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Solution of Telegraph Equation by Using Double Mahgoub Transform

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Abstract : In this paper, we apply Double Mahgoub transform to solve the general linear telegraph equation. The applicability of this new transform is demonstrated using some functions.

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Key Words : Telegraph equation; Double Mahgoub Transform

1 Introduction :

A lot of problems have been solved by integral transforms such as Laplace [1], Fourier, Mellin, and Sumudu [2, 3], Elzaki and Aboodh.

Also these problems have been solved by differential transform method [4-11]. In this paper we derive, we believe for the first time and solve telegraph equations by using Double Mahgoub transform.

1.1 Mahgoub Transform

Definition 1.1.1 Let function f(t) defined for $t \geq 0$ then Mahgoub transform of f(t) is the function H defined as follows:

$$M[f(t)] = H(v) = v\int_0^\infty f(t)e^{-vt}dt, t \geq 0$$

1.2 Double Mahgoub transform

Definition 1.2.1 Let f(x,t), where $x, t \in R^{+}$ be a function, which can be expressed as a convergent infinite series then, its double Mahgoub transform given by:

$$M_2[f(x,t),u,v] = H(u,v) = uv\int_0^\infty\int_0^\infty f(x,t)e^{-(ux+vt)}dxdt, x,t \geq 0,$$

where, u and v are complex values.

1.3 Theorem

Double Mahgoub Transform of First and Second Order partial derivatives are in the form :

$$(i)M_2\left(\frac{\partial f}{\partial x}\right) = uH(u,v) - uH(0,v)$$

$$(ii)M_2\left(\frac{\partial^2 f}{\partial x^2}\right) = u^2H(u,v) - u^2H(0,v) - u\frac{\partial[H(0,v)]}{\partial x}$$

$$(iii)M_2\left(\frac{\partial f}{\partial t}\right) = vH(u,v) - vH(u,0)$$

$$(iv)M_2\left(\frac{\partial^2 f}{\partial t^2}\right) = v^2H(u,v) - v^2H(u,0) - v\frac{\partial[H(u,0)]}{\partial t}$$

$$(v)M_2\left(\frac{\partial^2 f}{\partial x\partial t}\right) = uvf(0,0) - uvH(u,0) + uvH(u,v) - uvH(0,v)$$

2 Applications:

In this section we establish the validity of the double Mahgoub transform by applying it to solve the general linear telegraph equations. To solve partial differential equations by double Mahgoub transform, we need the following steps.

(i) Take the double Mahgoub transform of partial differential equations.

(ii) Take the single Mahgoub transform of the conditions.

(iii) Substitute (ii) in (i) and solve the algebraic equation.

(iv) Take the double inverse of Mahgoub transform to get the solution

Here we need the main equation:

$$M_2\left(e^{(ax+bt)}\right) = \frac{uv}{(u-a)(v-b)}$$

2.1 Example

Consider the general linear telegraph equation in the form

$$c^2 u_{xx} = u_{tt} + a u_t + b u \tag{2.1}$$

with the boundary conditions

$$u(0,t) = f_1(t), \quad u_x(0,t) = g_1(t) \tag{2.2}$$

and the initial conditions

$$u(x,0) = f_2(x), \quad u_t(x,0) = g_2(x) \tag{2.3}$$

Solution Take the Double Mahgoub Transform of (2.1), then we have

$$c^2 \left[u^2 H(u,v) - u^2 H(0,V) - u \frac{\partial [H(0,v)]}{\partial x} \right] = v^2 H(u,v) - v^2 H(u,0)$$
$$-v \frac{\partial [H(u,0)]}{\partial t} + a\left[vH(u,v) - vH(u,0) \right] + bH(u,v) \tag{2.4}$$

Taking single Mahgoub transform of conditions (2.2),(2.3)

$$H(0,v) = F_1(v), \quad H_x(0,v) = G_1(v), \quad H(u,0) = F_2(u), \; H_t(u,0) = G_2(u)$$

$$c^2 u^2 H(u,v) - c^2 u^2 F_1(v) - uG_1(v) = v^2 H(u,v) - v^2 F_2(u) - vG_2(u)$$
$$+avH(u,v) - avF_2(u) + bH(u,v)$$
$$\left[c^2u^2 - v^2 - av - b \right] H(u,v) = c^2 u^2 F_1(v) + uG_1(v) - v^2 F_2(u)$$
$$-vG_2(u) - avF_2(u)$$
$$\Rightarrow H(u,v) = \frac{c^2 u^2 F_1(v) + uG_1(v) - v^2 F_2(u) - vG_2(u) - avF_2(u)}{[c^2u^2 - v^2 - av - b]}$$

Take double inverse Mahgoub transform to obtain the solution of general linear telegraph equation (2.1) in the form

$$u(x,t) = M_2^{-1}\left[H(u,v) \right] = H(x,t)$$

2.2 Example

Consider the telegraph Equation,

$$u_{xx} = u_{tt} + u_t + u \tag{2.5}$$

with initial conditions

$$u(x,0) = e^x, \quad u_t(x,0) = -e^x \tag{2.6}$$

and boundary condition

$$u(0,t)=e^{-t},\quad u_x(0,t)=e^{-t} \tag{2.7}$$

Solution Take the Double Mahgoub Transform of (2.5)

$$u^2H(u,v)-u^2H(0,v)-u\frac{\partial[H(0,v)]}{\partial x}=v^2H(u,v)-v^2H(u,0)$$
$$-v\frac{\partial[H(u,0)]}{\partial t}+vH(u,v)-vH(u,0)+H(u,v) \tag{2.8}$$

Taking single Mahgoub Transform of (2.6) and (2.7)

$$H(u,0)=\frac{u}{u-1},\quad H_t(u,0)=\frac{-u}{u-1},\quad H(0,v)=\frac{v}{v+1},\quad H_x(0,v)=\frac{v}{v+1}$$

The equation (2.8) becomes,

$$u^2H(u,v)-u^2\frac{v}{v+1}-u\frac{v}{v+1}=v^2H(u,v)-v^2\frac{u}{u-1}$$
$$+v\frac{u}{u-1}+vH(u,v)-v\frac{u}{u-1}+H(u,v)$$
$$\Rightarrow\left(u^2-v^2-v-1\right)H(u,v)=u^2\frac{v}{v+1}+u\frac{v}{v+1}-v^2\frac{u}{u-1}$$
$$\Rightarrow\left(u^2-v^2-v-1\right)H(u,v)=\frac{(u-1)(u^2v+uv)-(v+1)uv^2}{(v+1)(u-1)}$$
$$\Rightarrow H(u,v)=\frac{uv}{(v+1)(u-1)}$$

Take double inverse Mahgoub transform to obtain the solution of
linear telegraph equation (2.5)

$$u(x,t)=e^{x}e^{-t}=e^{x-t}$$

3 Conclusion

In this work, double Mahgoub transform is applied to obtain the solution of general linear Telegraph Equation. It may be concluded that double Mahgoub transform is very powerful and efficient in finding the analytical solution for a wide class of partial differential equations.

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