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HEAT TRANSFER BY FREE CONVECTION FLOW WITH RADIATION ALONG A POROUS HOT VERTICAL PLATE IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD

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Abstract: An analysis of radiation effect on steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate has been discussed. Approximate solutions have been derived for the velocity, temperature field, skin friction and rate of heat transfer using multi-parameter perturbation technique. The obtained results are discussed with the help of graphs and tables to observe the effects of Prandtl number, radiation parameter, magnetic field parameter and Grashof number on velocity, temperature, skin-friction and the Nusselt number.

Keywords and Phrases: Free convection, radiation, magnetic field, heat transfer
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1. Introduction

Free convection phenomenon has been the object of extensive research. The importance of this phenomenon is due to enhanced concern in science and technology about buoyancy induced motion in the atmosphere, in bodies of water and quasi solid bodies such as earth. Free convection flow past a vertical plate has been studied extensively by Ostrach [9-10], Riley *et al.* [11], Martynenko *et al.* [8] and Weiss *et al.* [15] in numerous ways to include various physical aspects. Magnetohydrodynamic flows have application in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. On account of their varied importance, these flows have been studied by several authors notable amongst them are Shercliff [14], Ferraro and Plumpton [5] and Cramer and Pai [3].

Many processes in engineering areas occur at high temperatures and acknowledge radiation heat transfer becomes very important for the design of pertinent equipment. Nuclear power planets, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. At high operating temperature, radiation effects can be quite significant [4,6,7]. Most of the existing analytical studies for this problem are based on the constant physical properties. However, it is known that these properties may change with temperature, especially fluid viscosity [13]. Recently, Abeoldahab and Elgendy [1] studied the radiation effect on MHD free convection flow of a gas past a semi-infinite vertical plate with variable thermophysical properties for high temperature differences.

The propagation of thermal energy through mercury and electrolytic solution in the presence of magnetic field and radiation has wide range of application. To the best knowledge of authors very little attention has been paid to the effect of the radiation in mercury and electrolytic solution. Hence our objective in the present paper is an extension of the work Sahoo *et al.* [12], for steady flow taking radiation effect in to account The governing equations of motions are solved by using a regular perturbation technique.

2. Formulation of the Problem

Consider an infinite long non-conducting hot vertical, thin porous plate in an electrically conducting viscous fluid such that the x^*-axis is taken along the plate in the upwards direction and y^*-axis is normal to it. A transverse constant magnetic field is applied in the direction of y^*-axis. Here we shall consider that all flow quantities are independent of the z^*-axis. Since the motion is two dimensional and the length of the plate is large, therefore, all the physical variables are independent of x^*. The governing equations of continuity, motion and energy for a free convection flow of an electrically conducting fluid along a hot, nonconducting porous vertical plate in the presence of radiation are given by [12]

$$\frac{dv^*}{dy^*} = 0 \tag{2.1}$$

$$v^* = -v_0 \quad \text{(constant)}, \tag{2.2}$$

$$\rho v^* \frac{du^*}{dy^*} = \mu \frac{d^2u^*}{dy^{*2}} + \rho g \beta (T^* - T_\infty) - \sigma B_0^2 u^*, \tag{2.3}$$

$$\frac{dp^*}{dy^*} = 0 \Rightarrow p^* \text{ is constant}, \tag{2.4}$$

$$\rho C_p v^* \frac{dT^*}{dy^*} = \kappa \frac{d^2T^*}{dy^{*2}} + \mu \left(\frac{du^*}{dy^*}\right)^2 - \frac{\partial q_r^*}{\partial y^*}, \tag{2.5}$$

where u^* and v^* are components of velocity along x^* and y^* directions, g the acceleration due to gravity, β the coefficient of thennal expansion. T^* the temperature of the plate, T_∞ the free stream temperature, $\rho, \nu, \kappa, C_p, \sigma, B_0, \mu, p^*, q_r^*, v_0$, respectively, are density, kinematic viscosity, thermal conductivity, specific heat of the fluid at constant pressure, magnetic permeability, magnetic field, viscosity of fluid, pressure radiation, heat flux and cross velocity, respectively.

The radiative heat flux q_r^* is given by [2]:

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty)I', \tag{2.6}$$

where $I' = \int_0^\infty k_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$, $k_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Planck's function.

The boundary conditions are

$$\left.\begin{array}{l} y^* = 0 : u^* = 0, \;\; T = T_w \\ y^* \to \infty : u^* \to 0, \;\; T^* \to T_\infty \end{array}\right\} \tag{2.7}$$

Introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} y = v_0 y^*/\nu, \; u = u^*/v_0, \; M^2 = (B_0^2 \nu^2 \sigma / v_0^2 \mu) \\ Pr = \mu C_p/\kappa, \; \theta = (T^* - T_\infty)/(T_w - T_\infty), \\ E = v_0^2 / C_p(T_w - T_\infty), \;\; Gr = \rho g \beta v^2 (T_w - T_\infty)/v_0^3 \mu \end{array}\right\} \tag{2.8}$$

and $F = 4vI'/\rho C_p v_0^2$ in the equations (2.3) and (2.5), we get

$$\frac{d^2u}{dy^2} + \frac{du}{dy} - M^2 u = -Gr\,\theta, \tag{2.9}$$

$$\frac{d^2\theta}{dy^2} + Pr\,\frac{d\theta}{dy} - FPr\,\theta + PrE\left(\frac{du}{dy}\right)^2 = 0, \tag{2.10}$$

where Gr = Grashoff number, Pr = Prandtl number, F = Radiation parameter, M = Magnetic parameter, E=Eckert number.

The corresponding boundary conditions in dimensionless form are reduced to

$$\left.\begin{array}{l} y = 0 : u = 0, \;\; \theta = 1 \\ y \to \infty : u \to 0, \;\; \theta \to 0 \end{array}\right\}. \tag{2.11}$$

Solution of the Problem. The physical variables u, θ can be expanded in the power of the Eckert number. This can be possible physically as E for the flow of an incompresible fluid is always less than unity. Hence, we can assume

$$\left.\begin{array}{rcl} u(y) & = & u_0(y) + Eu_1(y) + 0(E^2) \\ \theta(y) & = & \theta_0(y) + E\theta_1(y) + 0(E^2) \end{array}\right\} \tag{2.12}$$

where $u_0,\ u_1$, θ_0 and θ_1 are components of the velocity and the temperature.

Using equation (2.12) equations (2.9) and (2.10) and equating the coefficient of like powers of E, we have

$$u_0'' + u_0' - M^2 u_0 \;=\; -\,Gr\,\theta_0, \tag{2.13}$$

$$\theta_0'' + Pr + \theta_0' - FPr\,\theta_0 \;=\; 0, \tag{2.14}$$

$$u_1'' + u_1' - M^2 u_1 \;=\; -\,Gr\,\theta_1, \tag{2.15}$$

$$\theta_1'' + Pr\,\theta_1' - FPr\,\theta_1 + Pr\,u_0'^2 \;=\; 0. \tag{2.16}$$

Now the corresponding boundary conditions are

$$\left.\begin{array}{l} y \;=\; 0 \;:\; u_0 \;=\; 0,\;\; u_1 \;=\; 0,\;\; \theta_0 \;=\; 1,\;\; \theta_1 \;=\; 0, \\ y \to \infty \;:\; u_0 \to 0,\;\; u_1 \to 0,\;\; \theta_0 \to 0,\;\; \theta_1 \to 0 \end{array}\right\}. \tag{2.17}$$

Equations (2.13) to (2.16) are second order linear differential equations with constant coefficients, the solutions of which are straightforward. The solutions of (2.13) to (2.16), satisfying the boundary conditions (1.17), are given by

$$u_0 \;=\; B_4(e^{-B_3 y} - e^{-B_1 y}), \tag{2.18}$$

$$u_1 \;=\; D_2 e^{-B_1 y} - D_3 e^{-C_4 y} - D_4 e^{-C_5 y} + D_5 e^{-C_6 y}, \tag{2.19}$$

$$\theta_0 \;=\; e^{-B_1 y}, \tag{2.20}$$

$$\theta_1 \;=\; D_{10} e^{-B_3 y} - D_6 e^{-B_1 y} + D_7 e^{-C_4 y} + D_8 e^{-C_5 y} - D_9 e^{-C_6 y}, \tag{2.21}$$

where

$$B_1 = \frac{Pr + \sqrt{Pr^2 + 4FPr}}{2},\;\; B_2 = \left(-1 + \frac{\sqrt{1+4M^2}}{2}\right),\;\; B_3 = \left(\frac{1+\sqrt{1+4M^2}}{2}\right),$$

$$B_4 = \frac{Gr}{(B_1+B_2)(B_1-B_3)},\;\; C_1 = B_3^2 B_4^2 Pr,\;\; C_2 = B_1^2 B_4^2 Pr,\;\; C_3 = 2B_1 B_3 B_4^2 Pr,$$

$$C_4 = 2B_3,\;\; C_5 = 2B_1,\;\; C_6 = B_1 + B_3,\;\; D_1 = \frac{-Pr + \sqrt{Pr^2 + 4FPr}}{2},$$

$$D_2 = \frac{C_1}{(C_4+D_1)(C_4-B_1)} + \frac{C_2}{(C_5+D_1)(C_5-B_1)} - \frac{C_3}{(C_6+D_1)(C_6-B_1)},$$

$$D_3 = \frac{C_1}{(C_4+D_1)(C_4-B_1)},\;\; D_4 = \frac{C_2}{(C_5+D_1)(C_5-B_1)},\;\; D_5 = \frac{C_3}{(C_6+D_1)(C_6-B_1)},$$

$$D_6 = \frac{GrD_2}{(B_1+B_2)(B_1-B_3)},\;\; D_7 = \frac{GrD_3}{(C_4+B_2)(C_4-B_3)},\;\; D_8 = \frac{GrD_4}{(C_5+B_2)(C_5-B_3)},$$

$$D_9 = \frac{GrD_5}{(C_6+B_2)(C_6-B_3)}, \ D_{10} = D_6 - D_7 - D_8 + D_9.$$

The skin-friction coefficient at the plate is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= [B_4(B_1-B_3) + E(-D_{10}B_3 + D_6B_1 - D_7C_4 - D_8C_9 + D_9C_{10})]. \quad (2.22)$$

The rate of heat transfer in terms of Nusselt number at the plate is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = [B_1 - E(-D_2B_1 + D_3C_4 + D_4C_5 - D_5C_6)]. \quad (2.23)$$

3. Discussion

In order to get physical insight into the problem, the velocity, temperature, skin-friction and rate of heat transfer have been discussed by assigning numerical values of M (Magnetic Parameter), Gr (Grashof number for heat transfer) and Pr (Prandtl number). The values of Pr are taken 0.025 (mercury) and 1.0 (electrolytic solution) at temperature 20^0C and one atmosphere pressure [12]. Grashof number for heat transfer is chosen to be $Gr = 5.0$ corresponding to cooling ($Gr > 0$) of the plate while $Gr = -5.0$ corresponding to heating ($Gr < 0$) of the plate. The values of magnetic parameter $M = 3.0$ and 5.0, radiation parameter $F = 1.0$ and 2.0 and $E = 0.2$ are chosen arbitrarily. The obtained numerical results are illustrated in figures 1 to 3 and table 1 and table 2.

Figure 1 depicts the variation of velocity component for $Gr = 5.0$ (cooling Newtonian fluid) and $Gr = -5.0$ (heating Newtonian fluid). The velocity profiles are drawn for $Pr = 0.025$ (mercury) and 1.0 (electrolytic solution), taking the different values of M and F. It is observed that an increase in M and F leads to decrease in the velocity for both mercury and electrolytic solution for cooling plate, while reverse effect is observed for heating plate. The velocity is greater for mercury than electrolytic solution for both cooling and heating plate.

Figures 2 and 3 represent the temperature profile for $Gr = 5.0$ and $Gr = -5.0$, respectively. It is noticed that the temperature decreases due to increase in M and F for both mercury and electrolytic solution for cooling plate. In case of heating of the plate, a comparison of temperature profile shows that the temperature increases with increase in M, while reverse phenomenon is observed for F for $Pr = 0.025$ and $Pr = 1.0$. The increase of the Prandtl number results in the decrease of temperature distribution. This is due to the fact that there would be a decrease of thermal boundary thickness with the increase of the Prandtl number.

Table 1: Skin-friction Coefficient τ for $E = 0.2$

M	GR	τ			
		$Pr = 0.025$ $F = 1.0$	$Pr = 0.025$ $F = 2.0$	$Pr = 1.0$ $F = 1.0$	$Pr = 1.0$ $F = 2.0$
2	5	2.87	2.77	1.39	1.20
4	5	1.35	1.32	0.96	0.89
6	5	0.87	0.86	0.69	0.66
2	-5	-2.87	-2.77	-1.39	-1.20
4	-5	-1.35	-1.32	-0.96	-0.89
6	-5	-0.87	-0.86	-0.69	-0.66

Table 2: Heat transfer coefficent Nu for $Gr = 5.0$ and $E = 0.2$

M	Nu			
	$Pr = 0.025$ $F = 1.0$	$Pr = 0.025$ $F = 2.0$	$Pr = 1.0$ $F = 1.0$	$Pr = 1.0$ $F = 2.0$
2	0.218	0.280	1.311	2.270
4	0.222	0.285	1.600	2.293
6	0.223	0.286	1.612	2.299

Table (1) gives the values of skin friction coefficient for different values of M, Gr, Pr and F. It is observed that an increase in M leads to decrease in the skin friction coefficient τ for cooling plate while reverse effects observed for heating of the plate. The values of skin friction are more for Mercury ($Pr = 0.025$) than electrolytic solution ($Pr = 1$) for cooling plate. Physically, this is true because the increase in the Prandtl number is due to increase in the viscosity of the fluid, which makes the fluid thick and hence a decrease in the velocity of the fluid. In the case of heating plate, reverse phenomenon is noticed. The values of τ decreases with increasing radiation parameter for cooling plate but increases with increase in radiation parameter for heating of the plate. The data in Table 2 represents rate of heat transfer for different values of Pr, M and F. It is noticed that increase in M and F leads to increase the heat transfer coefficient. The values are less for Mercury ($Pr = 0.025$) than electrolytic solution ($Pr = 1$).

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