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Relation Between the Orlicz Space of χ_M^π and $\chi_M^\pi(a)$

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Abstract : This paper is developed to the study on the general properties of the relation between the orlicz space of χ_M^π and $\chi_M^\pi(a)$.

Key words : Gai sequence, analytic sequence, rate sequence, modulus function, semi norm, difference sequence

1. Introduction :

Let ω denote the set of all real or complex sequences $x = (x_k)$ and $M:[0,\infty) \to [0,\infty)$ be an orlicz function (or) a modulus function.

An orlicz function is continuous, non-decreasing and convex with
$M(0) = 0,\ M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$

Nakano introduced "Modulus function" if the convexity of orlicz function is replaced by
$M(x+y) \leq M(x) + M(y)$

Lindenstraus and Tzafari used the idea of orlicz function to orlicz sequence space

$$\ell_M = \left\{ x \in \omega; \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}$$

The space ℓ_M with the norm,

$$\|x\| = \inf\left\{ \rho > 0; \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}$$

1. Definition :

Let ℓ_∞, c, c_o be the sequence spaces of bounded ,convergent and null sequence $x = (x_k)$ respectively. In respect of ℓ_∞, c, c_o we have $\|x\| = \sup_k |x_k|$, where $x = (x_k) \in c_0 < c < \ell_\infty$.

2.1 Definition :

A sequence $x=(x_k)$ is said to be analytic if $\sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequence will be denoted by Λ.

2.2 Definition :

A sequence $x=(x_k)$ is called entire sequence if $\lim_{k\to\infty} |x_k|^{1/k} = 0$.The vector space of all entire sequence will be denoted by Γ.

2.3 Definition :

Let χ be the set of all sequences $x=\{x_k\}$ such that $(k!=|x_k|)^{\frac{1}{k}} \to 0$ as $k\to\infty$. The metric d on χ is defined by

$$d(x,y)=\sup_k \left\{(k!|x_k - y_k|)^{1/k}; k=1,2,3,\dots\dots\right\}$$

2.4 Definition :

A semi norm p, on a linear space x , is a function $p: X\to R$ such that

(1) $p(\lambda x)=|\lambda| p(x)$

(2) $p(x+y)\le p(x+y)$

Property 1 is called absolute homogeneity of p , and property 2 is called subadditivity of p.

(1) Thus , a seminorm is a real, subadditivity , absolute homogeneous function on χ .
Moreover by (1) and (2)
$0= p(\theta)\le p(x)+p(-x)=2p(x)$, where p is always non-negative.

2.5 Definition :

Frechet space can be defined in two ways. The first employs a translation – invariant metric and the second a countable family of seminorms.

A topological vector space x is a Frechet space if and only if it satisfies the following three properties:

I. It is locally convex.

II. Its topological can be induced by a translation –invariant metric (ie) a metric $d: X x X\to R$ Such that $d(x,y)=(x+a, y+a)$ for all a $x, y\in x$.This means that a subset U of X is open if and only if for every u in U there exists an $\in>0$. such that $\{v; d(u,v)<\in\}$ is a subsetof U

III. Any translation –invariant metric inducting the topology is complete, is other words , is a complete topological vector space.

2.6 Definition :

The orlicz space of χ^{π} is denoted as χ_M^{π} and defined as

$$\chi_M^{\pi} = \left\{ x \in \omega; \lim_{k \to \infty} \left[M\left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right) \right] > 0 \text{ for some } \rho > 0 \right\}$$

The space χ_M^{π} is a metric space with the metric,

$$d(x, y) = \inf\left\{ \rho > 0; \sup_k \left[M\left(\frac{k!}{\rho} \left| \frac{x_k - y_k}{\pi_k} \right|^{1/k} \right) \right] \le 1 \right\}]$$

2.7 Definition :

The Orlicz space of $\chi^{\pi}(p)$ is denoted by $\chi_M^{\pi}(p)$ and defined as,

$$\chi_M^{\pi}(p) = \left\{ x = (x_k); \lim_{k \to \infty} \left[M\left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right)^{p_k} \right] = 0, \text{ for some } \rho > 0 \right\}$$

The metric of the space is defined as,

$$d(x, y) = \inf\left\{ \rho > 0; \sup_k \left[M\left\{ \left(\frac{k!}{\rho} \left| \frac{x_k - y_k}{\pi_k} \right|^{1/k} \right)^{p_k} \right\} \right] \le 1 \right\}$$

Suppose if p_k is constant, then $\chi_M^{\pi}(p) = \chi_M^{\pi}$

3.1 Theorem :

Let $0 \le a_k \le b_k$ and let $\left\{ \frac{b_k}{a_k} \right\}$ be bounded .Then $\chi_M^{\pi}(b) = \chi_M^{\pi}(a)$

Proof: Let

$x \in \chi_M^{\pi}(b)$

$$\Rightarrow \lim_{k \to \infty} \left(M\left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right) \right)^{b_k} = 0 \to (A)$$

Here $0 \le a_k \le b_k \Rightarrow 0 \le \frac{a_k}{b_k} \le 1$

Since M is non-decreasing function,

$$\left[M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)^{b_k}\right]^{\frac{a_k}{b_k}} \le M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)^{b_k} \Rightarrow M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)^{a_k} \le M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)^{b_k}$$

By A, R.H.S tends to zero . Hence

$$\text{L.H.S, } M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)^{a_k} = 0$$

Which implies $x \in \chi_M^{\pi}(a)$ $\therefore$ $\chi_M^{\pi}(b) \subset \chi_M^{\pi}(a)$

3.2 Theorem :

If $0 < \inf a_k \le a_k \le 1$, then $\chi_M^{\pi}(a) \subset \chi_M^{\pi}$.

Proof:

Let

$x \in \chi_M^{\pi}(a)$

$$\Rightarrow \lim_{k\to\infty}\left(M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)\right)^{a_k} = 0 \;\; \to (B)$$

Given If $0 < \inf a_k \le a_k \le 1$,

$$\Rightarrow M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right) \le \left[M\left(\frac{k!}{\rho}\left|\frac{x_k}{\pi_k}\right|^{1/k}\right)\right]^{a_k}$$

As $k \to \infty$, by B ,R.H.S tends to zero, then

$$\text{LHS, } \lim_{k\to\infty} M\left(\frac{k!}{\rho}|x_k|^{1/k}\right) = 0.$$

Which implies, $x \in \chi_M^{\pi}$. $\therefore \chi_M^{\pi}(a) \subset \chi_M^{\pi}$.

3.3 Theorem :

If $1 \le a_k \le \sup_{a_k} \le \infty$ then $\chi_M^{\pi} \subset \chi_m^{\pi}(a)$

Proof:

Let

$x \in \chi_M^{\pi}$

$$\Rightarrow \lim_{k \to \infty} \left(M \left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right) \right) = 0 \rightarrow (C)$$

Given $1 \leq a_k \leq \sup_{a_k} \leq \infty$, we have

$$\Rightarrow M \left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right)^{a_k} \leq \left[M \left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right) \right]$$

By C, as $k \rightarrow \infty$, R.H.S $\rightarrow o$, then

$$\text{L.H.S} \lim_{k \to \infty} \left[M \left(\frac{k!}{\rho} \left| \frac{x_k}{\pi_k} \right|^{1/k} \right) \right]^{a_k} = 0$$

$$\Rightarrow x \in \chi_M^{\pi}(a)$$

$$\chi_M^{\pi} \subset \chi_M^{\pi}(a)$$

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