards soft sets and we define the notion of a soft near-ring and investigate the algebraic properties of soft near-rings in detail. We introduce the notions of soft (left, right) ideals and (left, right) idealistic soft near-rings, soft homomorphism, soft near-ring homomorphism and derive some important properties. Main purpose of this paper is to extend the study of soft near-rings from a theoretical aspect. In [28], the authors investigate the properties of idealistic soft near-rings with respect to the near-ring mappings. This study can be regarded as an extension of this study.

2. Preliminaries

By a near-ring, we shall mean an algebraic system $(N,+,.)$, where

- $(N,+)$ forms a group (not necessarily abelian)
- $(N,.)$ forms a semi-group and
- $(a+b)c=ac+bc$ for all $a,b,c\in N$ (i.e. we study on right near-rings.)

Throughout this paper, N will always denote a right near-ring. A subgroup M of N with $MM\subseteq M$ is called a subnear-ring of N. A normal subgroup I of N is called a right ideal if $IN\subseteq I$ and denoted by $I\lhd_r N$. It is called a left ideal if $n(s+i)-ns\in I$ for all $n,s\in N$ and $i\in I$ and denoted by $I\lhd_\ell N$. If such a normal subgroup I is both left and right ideal in N, then it is called an ideal in N and denoted by $I\lhd N$. For all undefined concepts and notions we refer to [29,30].
Molodtsov [1] defined the soft set in the following manner: Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A\subseteq E$.

Definition 1. *[1] A pair (F,A) is called a soft set over U, where F is a mapping given by $F:A\rightarrow P(U)$.*
In other words, a soft set over U is a parameterized family of subsets of U. For $\varepsilon\in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F,A) or as the set of ε-approximate elements of the soft set.

Definition 2. *[3] Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\emptyset$. The restricted intersection of (F,A) and (G,B) is denoted* *by $(F,A) \cap_{\mathcal{R}} (G,B)$, and is defined as $(F,A) \cap_{\mathcal{R}} (G,B) = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.*

Definition 3. *[3] Let (F,A) and (G,B) be two soft sets over a common universe U. The extended union of (F,A) and (G,B) is defined to be the soft set (H,C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

This relation is denoted by $(F,A) \sqcup_{\varepsilon} (G,B) = (H,C)$.

Definition 4. *[2] If (F,A) and (G,B) are two soft sets over a common universe U, then "(F,A) AND (G,B)" denoted by $(F,A)\widetilde{\wedge}(G,B)$ is defined by $(F,A)\widetilde{\wedge}(G,B) = (H, A \times B)$, where $H(x,y) = F(x) \cap G(y)$ for all $(x,y) \in A \times B$.*

Definition 5. *[2] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B), denoted by $(F,A)\widetilde{\subset}(G,B)$, if it satisfies: (i) $A \subset B$; (ii) for every $\varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.*

Definition 6. *[7] For a soft set (F,A), the set $Supp(F,A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the support of the soft set (F,A). The null soft set is a soft set with an empty support, and a soft set (F,A) is non-null if $Supp(F,A) \neq \emptyset$.*

3. Soft Near-rings

In this paper, we define soft near-rings, soft subnear-rings by using the definition of the soft sets also we investigate their related properties.

Definition 7. *Let (F,A) be a non-null soft set over a near-ring N. Then (F,A) is called a soft near-ring over N if $F(x)$ is a subnear-ring of N for all $x \in Supp(F,A)$.*

Example 1. *Consider the additive group $(\mathbb{Z}_6,+)$. Under a multiplication defined by following table, $(\mathbb{Z}_6,+,.)$ is a (right) near-ring.* Let (F,A) be a soft set over $\mathbb{Z}_6$, where $A = \mathbb{Z}_6$ and $F : A \to P(\mathbb{Z}_6)$ is a set-valued function defined by

$$F(x) = \{y \in \mathbb{Z}_6 \mid xy \in \{0,3\}\}$$

for all $x \in A$. Then $F(0) = F(3) = \mathbb{Z}_6$ and $F(1) = F(2) = F(4) = F(5) = \{0,3\}$ are subnear-rings of $\mathbb{Z}_6$. Hence (F,A) is a soft near-ring over $\mathbb{Z}_6$.
Let (G,A) be a soft set over $\mathbb{Z}_6$, where $G : A \to P(\mathbb{Z}_6)$ is defined by

$$G(x) = \{y \in \mathbb{Z}_6 \mid xy \in \{1,2,3\}\}$$

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	1	5	3	1	5
2	0	2	4	0	2	4
3	3	3	3	3	3	3
4	0	4	2	0	4	2
5	3	5	1	3	5	1

for all $x \in A$. Then $G(1) = \{0, 1, 3, 4\}$ is not a subnear-ring of $\mathbb{Z}_6$ and hence (G, A) is not a soft near-ring over $\mathbb{Z}_6$.

Theorem 1. *Let (F, A), (G, B) and (K, A) be soft near-rings over N. Then*

a) If it is non-null, then the soft set $(F, A)\widetilde{\wedge}(G, B)$ is a soft near-ring over N.

b) If it is non-null, then the bi-intersection $(F, A) \cap_{\mathcal{R}} (K, A)$ is a soft near-ring over N.

c) If A and B are disjoint, then $(F, A) \sqcup_{\varepsilon} (G, B)$ is a soft near-ring over N.

Proof. (a) By Definition 4, let $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$, where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Then by hypothesis, $(H, A \times B)$ is a non-null soft set over N. If $(x, y) \in Supp(H, A \times B)$, then $H(x, y) = F(x) \cap G(y) \neq \emptyset$. It follows that $\emptyset \neq F(x)$ and $\emptyset \neq G(y)$ are both subnear-rings of N. Hence $H(x, y)$ is a subnear-ring for all $(x, y) \in Supp(H, A \times B)$. Therefore $(H, A \times B)$ is a soft near-ring over N.

(b) By Definition 2, let $(F, A) \cap_{\mathcal{R}} (K, A) = (H, A)$, where $H(x) = F(x) \cap K(x)$ for all $x \in A$. Suppose that (H, A) is a non-null soft set over N. If $x \in Supp(H, A)$, then $H(x) = F(x) \cap K(x) \neq \emptyset$. Thus $\emptyset \neq F(x)$ and $\emptyset \neq K(x)$ are both subnear-rings of N. Hence $H(x)$ is a subnear-ring of N for all $x \in Supp(H, A)$. Therefore (H, A) is a soft near-ring over N as required.

(c) By Definition 3, we can write $(F, A) \sqcup_{\varepsilon} (G, B) = (H, A \cup B)$, where

$$H(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B, \\ G(x) & \text{if } x \in B \setminus A, \\ F(x) \cup G(x) & \text{if } x \in A \cap B \end{cases}$$

for all $x \in A \cup B$. Since $A \cap B = \emptyset$, it follows that either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in A \cup B$. If $x \in A \setminus B$, then $H(x) = F(x)$ is a subnear-ring of N and if

$x \in B \setminus A$, then $H(x) = G(x)$ is a subnear-ring of N. Thus, $(H, A \cup B)$ is a soft near-ring over N.

Definition 8. *Let (F, A) and (G, B) be two soft near-rings over N_1 and N_2, respectively. The product of soft near-rings (F, A) and (G, B) is defined as $(F, A) \times (G, B) = (U, A \times B)$, where $U(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$.*

Theorem 2. *Let (F, A) and (G, B) be two soft near-rings over N_1 and N_2, respectively. If it is non-null, then the product $(F, A) \times (G, B)$ is a soft near-ring over $N_1 \times N_2$.*

Proof. By Definition 8, let $(F, A) \times (G, B) = (U, A \times B)$, where $U(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$. Then by hypothesis, $(U, A \times B)$ is a non-null soft set over $N_1 \times N_2$. If $(x, y) \in Supp(U, A \times B)$, then $U(x, y) = F(x) \times G(y) \neq \emptyset$. Since $\emptyset \neq F(x)$ is a subnear-ring of N_1 and $\emptyset \neq G(y)$ is a subnear-ring of N_2, it follows that $U(x, y)$ is a subnear-ring of $N_1 \times N_2$ for all $(x, y) \in Supp(U, A \times B)$. Therefore $(U, A \times B)$ is a soft near-ring over $N_1 \times N_2$.

For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in N \mid n0 = 0\}$, and the constant part of N denoted by N_c is defined by $N_c = \{n \in N \mid n0 = n\}$. It is well known that N_0 and N_c are subnear-rings of N [29]. For a near-ring N, we can obtain at least two soft near-rings over N using N_0 and N_c. We give these soft near-rings by the following examples:

Example 2. *Let N be a near-ring, $A = N_0$ and let $F_0 : A \to P(N)$ be a set-valued function defined by $F_0(x) = \{y \in A \mid yx \in N_0\}$ for all $x \in A$. Then (F_0, N_0) is a soft near-ring over N. To see this, we need to show the following:*

1. *$a - b \in F_0(x)$*
2. *$ab \in F_0(x)$*

for all $x \in Supp(F_0, N_0)$ and for all $a, b \in F_0(x)$. Since $a, b \in F_0(x)$, then $a \in N_0$, $b \in N_0$, $ax \in N_0$ and $bx \in N_0$. Since $(N_0, +)$ is a subgroup of $(N, +)$, then $a - b \in N_0$ and $(a - b)x = ax - bx \in N_0$, i.e. (1) is satisfied.
To prove (2), we need to show that $ab \in N_0$ and $(ab)x \in N_0$. Since a, b, ax and $bx \in N_0$, then $(ab)0 = a(b0) = a0 = 0$ and $((ab)x)0 = a(bx)0 = a0 = 0$. Hence $ab \in N_0$ and $(ab)x \in N_0$, i.e. (2) is satisfied. Therefore $F_0(x)$ is a subnear-ring of N for all $x \in Supp(F_0, N_0)$, i.e. (F_0, N_0) is a soft near-ring over N.

Example 3. *Now let $B = N$ and let $F_c : B \to P(N)$ be a set-valued function defined by $F_c(x) = \{y \in B \mid yx \in N_c\}$ for all $x \in B$. Then (F_c, N) is a soft near-ring over N. In fact, for all $x \in Supp(F_c, N)$ and for all $a, b \in F_c(x)$;*

$((a-b)x)0 = (ax)0 - (bx)0 = ax - bx = (a-b)x$, since $ax \in N_c$ and $bx \in N_c$. Then $(a-b)x \in N_c$, i.e. $a-b \in F_c(x)$. And $((ab)x)0 = a((bx)0) = a(bx) = (ab)x$, since $bx \in N_c$. Then $(ab)x \in N_c$, i.e. $ab \in F_c(x)$. Therefore $F_c(x)$ is a subnear-ring of N for all $x \in Supp(F_c, N)$, i.e. (F_c, N) is a soft near-ring over N.

Definition 9. *Let (F, A) be a soft near-ring over N. We have the following:*

- *a) (F, A) is called trivial if $F(x) = \{0_N\}$ for all $x \in Supp(F, A)$.*
- *b) (F, A) is said to be whole if $F(x) = N$ for all $x \in Supp(F, A)$.*

Proposition 1. *Let (F, A) and (G, B) be soft near-rings over N, where $A \cap B \neq \emptyset$. Then,*

- *i) If (F, A) and (G, B) are trivial soft near-rings over N, then $(F, A) \cap_{\mathcal{R}} (G, B)$ is a trivial soft near-ring over N.*
- *ii) If (F, A) and (G, B) are whole soft near-rings over N, then $(F, A) \cap_{\mathcal{R}} (G, B)$ is a whole soft near-ring over N.*
- *iii) If (F, A) is a trivial soft near-ring over N and (G, A) is a whole soft near-rings over N, then $(F, A) \cap_{\mathcal{R}} (G, B)$ is a trivial soft near-rings over N.*

Proof. The proof is easily seen by Definition 2, Definition 9, Theorem 1 (b).

Proposition 2. *Let (F, A) and (G, B) be two soft near-rings over N_1 and N_2, respectively. Then,*

- *i) If (F, A) and (G, B) are trivial soft near-rings over N_1 and N_2, respectively, then $(F, A) \times (G, B)$ is a trivial soft near-ring over $N_1 \times N_2$.*
- *ii) If (F, A) and (G, B) are whole soft near-rings over N_1 and N_2, respectively, then $(F, A) \times (G, B)$ is a whole soft near-ring over $N_1 \times N_2$.*

Proof. The proof is easily seen by Definition 8, Definition 9 and Theorem 2.

Definition 10. *Let (F, A) and (G, B) be soft near-rings over N. Then the soft near-ring (F, A) is called a soft subnear-ring of (G, B) if it satisfies:*

- *i) $A \subset B$*
- *ii) $F(x)$ is a subnear-ring of $G(x)$ for all $x \in Supp(F, A)$.*

Proposition 3. *Let (F, A) and (G, A) be soft near-rings over N. Then we have the following:*

a) If $F(x) \subset G(x)$ for all $x \in A$, then (F, A) is a soft subnear-ring of (G, A).

b) $(F, A) \cap_{\mathcal{R}} (G, A)$ is a soft subnear-ring of both (F, A) and (G, A) if it is non-null.

c) If $(G, B)\widetilde{\subset}(F, A)$, then (G, B) is a soft subnear-ring of (F, A).

Proof. a) If $F(x) \subset G(x)$ for all $x \in A$, it is obvious that $F(x)$ is a subnear-ring of $G(x)$. Hence the result is seen by Definition 10.
b) It follows from Proposition 3(a) and Theorem 1(b).
c) Since $F(x)$ and $G(x)$ are subnear-rings of N for all $x \in Supp(F, A)$ and for all $x \in Supp(G, B)$, respectively and $G(x)$ and $F(x)$ are identical approximations for all $x \in Supp(G, B)$ and $B \subseteq A$, the proof is completed by Proposition 3(a).

4. Soft ideals and idealistic soft near-rings

Definition 11. *Let (F, A) be a soft near-ring over N. A non-null soft set (G, I) over N is called a soft left (resp. right) ideal of (F, A) denoted by $(G, I)\widetilde{\lhd_\ell}(F, A)$ (resp. $(G, I)\widetilde{\lhd_r}(F, A)$) if it satisfies:*

i) $I \subset A$

ii) $G(x) \lhd_\ell F(x)$ (resp. $G(x) \lhd_r F(x)$) for all $x \in Supp(G, I)$.

If (G, I) is both soft left and soft right ideal of (F, A), then it is said that (G, I) is a soft ideal of (F, A) and denoted by $(G, I)\widetilde{\lhd}(F, A)$.

Example 4. *Let $N = (\mathbb{Z}_6, +, .)$ be the near-ring given in Example 1. Let $A = \mathbb{Z}_6$ and let $F : A \to P(N)$ be a set-valued function defined by*

$$F(x) = \{y \in A \mid xy \in \{0, 2, 4\}\}$$

for all $x \in A$. Then (F, A) is a non-null soft set over N. Let $I = \{0, 2, 4\}$ and $G : I \to P(N)$ be a set-valued function defined by

$$G(x) = \{y \in I \mid xy \in \{0, 2, 4\}\}$$

for all $x \in I$. Then we have $F(0) = F(2) = F(4) = \mathbb{Z}_6$ and $F(1) = F(3) = F(5) = \emptyset$, $G(0) = G(2) = G(4) = \{0, 2, 4\}$. It is easily seen that for all $x \in Supp(G, I) = \{0, 2, 4\}$, $G(x) \lhd F(x)$ and hence $(G, I)\widetilde{\lhd}(F, A)$.

Theorem 3. *Let (G_1, I_1) and (G_2, I_2) be soft left ideals (resp. soft right ideals, soft ideals) of a soft near-ring (F, A) over a near-ring N. Then the soft set $(G_1, I_1) \cap_{\mathcal{R}} (G_2, I_2)$ is a soft left ideal (resp. soft right ideal, soft ideal) of (F, A) if* *it is non-null.*

Proof. We give the proof for soft left ideals; the same proof can be seen for soft right ideals and hence for soft ideals. Assume that $(G_1, I_1)\widetilde{\lhd_\ell}(F, A)$ and $(G_2, I_2)\widetilde{\lhd_\ell}(F, A)$. By Definition 2, $(G_1, I_1) \cap_{\mathcal{R}} (G_2, I_2) = (G, I)$, where $I = I_1 \cap I_2$ and $G(x) = G_1(x) \cap G_2(x)$ for all $x \in I$. Since $I_1 \subset A$ and $I_2 \subset A$, it is clear that $I \subset A$. Suppose that the soft set (G, I) is non-null. If $x \in Supp(G, I)$, then $G(x) = G_1(x) \cap G_2(x) \neq \emptyset$. Since $G_1(x) \lhd_\ell F(x)$, $G_2(x) \lhd_\ell F(x)$, and the intersection of left ideals is a left ideal in near-rings, $G(x) \lhd_\ell F(x)$ for all $x \in Supp(G, I)$. Therefore $(G_1, I_1) \cap_{\mathcal{R}} (G_2, I_2)\widetilde{\lhd_\ell}(F, A)$.

Theorem 4. *Let (G_1, I_1) and (G_2, I_2) be soft left ideals (resp. soft right ideals, soft ideals) of a soft near-ring (F, A) over a near-ring N. Then the soft set $(G_1, I_1) \sqcup_\varepsilon (G_2, I_2)$ is a soft left ideal (resp. soft right ideal, soft ideal) of (F, A) if I_1 and I_2 are disjoint.*

Proof. Assume that $(G_1, I_1)\widetilde{\lhd_\ell}(F, A)$ and $(G_2, I_2)\widetilde{\lhd_\ell}(F, A)$. By Definition 3, $(G_1, I_1) \sqcup_\varepsilon (G_2, I_2) = (G, I)$, where $I = I_1 \cup I_2$ and for all $x \in I$

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in I_1 \setminus I_2, \\ G_2(x) & \text{if } x \in I_2 \setminus I_1, \\ G_1(x) \cup G_2(x) & \text{if } x \in I_1 \cap I_2. \end{cases}$$

Since $I_1 \subset A$ and $I_2 \subset A$, it is clear that $I \subset A$. If $I_1 \cap I_2 = \emptyset$, then for all $x \in Supp(G, I)$, we know that either $x \in I_1 \setminus I_2$ or $x \in I_2 \setminus I_1$. If $x \in I_1 \setminus I_2$, then $\emptyset \neq G_1(x) = G(x) \lhd_\ell F(x)$ and if $x \in I_2 \setminus I_1$, then $\emptyset \neq G_2(x) = G(x) \lhd_\ell F(x)$ for all $x \in Supp(G, I)$. Therefore $(G_1, I_1) \sqcup_\varepsilon (G_2, I_2)\widetilde{\lhd_\ell}(F, A)$. The proof can be seen for soft right ideals and hence for soft ideals in the same way.

Example 5. *Let (F, A) be the soft near-ring over the near-ring $N = (\mathbb{Z}_6, +, .)$ and let $(G, I)\widetilde{\lhd}(F, A)$ be the ones given in Example 4. Let $K : A \to P(N)$ be a set-valued function defined by*

$$K(x) = \{y \in A \mid xy = 0\}$$

for all $x \in A$. Then $K(0) = \mathbb{Z}_6$, $K(1) = \emptyset$, $K(2) = \{0, 3\}$, $K(3) = \emptyset$, $K(4) = \{0, 3\}$ and $K(5) = \emptyset$. Since $K(x) \lhd F(x)$ for all $x \in Supp(K, A)$, then $(K, A)\widetilde{\lhd}(F, A)$. We consider the bi-intersection of the soft ideals (K, A) and (G, I). Then, $(K, A) \cap_{\mathcal{R}} (G, I) = (H, A \cap I = I)$ where $H(x) = K(x) \cap G(x)$ for all $x \in I$. Since $Supp(H, I) = \{0, 2, 4\}$, it is non-null. For all $x \in Supp(H, I)$, we see that $H(0) = \{0, 2, 4\} \lhd F(0)$, $H(2) = \{0\} \lhd F(2)$ and $H(4) = \{0\} \lhd F(4)$. Therefore

$(K, A) \cap_{\mathcal{R}} (G, I) \widetilde{\lhd} (F, A)$.
Now we consider $(K, A) \sqcup_{\varepsilon} (G, I)$. Then $(K, A) \sqcup_{\varepsilon} (G, I) = (T, A \cup I = I)$, where

$$T(x) = \begin{cases} K(x) & \text{if } x \in A \setminus I = \{1, 3, 5\}, \\ G(x) & \text{if } x \in I \setminus A = \emptyset, \\ K(x) \cup G(x) & \text{if } x \in A \cap I = \{0, 2, 4\} \end{cases}$$

for all $x \in A$. Then Supp$(T, A) = \{0, 2, 4\}$ and $T(0) = \mathbb{Z}_6$, $T(2) = \{0, 2, 3, 4\} = T(4)$. Nevertheless, $T(2)$ is not an ideal of $F(2)$ and hence $(K, A) \sqcup_{\varepsilon} (G, I)$ is not a soft ideal of (F, A). Namely, we see that the condition 'disjoint' can not be removed from Theorem 4.

Definition 12. *Let (F, A) be a soft near-ring over N. If for all $x \in$ Supp(F, A) $F(x) \lhd_{\ell} N$ (resp. $F(x) \lhd_r N$, $F(x) \lhd N$), then (F, A) is called a left idealistic (resp. right idealistic, idealistic) soft near-ring over N.*

Example 6. *Let the soft near-rings (F, A) and (K, A) be the ones given in Example 5 over the near-ring $N = (\mathbb{Z}_6, +, .)$. Then for all $x \in$ Supp$(F, A) = \{0, 2, 4\}$, $F(x) \lhd N$, i.e. (F, A) is an idealistic soft near-ring over N. Since Supp$(K, A) = \{0, 2, 4\}$ and $K(0) = \mathbb{Z}_6 \lhd N$, $K(2) = K(4) = \{0, 3\} \lhd N$, then (K, A) is also an idealistic soft near-ring over N.*

Theorem 5. *Let (F, A) and (G, B) be idealistic soft near-rings over N. Then we have the following:*

a) If it is non-null, $(F, A) \cap_{\mathcal{R}} (G, B)$ is an idealistic soft near-ring over N.

b) If A and B are disjoint, then $(F, A) \sqcup_{\varepsilon} (G, B)$ is an idealistic soft near-ring over N.

c) If it is non-null, $(F, A) \widetilde{\wedge} (G, B)$ is an idealistic soft near-ring over N.

Proof. When considering the definition of ideal of N, the proof is similar to the proof of 1, hence omitted.

Definition 13. *A near-ring N is said to satisfy the condition (C) if $I \lhd J \lhd N$, then $I \lhd N$.*

Example 7. *Consider the near-ring $N = (\mathbb{Z}_6, +, .)$ in Example 1. It can be easily seen that N satisfies the condition (C).*

Proposition 4. *Let N be a near-ring which satisfies the condition (C) and let (F, A) be an idealistic soft near-ring over N. If (G, I) is a soft ideal of (F, A), then (G, I) is also an idealistic soft near-ring over N.*

Proof. If $(G,I)\widetilde{\lhd}(F,A)$, then for all $x\in Supp(G,I)$, $G(x)\lhd F(x)$. Since (F,A) is an idealistic soft near-ring over N, then for all $x\in Supp(F,A)$, $F(x)\lhd N$. So we have $G(x)\lhd F(x)\lhd N$ for all $x\in Supp(G,I)$. Since N satisfies condition (C), $G(x)\lhd N$ for all $x\in Supp(G,I)$. $G(x)$ is also a subnear-ring of N for all $x\in Supp(G,I)$, since every ideal of N is also a subnear-ring of N. Therefore (G,I) is a soft near-ring over N. Furthermore, (G,I) is an idealistic soft near-ring over N.

Example 8. *Let (F,A) be the soft near-ring over N and let $(G,I)\widetilde{\lhd}(F,A)$ be the ones given in Example 4. It is seen that (G,I) is also an idealistic soft near-ring over N.*

Definition 14. *Let (F,A) and (G,B) be soft near-rings over two near-rings N_1 and N_2, respectively. Let $f:N_1\to N_2$ and $g:A\to B$ be two mappings. Then the pair (f,g) is called a soft mapping from (F,A) to (G,B). A soft mapping (f,g) is called a soft homomorphism if it satisfies the conditions below:*

i) f is a near-ring homomorphism.

ii) g is a mapping.

iii) $f(F(x))=G(g(x))$ for all $x\in A$.

If (f,g) is a soft homomorphism and f and g are both surjective, then we say that (F,A) is softly near-ring homomorphic to (G,B) under the soft homomorphism (f,g), which is denoted by $(F,A)\sim(G,B)$. Then, (f,g) is called a soft near-ring homomorphism. Furthermore, if f is an isomorphism of near-rings and g is a bijective mapping, then (f,g) is said to be a soft near-ring isomorphism. In this case, we say that (F,A) is soft isomorphic to (G,B), which is denoted by $(F,A)\simeq(G,B)$.

Example 9. *Consider the near-ring $(\mathbb{Z}_6,+,.)$ given in Example 1. And let the subnear-ring $\{0,2,4\}$ of $(\mathbb{Z}_6,+,.)$. Let $f:\mathbb{Z}_6\to\{0,2,4\}$ be the mapping defined by $f(x)=4x$. Obviously, f is an epimorphism of near-rings. Let $g:\mathbb{Z}_6\to\{0,2,4\}$ by $g(x)=2x$ for all $x\in\mathbb{Z}_6$. Then one can easily say that g is surjective. Let $(F,\mathbb{Z}_6)$ be a soft set over $\mathbb{Z}_6$, where $F:\mathbb{Z}_6\to P(\mathbb{Z}_6)$ is a function by $F(x)=\{0\}\cup\{y\in\mathbb{Z}_6\mid 3x=y\}$ for all $x\in\mathbb{Z}_6$. It can be easily illustrated that $F(x)=\{0,3\}$ is a subnear-ring of $\mathbb{Z}_6$ for all $x\in\mathbb{Z}_6$. Thus $(F,\mathbb{Z}_6)$ is a soft near-ring over $\mathbb{Z}_6$. Let $(G,\{0,2,4\})$ be a soft set over $\{0,2,4\}$, where $G:\{0,2,4\}\to P(\{0,2,4\})$ is a function with $G(x)=\{y\in\{0,2,4\}\mid x0=y\}$ for all $x\in\{0,2,4\}$. Then one can show that $(G,\{0,2,4\})$ is a soft near-ring over $\{0,2,4\}$. Furthermore, $f(F(x))=f(\{0,3\})=\{0\}$ and $G(g(0))=G(0)=\{0\}$, $G(g(1))=G(2)=\{0\}$,*

$G(g(2)) = G(4) = \{0\}$, $G(g(3)) = G(0) = \{0\}$, $G(g(4)) = G(2) = \{0\}$, $G(g(5)) = G(4) = \{0\}$ for all $x \in \mathbb{Z}_6$, so it is to say that $f(F(x)) = G(g(x))$ for all $x \in \mathbb{Z}_6$. Therefore (f, g) is a soft near-ring homomorphism and $(F, \mathbb{Z}_6) \sim (G, \{0, 2, 4\})$.

Theorem 6. *Let (F, A), (G, B) and (H, C) be soft near-rings over N_1, N_2 and N_3, respectively. Let the soft mapping (f, g) from (F, A) to (G, B) is a soft homomorphism from N_1 to N_2 and the soft mapping (f^*, g^*) from (G, B) to (H, C) a soft homomorphism from N_2 to N_3. Then the soft mapping $(f^* \circ f, g^* \circ g)$ from (F, A) to (H, C) is a soft homomorphism from N_1 to N_3.*

Proof. Let the soft mapping (f, g) from N_1 to N_2 be a soft homomorphism from (F, A) to (G, B), then there exists a near-ring homomorphism f such that $f : N_1 \rightarrow N_2$, and a mapping g such that $g : A \rightarrow B$ which satisfy $f(F(x)) = G(g(x))$ for all $x \in A$. And let the soft mapping (f^*, g^*) from N_2 to N_3 be a soft homomorphism from (G, B) to (H, C), then there exists a near-ring homomorphism f^* such that $f^* : N_2 \rightarrow N_3$, and a mapping g^* such that $g^* : B \rightarrow C$ which satisfy $f^*(G(x)) = H(g^*(x))$ for all $x \in B$. We need to show that $(f^* \circ f)(F(x)) = H((g^* \circ g)(x))$ for all $x \in A$. Let $x \in A$, then

$$(f^* \circ f)(F(x)) = f^*(f(F(x))) = f^*(G(g(x))) = H(g^*(g(x))) = H((g^* \circ g)(x))$$

Therefore, the proof is completed.

Theorem 7. *The relation $\simeq$ is an equivalence relation on soft near-rings.*

Proof. Straightforward, hence omitted.

Theorem 8. *Let N_1 and N_2 be near-rings and (F, A), (G, B) be soft sets over N_1 and N_2, respectively. If (F, A) is a soft near-ring over N_1 and $(F, A) \simeq (G, B)$, then (G, B) is a soft near-ring over N_2.*

Proof. We need to show that $G(y)$ is a subnear-ring of N_2 for all $y \in Supp(G, B)$. Since $(F, A) \simeq (G, B)$, there exists a near-ring epimorphism f from N_1 to N_2 and a bijective mapping g from A to B which satisfies $f(F(x)) = G(g(x))$ for all $x \in A$. Assume that (F, A) is a soft near-ring over N_1. Then $F(x)$ is a subnear-ring of N_1 for all $x \in Supp(F, A)$, therefore $f(F(x))$ is a subnear-ring of N_2 for all $x \in Supp(F, A)$. Since g is a bijective mapping, for all $y \in Supp(G, B) \subseteq B$, there exists an $x \in A$ such that $y = g(x)$. Hence $G(y)$ is a subnear-ring of N_2 for all $y \in Supp(G, B)$ since $f(F(x)) = G(y)$.

Theorem 9. *Let $f : N_1 \rightarrow N_2$ be an epimorphism of near-rings and (F, A), (G, B) be two soft near-rings over N_1 and N_2, respectively.*

i) The soft mapping (f, I_A) from (F, A) to (H, A) is a soft near-ring homomor-

phism from N_1 *to* N_2*, where* $I_A : A \to A$ *is the identity mapping and the set valued function* $H : A \to P(N_2)$ *is defined by* $H(x) = f(F(x))$ *for all* $x \in A$.

ii) If $f : N_1 \to N_2$ *is an isomorphism of near-rings, then the soft mapping* (f^{-1}, I_B) *from* (G, B) *to* (T, B) *is soft near-ring homomorphism from* N_2 *to* N_1*, where* $I_B : B \to B$ *is the identity mapping and the set valued function* $T : B \to P(N_1)$ *is defined by* $T(x) = f^{-1}(G(x))$ *for all* $x \in B$.

Proof. It follows from Definition 14, therefore omitted.

5. Conclusion

Throughout this paper, in a near-ring structure we study the algebraic properties of soft sets. This work bears on soft near-rings, soft subnear-rings, soft (left, right) ideals, soft ideals, (left, right) idealistic soft near-rings and soft homomorphisms. To extend this work, one could study the ideals of soft near-rings.

References

[1] Molodtsov D, Soft set theory-first results, *Comput Math Appl*, 37(1999), 19-31.

[2] Maji PK, Biswas R, Roy AR, Soft set theory, *Comput Math Appl*, 45(2003), 555-562.

[3] Ali MI, Feng F, Liu X, Min WK, Shabir M, On some new operations in soft set theory, *Comput Math Appl*, 57(2009), 1547-1553.

[4] Sezgin A, Atagün AO, On operations of soft sets, *Comput Math Appl*, 61(5)(2011), 1457-1467.

[5] Aktaṣ H, Çağman N, Soft sets and soft groups, *Inform Sci*, 177(2007), 2726-2735.

[6] Sezgin A, Atagün AO, Soft groups and normalistic soft groups, *Comput Math Appl*, 62(2)(2011), 685-698.

[7] Feng F, Jun YB, Zhao X, Soft semirings, *Comput Math Appl*, 56(2008), 2621-2628.

[8] Zhan J, Jun YB, Soft BL-algebras based on fuzzy sets, *Comput. Math. Appl.*, 59(6)(2010), 2037-2046.

[9] Çağman N, Enginoğlu S, Soft matrix theory and its decision making, *Comput Math Appl*, 59(2010), 3308-3314.

[10] Acar U, Koyuncu F, Tanay B, Soft sets and soft rings, *Comput Math Appl*, 59(2010), 3458-3463.

[11] Atagün AO, Sezgin A, Soft substructures of rings, fields and modules, *Comput Math Appl*, 61(3) (2011), 592-601.

[12] Sezgin A and Atagün A O, Çağman N, Union soft substructures of near-rings and N-groups, *Neural Comput. Appl.* (2011) DOI: 10.1007/s00521-011-0732-1.

[13] Ali MA, Soft ideals and soft filters of soft ordered semigroups, *Computers and Mathematics with Applications*, 62(2011), 3396-3403.

[14] Ali MA, Shabir M, Naz M, Algebraic structures of soft sets associated with new operations, *Computers and Mathematics with Applications*, 61(2011), 2647-2654.

[15] Feng F, Ali MA, Shabir M, Soft relations applied to semigroups, *Filomat*, 27(2013), 1183-1196.

[16] Çağman N, Enginoğlu S, Soft set theory and uni-int decision making, *Eurepean Journal of Operational Research*, 207(2010), 848-855.

[17] Maji PK, Roy AR and Biswas R, An application of soft sets in a decision making problem, *Computers and Mathematics with Applications*, 44(2002), 1077-1083.

[18] Riaz M and Hashmi MR, Fixed points of fuzzy neutrosophic soft mapping with decision-making, *Fixed point theory and applications*, 7(2018), 1-10.

[19] Feng F, Jun YB, Liu XY, Li LF, An adjustable approach to fuzzy soft set based decision making, *Journal of Computational and Applied Mathematics*, 234(2010), 10-20.

[20] Feng F, Li C, Davvas B, Ali MI, Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Computing*, 14(2010)(6), 899-911.

[21] Feng F, Liu XY, Leoreanu-Fotea V, Jun YB, Soft sets and soft rough sets, *Information Sciences* 181(2011), 1125-1137.

[22] Zou Y, Xiao Z, Wang, X, Introduction to linear representations of soft groups, *Journal of Intelligent and Fuzzy Systems*, 29(2015), 1511-1519.

[23] Jianming Z, Wieslaw AD, Neggers J, A new soft union set: characterizations of hemirings, *International Journal of Machine Learning and Cybernetics*, DOI 10.1007/s13042-015-0343-8,(2015) 1-11.

[24] Atagn, AO, Aygn, Emin, Groups of soft sets, *Journal of Intelligent and Fuzzy Systems*, 30(2016), 729-733.

[25] Zorlutuna I, Ç̣akır H, On Continuity of Soft Mappings, *Applied Mathematics and Information Science*, 9(2015), 403-409.

[26] Riaz M and Hashmi MR, Fuzzy Parameterized Fuzzy Soft Compact Spaces with Decision-Making, *Punjab University Journal of mathematics*, 50(2)(2018), 131-145.

[27] Riaz M, Hashmi MR and Farooq A, Fuzzy Parameterized Fuzzy Soft Metric spaces, *Journal of Mathematical Analysis*, 9(2)(2018), 25-36.

[28] Sezgin A, Atagün AO, Aygün E, A note on soft near-rings and idealistic soft near-rings, *Filomat*, 25(1)(2011), 53-68.

[29] Pilz, *G Near-rings*, North Holland Publishing Company, Amsterdam-New York-Oxford, 1983.

[30] Taşdemir F, Completely Equi prime Ideals of Near-Ring Modules, *Karaelmas Science and Engineering Journal*, 8(1)(2018), 79-83.