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# EFFECTS OF SLIP PARAMETERS ON MAGNETOPOLAR FREE CONVECTION FLOW

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**Abstract :** This paper is concerned with steady free convection flow of an electrically conducting viscous incompressible magneto polar fluid through a porous medium, over a semi-infinite vertical porous plate under slip boundary conditions for velocity and temperature. Using perturbation technique expressions for velocity field, temperature field, skin friction and rate of heat transfer have been obtained. Effects of Grashof number ($G$), permeability parameter ($K$), magnetic parameter ($M$), rarefaction parameters ($h_1$ and $r_1$) and rotational parameters ($\alpha$ and $\lambda$) are discussed in detail and also shown graphically.

## 1. Introduction

It is a known fact that the fluid in geothermal region is electrically conducting. Flow through a porous medium is of great interest to geophysicists and fluid dynamicists. Brinkman [2], Yamamoto [12], Raptis et al. [8,9] have studied flow through a porous medium considering generalized Darcy's law. In all above research papers generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. Aero et al. [1], D'ep [3] derived flow equations with angular velocity. Such fluids are known as polar fluids in the literature. Raptis [7], Jain and Taneja [5], Taneja and Jain [11] have considered magnetic effects on a polar fluid through a porous medium.

In geothermal region situation may arise when slip at the boundary may take place. In such a situation of slip flow, ordinary continuum approach fails to yield satsifactory results. Many authors including Mittal et al. [6], Singh [10] solved problems taking slip conditions at the boundary. Results agreeing with the observed physical phenomena, can be obtained by solving usual equations of motion together with modified boundary conditions.

In the present paper an attempt has been made to study the effects of rotational and slip parameters (Eckert and Drake [4]) on free convection flow of a magnetopolor fluid through a porous medium. The effects of Grashof number ($G$), permeability parameter ($K$), magnetic parameter ($M$), rarefaction parameters ($h_1$ and $r_1$) and rotational parameters ($\alpha$ and $\lambda$), on the velocity field, temperature field, skin friction and the rate of heat transfer are discussed numerically.

## 2. Formulation and Solution of the Problem

We consider two dimensional steady hydromagnetic free convection flow of an electrically conducting polar fluid through a porous medium occupying a semi-infinite region of the space bounded by an infinite vertical porous plate. A magnetic field of uniform strength is applied transversely to the direction of the flow and induced magnetic field is neglected. Taking the $x$-axis along the vertical porous plate in upward direction and $y$-axis normal to it, the equations of the fluid motion which govern the flow are

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = g\beta(T-T_\infty)+(\nu+\nu_r)\frac{\partial^2 u}{\partial y^2}+2\nu_r\frac{\partial \omega}{\partial y}-\frac{\nu}{K}u-\frac{\sigma\, B_0^2}{\rho}\,u \tag{2}$$

$$v\frac{\partial \omega}{\partial y} = \frac{\gamma}{I}\,\frac{\partial^2 \omega}{\partial y^2} \tag{3}$$

$$v\frac{\partial T}{\partial y} = \frac{k}{\rho\, C_p}\,\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p}\,\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

where appropriate boundary conditions are

$$\left.\begin{aligned} u &= L_1\frac{\partial u}{\partial y}\ , \ \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}\ , \ T = T_w+\xi\frac{\partial T}{\partial y} \ \text{ at } \ y=0 \\ u &\to 0 \ , \ \omega \to 0 \ , \ T \to T_\infty \ \text{ as } \ y\to\infty, \end{aligned}\right\} \tag{5}$$

where $\nu_r$ is the kinematic rotational viscosity, $\omega$ the mean angular velocity of rotaion of the particles, $K$ the permeability of the porous medium, $\sigma$ the electrical conductivity, $B_0$ the magnetic induction, $\beta$ the coefficient of volume expansion, $k$ the thermal conductivity, $I$ a scalar constant of dimension equal to that of the moment of inetria of unit mass and

$$\gamma = \frac{C_a+C_d}{I}$$

where $C_a$ and $C_d$ are coefficients of couple stress viscosities. Remaining symbols have their usual meanings.

Above boundary conditions are derived with the assumptions that the couple stress are dominant during the rotation of the particles [D'ep [3]].

The continuity equation (1) gives

$$v = -v_0 \tag{6}$$

We introduce the following non-dimensional quantities

$$u^\star = \frac{u}{v_0}, \quad y^\star = \frac{yv_0}{\nu}, \quad T^\star = \frac{T - T_\infty}{T_w - T_\infty}$$

$K^\star = \dfrac{kv_0^2}{\nu^2}$; permeability parameter

$E = \dfrac{v_0^2}{C_p(T_w - T_\infty)}$; Eckert number

$P = \dfrac{\rho\nu C_p}{k}$; Prandtl number

$M = \left(\dfrac{\sigma B_0^2 \nu}{\rho v_0^2}\right)^{\frac{1}{2}}$; magnetic parameter

$G = \dfrac{\nu g\beta(T_w - T_\infty)}{v_0^3}$; Grashof number

$$\left.\begin{aligned} \alpha &= \frac{\nu_r}{\nu} \\ \lambda &= \frac{I\nu}{\gamma} \\ \omega_1 &= \frac{\omega\nu}{v_0^2} \end{aligned}\right\} ;\ \text{rotational parameters}$$

$h_1 = \dfrac{L_1 v_0}{\nu}$; rarefaction number

$r_1 = \dfrac{\xi v_0}{\nu}$; temperature jump parameter

Substituting the non-dimensional quantities in (2), (3) and (4) these equations in view of (6) after dropping the asterisks over them reduce to

$$(1+\alpha)u'' + u' = -GT - 2\alpha\omega_1' + \left(\frac{1}{K} + M^2\right)u \tag{7}$$

$$\omega'' + \lambda\omega_1' = 0 \tag{8}$$

$$T'' + PT' = -PEu'^2 \tag{9}$$

where primes denote differentiation with respect to $y$.

The boundary conditions (5) become:

$$\left.\begin{array}{llll} u = h_1\left(\dfrac{\partial u}{\partial y}\right) & , \dfrac{\partial \omega_1}{\partial y} = -\dfrac{\partial^2 u}{\partial y^2} & , T = 1 + r_1\left(\dfrac{\partial T}{\partial y}\right) & \text{at } y = 0 \\ u \to 0 & , \omega_1 \to 0 & , T \to 0 \text{ as } y \to \infty & \end{array}\right\} \tag{10}$$

The solution of (8) under the boundary conditions (10) is

$$\omega_1 = A_1 e^{-\lambda y} \tag{11}$$

In order to solve the system of equations (8) and (10) which are coupled and nonlinear, we expand $u$ and $T$ into the power of Eckert number $E$, which is for incompressible fluid $<< 1$. We then have

$$\left.\begin{array}{l} u = u_0 + Eu_1 + O(E^2) \\ T = T_0 + ET_1 + O(E^2) \end{array}\right\} \tag{12}$$

Substituting (12) in (7) and (9) and collecting the coefficients of like powers of $E$, we get the following equations:

$$(1+\alpha)u_0'' + u_0' = -GT_0 - 2\alpha\omega_1' + \left(\frac{1}{K} + M^2\right)u_0 \tag{13}$$

$$(1+\alpha)u_1'' + u_1' = -GT_1 + \left(\frac{1}{K} + M^2\right)u_1 \tag{14}$$

$$T_0'' + PT_0' = 0 \tag{15}$$

$$T_1'' + PT_1' = -Pu_0'^2 \tag{16}$$

while the boundary conditions become

$$\left.\begin{array}{lllll} u_0 = h_1\left(\dfrac{\partial u_0}{\partial y}\right), & u_1 = h_1\left(\dfrac{\partial u_1}{\partial y}\right), & T_0 = 1 + r_1\left(\dfrac{\partial T_0}{\partial y}\right), & T_1 = r_1\left(\dfrac{\partial T_1}{\partial y}\right) & \text{at } y = 0 \\ u_0 \to 0, & u_1 \to 0, & T_0 \to 0, & T_1 \to 0 & \text{as } y \to \infty \end{array}\right\} \tag{17}$$

The solution of equations (13) to (16) under the boundary conditions (17) are given by

$$u = (L_1+A_1L_2)e^{R_1y}+L_3e^{-Py}+L_4A_1e^{-\lambda y}+E\Big\{L_{16}e^{2R_1y}+L_{17}e^{-2Py}+L_{18}e^{-2\lambda y}$$
$$+L_{19}e^{-(P+\lambda)y}+L_{20}e^{(R_1-P)y}+L_{21}e^{(R_1-\lambda)y}+L_{22}e^{-Py}+L_{23}e^{R_1y}\Big\} \quad (18)$$

$$T = \frac{e^{-Py}}{(1+r_1P)}+E\Big\{(L_5+L_6+L_7)e^{2R_1y}+L_8e^{-2Py}+L_9e^{-2\lambda y}+L_{10}e^{-(P+\lambda)y}$$
$$+(L_{11}+L_{13})e^{(R_1-P)y}+(L_{12}+L_{14})e^{(R_1-\lambda)y}+L_{15}e^{-Py}\Big\} \quad (19)$$

where the constants $L_i$; $i = 1, 2, \cdots 23$, $R_j$, $j = 1, 2$ and $A_1$ are defined in Appendix.

When we put $r_1 = 0$, i.e. no jump in tempertaure, equations (18) and (19) reduce to Taneja and Jain [11]. From equation (18) we can calculate the dimensionless skin friction as:

$$\tau = \frac{\tau_w}{\rho v_0^2} = (1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau = (1+\alpha)[(L_1+AL_2)R_1-L_3P-L_4A_1\lambda+E\{2L_{16}R_1-2L_{17}P-2L_{18}\lambda$$
$$-L_{19}(P+\lambda)+L_{20}(R_1-P)+L_{21}(R_1-\lambda)-L_{22}P+L_{23}R_1\}] \quad (20)$$

From equation (19) we can calculate the rate of heat transfer i.e. Nusselt number

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$N_u = \frac{P}{(1+r_1P)}+E\{-(L_5+L_6+L_7)2R_1+2L_8P+2L_9\lambda+L_{10}(P+\lambda)$$
$$-(L_{11}+L_{13})(R_1-P)-(L_{12}+L_{14})(R_1-\lambda)+L_{15}P\} \quad (21)$$

## 3. Discussion and Conclusion

In order to understand the solution physically, we have calculated the numerical values of velocity distribution, temperature distribution, skin friction and the rate of heat transfer at the plate for differnt values of $K$ (permeability parameter), $M$ (magnetic parameter), $G$ (Grashof number), $h_1$ and $r_1$ (rarefaction parameters), $\alpha$ and $\lambda$ (rotational parameters).

In fig. 1 the velocity distribution is plotted against $y$ for fixed values of $E = 0.01$ and $P = 7.0$. It is being observed that when $K, \alpha$ and $G$ are increased rate of heat transfer is increased but the phenomena reverses for the case of $M$ and $r_1$. It is interesting to note that when $h_1$ increases velocity increases for small values of $y$ and for large values of $y$ velocity decreases. It is also being noted that when there is no slip in velocity but there is jump in temperature then the velocity is increased (curves 9 and 10).

In fig. 2 the temperature is plotted against $y$ for fixed values of $E = 0.01$ and $P = 7.0$. It is being observed that when $h_1,\ M,\ \lambda$ and $r_1$ are increased temperature is decreased but the temperature is increased with the increase in the values of $K,\ \alpha$ and $G$. It is also being observed that if there is no slip in velocity but there is tempertaure jump, the temperature is decreased.

In fig. 3 the skin friction is plotted against $K$ for fixed values of $E = 0.01$, $P = 7.0$ and $G = 5.0$. It is being observed that except $\alpha$ if there is increase in $h_1,\ M,\ \lambda$ and $r_1$ skin friction is increased at the plate. It is further observed that jump in temperature decreases the skin friction when there is no slip in velocity (curves 7 and 8).

In fig. 4 the rate of heat transfer is plotted against $K$ for fixed values of $E = 0.01$, $P = 7.0$ and $G = 5.0$. It is being observed that when $h_1,\ M$ and $\lambda$ are increased rate of heat transfer is increased but the phenomena is reversed for the case of $\alpha$ and $r_1$. Here it is interesting to note that the Nusselt number increases when there is no jump in temperature (curves 7 and 8).

## APPENDIX

$$R_1 = \frac{-1 - \sqrt{1 + 4(1+\alpha)\left(\dfrac{1}{K} + M^2\right)}}{2(1+\alpha)}, \quad R_2 = \frac{-1 + \sqrt{1 + 4(1+\alpha)\left(\dfrac{1}{K} + M^2\right)}}{2(1+\alpha)}$$

$$A_1 = \frac{L_1 R^2 + L_3 P^2}{(\lambda - L_2 R_1^2 - L_4 \lambda^2)}, \quad L_1 = \frac{G(1 + h_1 P)}{(1 - h_1 R_1)(1 + r_1 P)(P + R_1)(P + R_2)},$$

$$L_2 = \frac{-2\alpha\lambda(1 + h_1\lambda)}{(1 - h_1 R_1)(\lambda + R_1)(\lambda + R_2)}, \quad L_3 = \frac{-G}{(1 + r_1 P)(P + R_1)(P + R_2)},$$

$$L_4 = \frac{2\alpha\lambda}{(\lambda + R_1)(\lambda + R_2)}, \quad L_5 = \frac{-L_1^2 R_1 P}{2(2R_1 + P)}, \quad L_6 = \frac{-A_1^2 L_2^2 R_1 P}{2(2R_1 + P)}, \quad L_7 = \frac{-2L_1 L_2 A_1 P R_1}{2(2R_1 + P)},$$

$$L_8 = -\frac{L_3^2 P}{2}, \quad L_9 = \frac{-L_4^2 A_1^2 \lambda P}{2(2\lambda - P)}, \quad L_{10} = \frac{2L_3 L_4 P^2 A_1}{(\lambda + P)}, \quad L_{11} = \frac{2L_1 L_3 P^2}{(R_1 - P)},$$

$$L_{12} = \frac{2L_1 L_4 A_1 \lambda P R_1}{(R_1 - \lambda)(R_1 - \lambda + P)}, \quad L_{13} = \frac{2L_1 L_3 P^2 A_1}{(R_1 - P)}, \quad L_{14} = \frac{2A_1^2 L_2 L_4 \lambda P R_1}{(R_1 - \lambda)(R_1 - \lambda + P)},$$

$$L_{15} = \frac{1}{(1 + r_1 P)}[(2R_1 r_1 - 1)(L_5 + L_6 + L_7) + (-2Pr_1 - 1)L_8 + (-2\lambda r_1 - 1)L_9$$
$$+(-Pr_1 - \lambda r_1 - 1)L_{10} + (R_1 r_1 - Pr_1 - 1)(L_{11} + L_{13}) + (R_1 r_1 - \lambda r_1 - 1)(L_{12} + L_{14})],$$

$$L_{16} = \frac{-G(L_5 + L_6 + L_7)}{R_1(2R_1 - R_2)}, \quad L_{17} = \frac{-GL_8}{(2P + R_1)(2P + R_2)}, \quad L_{18} = \frac{-GL_9}{(2\lambda + R_1)(2\lambda + R_2)},$$

$$L_{19} = \frac{-GL_{10}}{(P + \lambda + R_1)(P + \lambda + R_2)}, \quad L_{20} = \frac{-G(L_{11} + L_{13})}{(-P)(R_1 - P - R_2)}, \quad L_{21} = \frac{-G(L_{12} + L_{14})}{(-\lambda)(R_1 - \lambda - R_2)},$$

$$L_{22} = \frac{-GL_{15}}{(P + R_1)(P + R_2)},$$

$$L_{23} = \frac{1}{(1 - h_1 R_1)}[(2R_1 h_1 - 1)L_{16} + (-2Ph_1 - 1)L_{17} + (-2\lambda h_1 - 1)L_{18} + (-Ph_1 - \lambda h_1 - 1)L_{19},$$
$$+(R_1 h_1 - Ph_1 - 1)L_{20} + (R_1 h_1 - \lambda h_1 - 1)L_{21} + (-Ph_1 - 1)L_{22}].$$

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