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A BILATERAL EXTENSION OF SECOND ORDER MOCK THETA FUNCTIONS

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Dedicated to Professor G. E. Andrews on his seventieth birthday

Abstract: We introduce a bilateral extension of two second mock theta functions and establish a new identity connecting the two functions.

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1. Preliminaries and Results

Recently, McIntosh [1] has introduced three second order mock theta functions defined by the following q-series

$$\begin{aligned} A(q) &= \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^{n+1}(-q^2;q^2)_n}{(q;q^2)_{n+1}} \\ B(q) &= \sum_{n=0}^{\infty} \frac{q^{n^2+n}(-q^2;q^2)_n}{(q;q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^{n}(-q;q^2)_n}{(q;q^2)_{n+1}} \\ C(q) &= \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2}(q;q^2)_n}{(-q^2;q^2)_n^2}, \end{aligned}$$

where

$$\begin{aligned} (a;q^k)_n &= (1-a)(1-aq)\cdots(1-aq^{k(n-1)}), \quad n>0 \\ (a)_0 &= 1. \end{aligned}$$

In the present short communication, we introduce the following bilateral extension of the first two functions

$$A_c(q) = \sum_{n=-\infty}^{\infty} \frac{q^{n+1}(-q^2;q^2)_n}{(q;q^2)_{n+1}}$$
$$B_c(q) = \sum_{n=-\infty}^{\infty} \frac{q^{n}(-q;q^2)_n}{(q;q^2)_{n+1}}$$

It is interesting to note that if we use the well known Ramanujan's sum

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n\, z^n}{(b)_n} = \frac{(q;q)_\infty\,(b/a;q)_\infty\,(az;q)_\infty\,(q/az;q)_\infty}{(b;q)_\infty\,(q/a;q)_\infty\,(z;q)_\infty\,(b/az;q)_\infty}, \tag{1.1}$$

we get

$$A_c(q) = \frac{(-q;q^2)^3_\infty\,(q^2;q^2)_\infty}{4(q;q^2)^2_\infty\,(q^2;q^2)^3_\infty}, \tag{1.2}$$

after replacing $q \to q^2$ and then taking $a = -q$, $b = z = q$. Next, replacing $q \to q^2$ and then taking $a = q^{-1}$, $b = z = q$, we get

$$B_c(q) = \frac{2(-q^2;q^2)^3_\infty\,(q^2;q^2)_\infty}{(q;q^2)_\infty\,(-q;q^2)^3_\infty}. \tag{1.3}$$

Finally, from (1.2) and (1.3), we get

$$8(q;q^2)_\infty(-q^2;q^2)^5_\infty A_c(q) = (-q;q^2)^6_\infty\, B_c(q). \tag{1.4}$$

Reference

[1] McIntosh, R.J., *Second order mock theta functions*, Cand. Bull. Math. **50**(2) (2007), 284–290.