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CERTAIN CLASSES GENERATING FUNCTIONS ASSOCIATED WITH THE ALEPH-FUNCTION OF SEVERAL VARIABLES II

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Abstract: In this paper, we present two new generating functions involving multivariable Aleph-function, the I-function of several variables and Aleph-function of two variables. The mains results of our document are quite general in nature and capable of yielding a very large number of generating functions involving polynomials and various special functions occurring in the problem of mathematical analysis and mathematical physics and mechanics.

Keywords and Phrases: Generalized multivariable Aleph-function, Aleph-function of two variables, generating functions, multivariable I-function.

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1. Introduction and preliminaries

The Aleph-function of several variables is an extension of the multivariable I-function defined by C.K. Sharma and Ahmad [4], itself is an a generalisation of G and H-functions of multiple variables defined by Srivastava et al [6]. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have,

$$\aleph(z_1,...,z_r) = \aleph^{0,n:m_1,n_1,...,m_r.n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};...;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}$$

$$\left(\begin{array}{c|c} z_1 & [(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,n}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},...,\alpha_{ji}^{(r)})_{n+1,p_i}]:[(c_j^{(1)},\gamma_j^{(1)})_{1,n_1}], \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & ....[\tau_i(b_{ji};\beta_{ji}^{(1)},...,\beta_{ji}^{(r)})_{m+1,q_i}]:[(d_j^{(1)},\delta_j^{(1)})_{1,m_1}], \end{array}\right.$$

$$[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}})_{n_1+1,p_i^{(1)}}];...;;[(c_j^{(r)},\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}})_{n_r+1,p_i^{(r)}}]$$

$$[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}})_{m_1+1,q_i^{(1)}}];...;;[(d_j^{(r)},\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}})_{m_r+1,q_i^{(r)}}]\Bigg)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\psi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}ds_1...ds_r \tag{1.1}$$

with $\omega=\sqrt{-1}$
For more details, see Ayant [1,2].
The reals numbers τ_i are positives for $i=1,...,R,\tau_i(k)$, are positives for $i^{(k)}=1,...,R^{(k)}$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|\arg z_k|<\frac{1}{2}A_i^{(k)}\pi,\ \text{where } A_i^{(k)}=\sum_{j=1}^{n}\alpha_j^{(k)}-\tau_i\sum_{j=n+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-$$

$$\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_i(k)}\gamma_{ji^{(k)}}^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_i(k)}\delta_{ji^{(k)}}^{(k)}>0, \tag{1.2}$$

with $k=1,...,r,i=1,...,R$ an $i^{(k)}=1,...,R^{(k)}$.
The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the asymptotic expansion in the following convenient form:

$$\aleph(z_1,...,z_r)=0(|z_1|^{\alpha_1},...,|z_r|^{\alpha_r}),max(|z_1|,...,|z_r|)\to 0$$

$$\aleph(z_1,...,z_r)=0(|z_1|^{\beta_1},...,|z_r|^{\beta_r}),min(|z_1|,...,|z_r|)\to \infty$$

where $k=1,...,r;\alpha_k=\min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,...,m_k$ and $\beta_k=\max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,...,n_k$.
We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R;V=m_1,n_1;...;m_r,n_r \tag{1.3}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},...,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.4}$$

$$A=\left\{(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)})_{1,n}\right\},\left\{\tau_i(a_{ji};\alpha_{ji}^{(1)},...,\alpha_{ji}^{(r)})_{n+1,p_i}\right\} \tag{1.5}$$

$$B=\left\{\tau_i(b_{ji};\beta_{ji}^{(1)},...,\beta_{ji}^{(r)})_{m+1,q_i}\right\} \tag{1.6}$$

$$C = \left\{ (c_j^{(1)}; \gamma_j^{(1)})_{1,n_1} \right\}, \tau_{i^{(1)}} (c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}} \right\},$$

$$..., \left\{ (c_j^{(r)}; \gamma_j^{(r)})_{1,n_r} \right\}, \tau_{i^{(r)}} (c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}} \right\} \tag{1.7}$$

$$D = \left\{ (d_j^{(1)}; \delta_j^{(1)})_{1,m_1} \right\}, \tau_{i^{(1)}} (d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}} \right\},$$

$$..., \left\{ (d_j^{(r)}; \delta_j^{(r)})_{1,m_r} \right\}, \tau_{i^{(r)}} (d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}} \right\} \tag{1.8}$$

The multivariable Aleph-function write

$$\aleph(z_1, ..., z_r) = \aleph_{U:W}^{0,n:V} \left(\begin{array}{c|c} z_1 & A : C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B : D \\ z_r & \end{array} \right) \tag{1.9}$$

2. Required result

Now using the following combinatorial identity, see Raina [3].

$$\sum_{k=0}^{\infty} \binom{\mu+k-1}{k}^{-1} \binom{\lambda+k-1}{k} \binom{\alpha+k-1}{k}$$

$$\times {}_2F_1[\lambda+k, \mu-\alpha; \mu+k; z] = (1-z)^{-\lambda} \tag{2.1}$$

with $|z| < 1$.

3. Main Results

Let $\Omega_a(z_1, ..., z_s)$ an identically nonvanishing function of s complex variables and of complex order a

Theorem 1. Let

$$\gamma_{a,\rho,m,\sigma,\lambda}^{(1)}[y_1, ..., y_r; z_1, ..., z_s; t] = \sum_{k=0}^{\infty} \frac{a_k \Omega_{a+\rho k}(z_1, ..., z_s) t^k}{(mk)!}$$

$$\aleph_{U_{10}:W}^{0,n+1:V} \left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk - mk; \epsilon_1, ..., \epsilon_r), A : C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B : D \\ y_r & \end{array} \right) \tag{3.1}$$

and

$$R_{n,m}^{a,\rho,\lambda,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}B_{k,n,m}^{\lambda,\sigma,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;t]$$

$$\begin{pmatrix}\mu+n+\sigma mk-1\\ n-mk\end{pmatrix}^{-1}\begin{pmatrix}\alpha+n+\sigma mk-1\\ n-mk\end{pmatrix}\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!(n-mk)!}\eta^k \tag{3.2}$$

where

$$B_{k,n,m}^{\lambda,\sigma,\mu,\alpha}(y_1,...,y_r;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$\aleph_{U_{10}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-n-l;\epsilon_1,...,\epsilon_r),A:C\\ \cdot & \cdots\\ \cdot & \\ \cdot & \\ y_r & B:D\end{array}\right) \tag{3.3}$$

then

$$\sum_{n=0}^{\infty}R_{n,m}^{a,\rho,\lambda,\sigma,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma_{a,\rho,m,\sigma,\lambda}^{(1)}\left[\frac{y_1}{(1-t)^{\epsilon_1}},...,\frac{y_r}{(1-t)^{\epsilon_r}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \tag{3.4}$$

where $U_{10}=p_i+1,q_i,\tau_i;R$.

Theorem 2. Let

$$\gamma_{a,\rho,m,\sigma,\lambda}^{(2)}[y_1,...,y_r;z_1,...,z_s;t]=\sum_{k=0}^{\infty}(-)^{mk}a_k\Omega_{a+\rho k}(z_1,...,z_s)t^k$$

$$\aleph_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-mk-\omega k-\sigma mk;\epsilon_1,...,\epsilon_r),A:C\\ \cdot & \cdots\\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,...\epsilon_r),B:D\\ y_r & \end{array}\right) \tag{3.5}$$

and

$$V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;t]$$

$$\begin{pmatrix}\mu+n+\sigma mk-1\\ n-mk\end{pmatrix}^{-1}\begin{pmatrix}\alpha+n+\sigma mk-1\\ n-mk\end{pmatrix}(-)^{mk}a_k\Omega(z_1,...,z_s)\eta^k \tag{3.6}$$

where

$$F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}(y_1,...,y_r;t) = \sum_{l=0}^{\infty} \frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$\aleph_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-n-\sigma mk-\omega k-l;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,...,\epsilon_r),B:D \\ y_r & \end{array}\right) \tag{3.7}$$

then

$$\sum_{n=0}^{\infty} V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]t^n$$

$$= (1-t)^{-\lambda}\gamma_{m,a,\rho,\lambda,\sigma,\omega}^{(2)}\left[\frac{y_1}{(1-t)^{\epsilon_1}},...,\frac{y_r}{(1-t)^{\epsilon_r}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right] \tag{3.8}$$

where $U_{11} = p_i+1, q_i, \tau_i; R$.

Proof of Theorem 1. Let

$$m\{\} = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)\{\} \tag{3.9}$$

We denote the left hand side of the the assertion (3.4) of theorem 1 by $P(y_1,...,y_r$ $;z_1,...,z_s,t)$ then use the definitions (3.2) and (3.3), we have

$$P(y_1,...,y_r;z_1,...,z_s,t) = \sum_{n,l=0}^{\infty}\sum_{k=0}^{[n/m]}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!(n-mk)!}\eta^k$$

$$\aleph_{U_{10}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-n-l;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B:D \\ y_r & \end{array}\right)t^n \tag{3.10}$$

Now using the definition of multivariable Aleph-function from (1.1) and changing the order of summation and integration and then on making series rearrangement therein, it takes following form,

$$P(y_1,...,y_r;z_1,...,z_s,t) = M\left[\sum_{n,l,k=0}^{\infty} \frac{(\mu-\alpha)_l t^l}{(\mu+n+mk+\sigma mk)_l l!} \frac{a_k \Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!n!}\right.$$

$$\left(\begin{array}{c}\mu+n+mk+\sigma mk-1\\ n\end{array}\right)^{-1}\left(\begin{array}{c}\alpha+n+\sigma mk-1\\ n\end{array}\right)$$

$$\Gamma(\lambda+n+mk+\sigma nk+l+\sum_{i=1}^{r}\epsilon_i s_i)\eta^k t^{n+mk}]ds_1,...,ds_r \tag{3.11}$$

Now in view of the relation

$$\frac{\Gamma(\rho+n+l)}{n!} = (\rho+n)_i \left(\begin{array}{c}\rho+n-1\\ n\end{array}\right)\Gamma(\rho) \tag{3.12}$$

and then interpreting the inner series into Gauss' hypergeometric function ${}_2F_1$,we have

$$P(y_1,...,y_r;z_1,...,z_s,t) = M\left[\sum_{k=0}^{\infty}\left\{\sum_{n=0}^{\infty}\left(\begin{array}{c}\mu+n+mk+\sigma mk+\sum_{i=1}^{r} s_i\epsilon_i-1\\ n\end{array}\right)\right.\right.$$

$$\left(\begin{array}{c}\mu+n+mk+\sigma mk-1\\ n\end{array}\right)^{-1}\left(\begin{array}{c}\alpha+n+mk+\sigma mk-1\\ n\end{array}\right)$$

$${}_2F_1\left[\lambda+n+mk+\sum_{i=1}^{r}\epsilon_i s_i, \mu-\alpha; \mu+n+mk; t\right] t^n\left.\right\} \frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!n!}$$

$$\left.\Gamma(\lambda+mk+\sigma km+\sum_{i=1}^{r}s_i\epsilon_i)\right] ds_1...ds_r \tag{3.13}$$

Now using the combinatorial identity (2.1) and then interpreting the resulting contour integral into the multivariable Aleph-function with the help of (1.1), we obtain the desired result (3.1).
Similarly the proof of the theorem 2 use the same method to that theorem 1.

4. Multivariable I-function

If $\tau_i, \tau_{i(1)}, ..., \tau_{i(r)} \to 1$, the Aleph-function of several variables reduces to the I-function of several variables. The two generating relationships involving multivariable I-function defined by Sharma et al [4] have been derived in this section.

Corollary 1. Let

$$\gamma^{(1)}_{a,\rho,m,\sigma,\lambda}[y_1,...,y_r;z_1,...,z_s;t]=\sum_{k=0}^{\infty}\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)t^k}{(mk)!}$$

$$I^{0,n+1:V}_{U_{10}:W}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-mk;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B:D \\ y_r & \end{array}\right) \tag{4.1}$$

and

$$R^{a,\rho,\lambda,\mu,\alpha}_{n,m}[y_1,...,y_r;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}B^{\lambda,\sigma,\mu,\alpha}_{k,n,m}[y_1,...,y_r;z_1,...,z_s;t]$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!(n-mk)!}\eta^k \tag{4.2}$$

where

$$B^{\lambda,\sigma,\mu,\alpha}_{k,n,m}(y_1,...,y_r;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$I^{0,n+1:V}_{U_{10}:W}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-n-l;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B:D \\ y_r & \end{array}\right) \tag{4.3}$$

then

$$\sum_{n=0}^{\infty}R^{a,\rho,\lambda,\sigma,\mu,\alpha}_{n,m}[y_1,...,y_r;z_1,...,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma^{(1)}_{a,\rho,m,\sigma,\lambda}\left[\frac{y_1}{(1-t)^{\epsilon_1}},...,\frac{y_r}{(1-t)^{\epsilon_r}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \tag{4.4}$$

under the same notations and conditions that theorem 1.

Corollary 2. Let

$$\gamma^{(2)}_{a,\rho,m,\sigma,\lambda}[y_1,...,y_r;z_1,...,z_s;t]=\sum_{k=0}^{\infty}(-)^{mk}a_k\Omega_{a+\rho k}(z_1,...,z_s)t^k$$

$$I_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-mk-\omega k-\sigma mk;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,...\epsilon_r),B:D \\ y_r & \end{array}\right) \tag{4.5}$$

and

$$V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;t]$$

$$\binom{\mu+n+\sigma mk-1}{n-mk}^{-1}\binom{\alpha+n+\sigma mk-1}{n-mk}(-)^{mk}a_k\Omega(z_1,...,z_s)\eta^k \tag{4.6}$$

where

$$F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}(y_1,...,y_r;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$I_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-n-\sigma mk-\omega k-l;\epsilon_1,...,\epsilon_r),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,...,\epsilon_r),B:D \\ y_r & \end{array}\right) \tag{4.7}$$

then

$$\sum_{n=0}^{\infty}V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;z_1,...,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma_{m,a,\rho,\lambda,\sigma,\omega}^{(2)}\left[\frac{y_1}{(1-t)^{\epsilon_1}},...,\frac{y_r}{(1-t)^{\epsilon_r}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right] \tag{4.8}$$

under the same notations and conditions that theorem 2.

Remark. we obtain the same results with the multivariable H-function defined by Srivastava et al [6]. These results are the extensions of the formulae due to Srivastava et al, for more detail, see [7].

5. Aleph-function of two variables

If $r=2$, the multivariable Aleph-function reduces Aleph-function of two variables defined by K.Sharma [5], and we have the following formulae.

Corollary 3. Let

$$\gamma_{a,\rho,m,\sigma,\lambda}^{(1)}[y_1,y_2;z_1,...,z_r;t]=\sum_{k=0}^{\infty}\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)t^k}{(mk)!}$$

$$\aleph_{U_{10}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-mk;\epsilon_1,\epsilon_2),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B:D \\ y_2 & \end{array}\right) \tag{5.1}$$

and

$$R_{n,m}^{a,\rho,\lambda,\mu,\alpha}[y_1,y_2;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}B_{k,n,m}^{\lambda,\sigma,\mu,\alpha}[y_1,y_2;z_1,...,z_s;t]$$

$$\left(\begin{array}{c}\mu+n+\sigma mk-1\\ n-mk\end{array}\right)^{-1}\left(\begin{array}{c}\alpha+n+\sigma mk-1\\ n-mk\end{array}\right)\frac{a_k\Omega_{a+\rho k}(z_1,...,z_s)}{(mk)!(n-mk)!}\eta^k \tag{5.2}$$

where

$$B_{k,n,m}^{\lambda,\sigma,\mu,\alpha}(y_1,y_2;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$\aleph_{U_{10}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-\sigma mk-n-l;\epsilon_1,\epsilon_2),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & B:D \\ y_2 & \end{array}\right) \tag{5.3}$$

then

$$\sum_{n=0}^{\infty}R_{n,m}^{a,\rho,\lambda,\sigma,\mu,\alpha}[y_1,y_2;z_1,...,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma_{a,\rho,m,\sigma,\lambda}^{(1)}\left[\frac{y_1}{(1-t)^{\epsilon_1}},\frac{y_2}{(1-t)^{\epsilon_2}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m}}\right] \tag{5.4}$$

under the same notations and conditions that theorem 1.

Corollary 4. Let

$$\gamma_{a,\rho,m,\sigma,\lambda}^{(2)}[y_1,y_2;z_1,...,z_r;t]=\sum_{k=0}^{\infty}(-)^{mk}a_k\Omega_{a+\rho k}(z_1,...,z_s)t^k$$

$$\aleph_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-mk-\omega k-\sigma mk;\epsilon_1,\epsilon_2),A:C \\ \cdot & \cdots \\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,\epsilon_2),B:D \\ y_2 & \end{array}\right) \tag{5.5}$$

and

$$V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,y_2;z_1,...,z_s;\eta]=\sum_{k=0}^{[n/m]}F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}[y_1,...,y_r;t]$$

$$\begin{pmatrix}\mu+n+\sigma mk-1\\ n-mk\end{pmatrix}^{-1}\begin{pmatrix}\alpha+n+\sigma mk-1\\ n-mk\end{pmatrix}(-)^{mk}a_k\Omega(z_1,...,z_s)\eta^k \tag{5.6}$$

where

$$F_{k,n,m}^{\lambda,\sigma,\omega,\mu,\alpha}(y_1,y_2;t)=\sum_{l=0}^{\infty}\frac{(\mu-\alpha)_l t^l}{(\mu+n+\sigma mk)_l l!}$$

$$\aleph_{U_{11}:W}^{0,n+1:V}\left(\begin{array}{c|c} y_1 & (1-\lambda-n-\sigma mk-\omega k-l;\epsilon_1,\epsilon_2),A:C\\ \cdot & \cdots\\ \cdot & \\ \cdot & (-\lambda-\omega k-mk-\sigma mk;\epsilon_1,\epsilon_2),B:D\\ y_2 & \end{array}\right) \tag{5.7}$$

then

$$\sum_{n=0}^{\infty}V_{n,m}^{a,\rho,\lambda,\sigma,\omega,\mu,\alpha}[y_1,y_2;z_1,...,z_s;\eta]t^n$$

$$=(1-t)^{-\lambda}\gamma_{m,a,\rho,\lambda,\sigma,\omega}^{(2)}\left[\frac{y_1}{(1-t)^{\epsilon_1}},\frac{y_2}{(1-t)^{\epsilon_2}};z_1,...,z_s;\frac{\eta t^m}{(1-t)^{(\sigma+1)m+\omega}}\right] \tag{5.8}$$

under the same notations and conditions that theorem 2.

5. Conclusion

Due to general nature of the aleph-function of several variables, our formulae are capable to be reduced into many known and new generating functions for special functions of one and several variables.

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