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GENERAL CLASS OF GENERATING FUNCTIONS AND ITS APPLICATIONS-I

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Abstract: In this paper, we introduce a general class of generating functions involving the product of modified Jacobi polynomials $P_n^{(\alpha,\beta-n)}(x)$ and the confluent hypergeometric functions ${}_1F_1[.]$ and then obtain its some more general class of generating functions by group-theoretic approach and discuss their applications. Earlier Chandel, Kumar and Senger [1] introduce a general class of generating functions involving the product of modified Bessel polynomials $Y_n^{(\alpha+n)}$ and the confluent hypergeometric functions ${}_1F_1[.]$.

Keywords and Phrases: Generating functions, Modified Jacobi polynomials, Confluent hypergeometric functions.

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1. Introduction

The modified Jacobi polynomials $P_n^{(\alpha,\ \beta)}(x)$ is introduced by Srivastava and Manocha [6] is defined as:

$$P_n^{(\alpha,\ \beta)}(x) = \frac{(1+\alpha)_n}{n!}\ {}_2F_1\left[-n, 1+\alpha+\beta+n; 1+\alpha; \frac{1-x}{2}\right] \tag{1.1}$$

The confluent hypergeometric functions ${}_1F_1$ can be replaced by many special functions such as the Bessel polynomials. Srivastava and Manocha [6] defined and studied various bilinear, bilateral and multilinear generating functions.
In the present paper, we introduce the following new general class of generating functions:

$$G(x,u,w) = \sum_{n=0}^{\infty} a_n P_n^{(\alpha,\ \beta-n)}(x)\ {}_1F_1[-n; m+1; u]w^n \tag{1.2}$$

where a_n is any arbitrary sequence independent of x, u and w.
Again in (1.2) setting various values of a_n, we may find several results on generating functions involving different special functions, hence (1.2) is a general class of generating functions.
In the present paper, we evaluate some more general class of generating functions and finally discuss their applications.

2. Group-Theoretic Operators

In our investigations, we use the following group-theoretic operators:
The operators R_1 due to Chongdar [2] is given by

$$R_1 = (1-x^2)y\frac{\partial}{\partial x} - 2y^2\frac{\partial}{\partial y} - [(1+\alpha+\beta+p)(1+x) - 2\beta]y \tag{2.1}$$

Such that

$$R_1\left[P_n^{(\alpha,\ \beta-n)}(x)y^n\right] = -2(n+1)P_{n+1}^{(\alpha,\ \beta-n-1)}(x)y^{n+1} \tag{2.2}$$

The operator R_2 due to Miller Jr. [5] is given by

$$R_2 = v\frac{\partial}{\partial t} + vut^{-1}\frac{\partial}{\partial u} - vut^{-1} \tag{2.3}$$

Such that

$$R_2[\ {}_1F_1[-n; m+1; v]v^n t^m] = m\ {}_1F_1[-n-1; m; u]v^{n+1}t^{m-1} \tag{2.4}$$

The actions of R_1 and R_2 on function f are obtained as follows,

$$e^{wR_1}F(x,y) = \{1+wy(1+x)\}^{-1-\alpha-\beta-p}(1+2wy)^{\beta}F\left[\frac{x+wy(1+x)}{1+wy(1+x)}, \frac{y}{1+2wy}\right] \tag{2.5}$$

[Chongdar; 2]

and

$$e^{wR_2}f(v,t,u) = exp\left(\frac{-uvw}{t}\right)f\left[v, t+wv, u\left(1+\frac{wv}{t}\right)\right] \tag{2.6}$$

[Miller Jr.; 5]

3. Some more general class of generating functions

In this sections, making an use of the general class of generating function (1.2) and group-theoretic operators R_1 and R_2 with their actions given in the section

2, we obtain some more general class of generating functions through following theorem:

Theorem 3.1. *If there exists a general class of generating functions involving the product of modified Jacobi polynomials $P_n^{(\alpha,\ \beta-n)}(x)$ and the confluent hypergeometric functions ${}_1F_1[-n;m+1;u]$ given by*

$$G(x,u,w)=\sum_{n=0}^{\infty}a_nP_n^{(\alpha,\ \beta-n)}(x)\ {}_1F_1[-n;m+1;u]w^n \tag{3.1}$$

Then the following more general class of generating functions holds:

$$\{1+wy(1+x)\}^{-1-\alpha-\beta-p}(1+2wy)^{\beta}(1+w)^{m}$$

$$\times\exp(-uw)G\left[\frac{x+wy(1+x)}{1+wy(1+x)},u(1+w),wyt\right]$$

$$=\sum_{n,p,q=0}^{\infty}\frac{a_n(-2)^p(n+1)_p}{p!q!}P_{n+p}^{(\alpha,\ \beta-n-p)}(x)\ {}_1F_1[-n-q;m-q+1;u](mw)^q(wy)^{n+p}t^n \tag{3.2}$$

Proof. In the general class of generating functions (3.1), replacing w by wyv and then multiplying by t^m on both sides, we get

$$G(x,u,wyv)t^m=\sum_{n=0}^{\infty}a_nP_n^{(\alpha,\ \beta-n)}(x)y^n\ {}_1F_1[-n;m+1;u]v^nt^mw^n \tag{3.3}$$

Now, operating both the sides of (3.3) with $e^{wR_1}e^{wR_2}$, we obtain

$$e^{wR_1}e^{wR_2}[G(x,u,wyv)t^m]=e^{wR_1}e^{wR_2}\sum_{n=0}^{\infty}a_nL_n^{(\alpha-n)}(x)y^n\ {}_1F_1[-n;m+1;u]v^nt^mw^n \tag{3.4}$$

The left hand side of (3.4) becomes

$$\{1+wy(1+x)\}^{-1-\alpha-\beta-p}(1+2wy)^{\beta}(t+wv)^{m}\exp\left(\frac{-uvw}{t}\right)$$

$$\times G\left[\frac{x+wy(1+x)}{1+wy(1+x)},u\left(1+\frac{wv}{t},wyv\right)\right] \tag{3.5}$$

and the right hand side of (3.4) becomes

$$\sum_{n,p,q=0}^{\infty} \frac{a_n(-2)^p(n+1)_p m^q w^{n+p+q}{}^{n+p}}{y} t^{m-q} v^{n+q} P_{n+p}^{(\alpha,\ \beta-n-p)}(x)\ {}_1F_1[-n-q; m-q+1; u] \tag{3.6}$$

Now equating (3.5) and (3.6), and setting $v = t$, we prove the result (3.2).

4. Special Case: Taking $u = 0$ in given theorem and proceeding as the proof of the main theorem, we get

$$\{1+wy(1+x)\}^{-1-\alpha-\beta-p}(1+2wy)^{\beta} G\left[\frac{x+wy(1+x)}{1+wy(1+x)}, wy\right]$$

$$= \sum_{n,p=0}^{\infty} \frac{a_n(-2)^p(n+1)_p w^{n+p}}{p!} y^{n+p} P_{n+p}^{(\alpha,\ \beta-n-p)}(x)$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{n} \frac{a_{n-p}(-2)^p(n-p+1)_p w^n}{p!} y^n P_n^{(\alpha,\ \beta-n)}(x) = \sum_{n=0}^{\infty} g_n(y) P_n^{(\alpha,\ \beta-n)}(x) \tag{4.1}$$

where $g_n(y) = \sum_{p=0}^{n} \frac{a_{n-p}(-2)^p(n-p+1)_p}{p!} y^n$ which is known result and as parallel to Chongdar [3].

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