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UNSTEADY MHD FLOW OF A GENERALIZED VISCO-ELASTIC OLDROYDIAN FLUID

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ABSTRACT : The aim of this paper is to study the unsteady MHD flow of generalised visco-elastic fluids through a porous rectangular duct. From this generalised investigation we have deduced the different problems of flow in cases of oldroyd first order, second order, n-th order fluids ; Maxwell first order, second order, n-th order fluids ; Rivlin-Ericksen first order, second order, n-th order fluids and ordinary viscous fluid.

The numerical calculation of the velocity profile for oldroyd fluid has been made in the forms of tables and graphs.

INTRODUCTION

The development of hydrodynamic motion of inviscid and viscous liquids has been presented in the informative works of Lamb (1), Milne-Thomson (2), Batchelor (3), Landau and Lifshitz (4) and others (5,6,7). Various hydromagnetic problems and the corresponding development of the theories will be found in the monographs of Cowling (8), Ferraro and Plumpton (9); Cabannes (10), Jeffrey (11) and others (12,13,14). The hydrodynamic and hydromagnetic stability problems were considered by Chandrasekhar (15) and Lin (16).

There are circumstances to consider a large variety of continua in which considerable impetus is given to the development of rheology as a science covering a wide range of study of material properties exhibiting both the properties of ideal elastic bodies and those of viscous liquids. It constitute the subjects of the theory of elasticity and hydromechanics of viscous liquids. In fact, there are materials, solid or liquid, which exhibit the properties of elasticity of solids and viscosity of liquids. It gives rise to the discipline of Rheology of continua, the continua may be solid, liquid or gases. These liquids are sometimes called as non-Newtonian liquids or non-Newtonian fluids or viscoelastic fluids. In this area we can cite good number of references (17-28) and some papers of Sengupta and his research Collaborators (29-31). Moreover, a survey monograph of non-Newtonian fluid flows of Kapur, Bhatt, Sacheti (32) may be referred. It contains a large number of research articles of various scientists. Applied Mathematicians and research workers who are working in different relevant fields. The works of Bhatnagar (33) are also very worthy to mention. Fluid dynamics of visco-elastic liquids are amply presented in the work of Joseph (34). In this paper the authors have considered unsteady MHD flow of a generalised visco-elastic oldroydian fluid in a porous rectangular duct.

GENERAL MODEL OF VISCO-ELASTIC LIQUIDS

A new general model of visco-elastic fluid has been suggested by Professor P. R. Sengupta in the following form

$$\left.\begin{aligned}
&\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \\
&\left(1+\sum_{j=0}^{n}\lambda_j \frac{\partial^j}{\partial t^j}\right)\tau'_{ij} = 2\mu\left(1+\sum_{j=0}^{n}\mu_j \frac{\partial^j}{\partial t^j}\right)e_{ij} \\
&e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})
\end{aligned}\right\}$$

where τ_{ij} is the stress tensor, τ'_{ij} is the deviatoric stress tensor, e_{ij} the rate of strain tensor, p the fluid pressure, λ_j are new material constants of which the greatest value λ_1, represents the relaxation time parameter and $\lambda_2,\lambda_3,...,\lambda_n$, are additional material constants ; μ_j are also new material constants of which the greatest value μ_1 represents the strain rate retardation time parameter and $\mu_2,\mu_3,...,\mu_n$, are additional material constants representing the behaviour of a very wide class of visco-elastic liquids, δ_{ij} the metric tensor in cartesian co-ordinates and μ, the co-efficient of viscosity and v_i the velocity components. The material constants λ_j and μ_j designating visco-elasticity satisfy the following conditions $\lambda_1 > \lambda_2 > \lambda_3 > ... > \lambda_n$ and $\mu_1 > \mu_2 > \mu_3 > ... > \mu_n$ i.e. they are arranged in descending order of magnitudes.

EQUATION OF MOTION

We consider the boundary of the rectangular duct is x = ± a, y = ± b; 0, 0, w(x, y, t) are the velocity components in x, y, z directions, where w (x, y, t) is axial velocity of the fluid. Here z-axis is parallel to the length of the duct.

Here we consider an uniform transverse magnetic field of constant strength Bo which acts perpendicularly to the non-conducting duct.

Since the magnetic field is of moderate strength, the induced effects and Hall currents may be neglected. The equation of continuity div $\vec{V}$ = 0 satisfied with this choice of the velocity.

The equation for unsteady motion following generalised Darcy's law is given by ;

$$\left(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}\right)\frac{\partial W}{\partial t} = -\frac{1}{\rho}\left(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}\right)\frac{\partial p}{\partial z} + \nu\left(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}\right)\nabla^2 W$$

$$-\frac{\sigma B_0^2}{\rho}\left(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}\right)w - \frac{\nu}{K}\left(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}\right)W \quad (1)$$

where K is the permeability of the medium, σ is the electrical conductivity of the fluid and $-\frac{\partial p}{\partial z}$ is the axial pressure gradient.

SOLUTION OF THE PROBLEM

Here we have used the fact that the momentum flux, the pressure gradient, the local velocity and the volume flow rate are all periodic in time with frequency ω.

We assume

$$\left.\begin{aligned} -\frac{\partial p}{\partial z} &= \mathrm{Re}(Ae^{i\omega t}) \\ W &= \mathrm{Re}\, W_1(x,y)e^{i\omega t} \end{aligned}\right\} \qquad (2)$$

Substituting for W from (2) in equation (1), we get,

$$\nabla^2 W_1 - \frac{\left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)\left(1 + \sum_{j=1}^{n} (i\omega)^j \lambda_j\right)}{a^2\left(1 + \sum_{j=1}^{n} (i\omega)^j \mu_j\right)} W_1 = -\frac{A}{\mu} \frac{\left(1 + \sum_{j=1}^{n} (i\omega)^j \lambda_j\right)}{a^2\left(1 + \sum_{j=1}^{n} (i\omega)^j \mu_j\right)} \qquad (3)$$

where $M = aB_0\sqrt{\frac{\sigma}{\mu}}$

For the symmetric condition, the flow in region $x \geq 0$, $y \geq 0$ is considered. Accordingly, the boundary conditions are

$$\left.\begin{aligned} &t > 0,\ W(1, y, t) = 0,\ 0 \le y \le \frac{b}{a} \\ &\frac{\partial W}{\partial x} = 0 \quad \text{at} \quad x = 0 \end{aligned}\right\} \qquad (4)$$

and

$$\left.\begin{aligned} &W\left(x, \frac{b}{a}, t\right) = 0,\ 0 \le x \le 1 \\ &\frac{\partial W}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{aligned}\right\} \qquad (5)$$

To solve the problem we choose the finite cosine transform defined by

$$\overline{W}(m,y,t)=\int_0^1 W(x,y,t)\cos q_m x dx \tag{6}$$

and $$\overline{W}(m,n,t)=\int_0^{b/a} W(x,y,t)\cos q_n y dy \tag{7}$$

where $$q_m=\frac{(2m+1)\pi}{2},\quad q_n=\frac{(2n+1)\pi a}{2b} \tag{8}$$

Multiplying equation (3) by $\cos q_m x$, $\cos q_n y$ and then integrating twice with respect to x and y in the limits from x=0 to x=1 and y=0 to y=b/a and using the boundary conditions (4) and (5), we get,

$$\overline{W}_1(m,n)=\frac{(-1)^{m+n}\dfrac{Aa^2}{\mu}\left[1+\sum_{j=1}^{n}\lambda_j(i\omega)^j\right]}{q_m q_n\left[\alpha_{mn}+a^2(q_m^2+q_n^2)\sum_{j=1}^{n}\mu_j(i\omega)^j\right]}$$

where

$$\alpha_{mm}=a^2(q_m^2+q_n^2)+\left(M^2+\frac{a^2}{K}+\frac{i\omega a^2}{\nu}\right)\left(1+\sum_{j=1}^{n}(i\omega)^j\lambda_j\right).$$

Applying the inversion formula for the finite cosine transform defined by

$$W_1(x,y)=\frac{4a}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\overline{W}_1(m,n)\cos q_m x\cos q_n y$$

$$=\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-)^{m+n}\left[1+\sum_{j=1}^{n}\lambda_j(i\omega)^j\right]}{q_m q_n\left[\alpha_{mn}+a^2(q_m^2+q_n^2)\sum_{j=1}^{n}\mu_j(i\omega)^j\right]}\cos q_m x\cos q_n y$$

Hence $W(x,y,t)=\mathrm{Re}W_1(x,y)e^{i\omega t}$

$$=\mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\left[1+\sum_{j=1}^{n}\lambda_j(i\omega)^j\right]\cos q_m x\cos q_n y}{q_m q_n\left[\alpha_{mn}+a^2(q_m^2+q_n^2)\sum_{j=1}^{n}\mu_j(i\omega)^j\right]}e^{i\omega t}$$

Deduction for Various Visco-Elastic fluids of Different Orders

Let us now consider various visco-elastic conducting fluids in presence of transverse magnetic field of different orders for the solutions of the titled problem. In fact we can pass on from the general solution of the generalized model of fluid to the particular cases of visco-elastic conducting fluids by suitable changing visco-elastic parameters. In the following we are deriving ten cases of fluids.

Case I. Oldroyd first order fluid

Here we put $\lambda_1 \neq 0,\ \lambda_2=\lambda_3=...=\lambda_n=0$ and $\mu_1\neq 0, \mu_2=\mu_3=...=\mu_n=0$ in equation (1) and (9) and the velocity is obtained in the following form

$$W(x,y,t)=\mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(1+i\lambda_j\omega)\cos q_m x\cos q_n y}{q_m q_n[\alpha_{mn}+a^2(q_m^2+q_n^2)\mu_j i\omega]}e^{i\omega t}$$

where $\alpha_{mn}=a^2(q_m^2+q_n^2)+\left(M^2+\frac{a^2}{K}+\frac{i\omega a^2}{\nu}\right)(1+i\omega\lambda_1)$.

Case II. Oldroyd second order fluid

Here we put $\lambda_1\neq 0,\ \lambda_2\neq 0,\ \lambda_3=\lambda_4=...=\lambda_n=0$ and

$\mu_1\neq 0,\mu_2\neq 0,\mu_3=\mu_4=...=\mu_n=0$ in equation (1) and (9) the velocity is obtained in the following form

$$W(x,y,t)=\mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(1+i\lambda_1\omega-\lambda_2\omega^2)\cos q_m x\cos q_n y\, e^{i\omega t}}{q_m q_n[\alpha_{mn}+a^2(q_m^2+q_n^2)(i\mu_1\omega-\mu_1\omega^2)]}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)(1 + i\omega\lambda_1 - \omega^2\lambda_2)$.

Case III. Oldroyd n-th order fluid

If all the parameters $\lambda_j, \mu_i \, (j = 1,2,...,n)$ are non-zero, then the velocity is clear in equation (9).

Case IV. Maxwell first order fluid

Here we put $\lambda_1 \neq 0$, $\lambda_j = 0 (j = 2,3,...,n)$ and $\mu_j \neq 0, (j = 1,2,...,n)$ in equation (1) and (9) the velocity is obtained in the following form

$$W(x,y,t) = \mathrm{Re}\frac{4a^3 A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(1 + i\lambda_1\omega)\cos q_m x \cos q_n y \, e^{i\omega t}}{q_m q_n \alpha_{mn}}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)(1 + i\omega\lambda_1)$.

Case V. Maxwell second order fluid

Here we put $\lambda_1 \neq 0$, $\lambda_2 \neq 0$, $\lambda_3 = \lambda_4 = ... = \lambda_n = 0$ and

$\mu_j \neq 0, (j = 1,2,...,n)$ in equation (1) and (9). Thus finally we get the corresponding solution as

$$W(x,y,t) = \mathrm{Re}\frac{4a^3 A}{b\mu} \times$$

$$\times \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(1 + i\lambda_1\omega - \lambda_2\omega^2)\cos q_m x \cos q_n y \, e^{i\omega t}}{q_m q_n \alpha_{mn}}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)(1 + i\omega\lambda_1)$.

Case VI. Maxwell n-th order fluid

Now we put $\lambda_1 \neq 0,\ (j=1,2,3,...,n)$ and $\mu_j = 0, (j=1,2,...,n)$ in equation (1) and (9). Thus finally we get the corresponding solution as follows

$$W(x,y,t) = \mathrm{Re}\frac{4a^3A}{b\mu}\times$$

$$\times\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\left[1+\sum_{j=1}^{n}(i\omega)^j\lambda_1\right]\cos q_m x\cos q_n y\, e^{i\omega t}}{q_m q_n \alpha_{mn}}$$

where $\alpha_{mn} = a^2(q_m^2+q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)\left(1+\sum_{j=1}^{n}(i\omega)^j\lambda_1\right).$

Case VII. Rivlin-Erocksen first order fluid

Here we put $\lambda_j - 0,\ (j=2,3,...,n)$ and $\mu_j \neq 0, \mu_2 = \mu_3 = \mu_4 = ... = \mu_n = 0$ in equation (1) and (9). As such the solution of the corresponding problem is derived in the following form

$$W(x,y,t) = \mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos q_m x\cos q_n y\, e^{i\omega t}}{q_m q_n[\alpha_{mn} + a^2(q_m^2+q_n^2)i\mu_1\omega]}$$

where $\alpha_{mn} = a^2(q_m^2+q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right).$

Case VIII. Rivlin-Ericksen second order fluid

Let us put $\lambda_j - 0,\ (j=1,2,...,n)$ and $\mu_j \neq 0, \mu_2 \neq 0, \mu_3 = \mu_4 = ... = \mu_n = 0$ in equation (1) and (9). So the solution is obtained as follows

$$W(x,y,t) = \mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos q_m x\cos q_n y\, e^{i\omega t}}{q_m q_n[\alpha_{mn} + a^2(q_m^2+q_n^2)(i\mu_1\omega - \mu_2\omega^2)]}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)$.

Case IX. Rivlin-Ericksen n-th order fluid

Now we put $\lambda_j - 0,\ (j = 1,2,...,n)$ and $\mu_j \neq 0, (j = 1,2,...,n)$ in equation (1) and (9) and the velocity is obtained as follows

$$W(x,y,t) = \mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos q_m x \cos q_n y e^{i\omega t}}{q_m q_n[\alpha_{mn} + a^2(q_m^2 + q_n^2)\sum_{j=1}^{n} i\mu_j (i\omega)^j]}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)\left(1 + \sum_{j=1}^{n}(i\omega)^j \lambda_j\right)$

Case X. Ordinary viscous fluid

Here we put $\lambda_j = \mu_j = 0, (j = 1,2,...,n)$ in the quation (1) and (9). Thus finally we get,

$$W(x,y,t) = \mathrm{Re}\frac{4a^3A}{b\mu}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos q_m x \cos q_n y}{q_m q_n \alpha_{mn}}$$

where $\alpha_{mn} = a^2(q_m^2 + q_n^2) + \left(M^2 + \frac{a^2}{K} + \frac{i\omega a^2}{\nu}\right)$.

Basic Theory and Equation of Motion for oldroyd Fluid

For slow motion, the rheological equations for oldroyd visco-elastic fluid are

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij}$$

$$\left(1 + \lambda'_1 \frac{\partial}{\partial t'}\right)\tau'_{ij} = 2\mu\left(1 + \mu'_1 \frac{\partial}{\partial t'}\right)e_{ij}$$

$$e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right).$$

We consider the flow in x’y’ plane. z’-axis is taken parallel to the length of the duct with impermeable boundary x’=±a, y’=±b; 0, 0, w’(x’,y’,t’) are respectively the velocity components along x’, y’, z’ direction, where w’(x’,y’t’) is asial velocity of the fluid.

The equation of continuity div $\vec{V} = 0$ is satisfied with the choice of the velocity. Following generalized Darcy’s law, the equation for unsteady motion is given by

$$\left(1+\lambda'_1\frac{\partial}{\partial t'}\right)\frac{\partial w'}{\partial t'} = -\frac{1}{\rho}\left(1+\lambda'_1\frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial z'} + \nu\left(1+\mu'_1\frac{\partial}{\partial t'}\right)\nabla^2 W'$$

$$-\frac{\nu}{K}\left(1+\lambda'_1\frac{\partial}{\partial t'}\right)W' - \frac{\sigma B_0^2}{\rho}\left(1+\lambda'_1\frac{\partial}{\partial t'}\right)W' \qquad (10)$$

We introduce the non-dimensional quantities in equation (10)

$$W = W'\frac{a}{\nu},\ p = p'\frac{a^2}{\rho\nu^2},\ t = t'\frac{\nu}{a^2},\ w = w'\frac{a^2}{\nu},\ (x,y,z) = \frac{1}{a}(x',y',z'),$$

$$\lambda_1 = \lambda'_1\frac{\nu}{a^2},\ \mu_1 = \mu'_1\frac{\nu}{a^2},\ M = aB_0\sqrt{\frac{\sigma}{\mu}} \qquad \text{(Hartmann number)}$$

The equation (1) reduces to

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \left(1+\mu_1\frac{\partial}{\partial t}\right)\nabla^2 W - \left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z}$$

$$-\frac{a^2}{K}\left(1+\lambda_1\frac{\partial}{\partial t}\right)W - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)W$$

where $-\frac{\partial p}{\partial z}$ is the axial pressure gradient.

Solution of the Problem

Case I. When the pressure gradient is periodic in nature.

In this case we assume $-\frac{\partial p}{\partial z} = \mathrm{Re}(Ae^{i\omega t})$

and $$W = \mathrm{Re}\, W_1(x,y)e^{i\omega t}$$

using the same boundary conditions and the same procedure as in the previous generalized article, we get the solution of the equation (11) as follows

$$W(x,y,t) = \frac{4aA}{b}$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^{m+n}\left[\left[\alpha_{mn} + \lambda_1 w^2\left\{1+\lambda_1\left(\frac{a^2}{K}+M^2\right)+\mu_1(q_m^2+q_n^2)\right\}\right]\cos\omega t}{q_m q_n\left[\alpha_{mn}^2 + \left\{1+\lambda_1\left(\frac{a^2}{K}+M^2\right)+\mu_1(q_m^2+q_n^2)\right\}^2\omega^2\right]}$$

$$+\frac{\left[1+\left\{\frac{a^2}{K}+M^2-\alpha_{mn}\right\}\lambda_1+\mu_1(q_m^2+q_n^2)\omega\sin\omega t\right]\Bigg]}{q_m q_n\left[\alpha_{mn}^2 + \left\{1+\lambda_1\left(\frac{a^2}{K}+M^2\right)+\mu_1(q_m^2+q_n^2)\right\}^2\omega^2\right]}$$

where $$\alpha_{mn} = (q_m^2+q_n^2)+\frac{a^2}{K}+M^2-\lambda_1 w^2. \qquad (12)$$

Case II. When the pressure gradient is transient in nature.

Here we assume $-\frac{\partial p}{\partial z} = Ae^{-\omega t}$

and $$W = W_1(x,y)e^{-\omega t}$$

In this case we have also used the same boundary conditions and we have proceeded in the similar fashion as before. Finally, we get the solution of the equation (11) as follows

$$W(x,y,t)=\frac{4aAe^{-\omega t}}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(1-\lambda_1\omega)\cos q_m x\cos q_n y}{q_m q_n[\beta_{mn}-\mu_1\omega(q_m^2+q_n^2)]}$$

where $\beta_{mn}=q_m^2+q_n^2+(1-\lambda_1\omega)\left(\frac{a^2}{K}-\omega+M^2\right)$. (13)

Numerical Calculations

For numerical calculations of equation (12), we take λ_1=0.0023, μ_1=0.0005, M=10, ω=430, K=0.5, b=0.25, a=0.5, x=0.75, y=0.45 and for numerical computations of the velocity distribution in the equation (13), we take λ_1=0.0023, μ_1=0.0005, M=10, ω=10, K=0.5, b=0.25, a=0.5, x=0.75, y=0.45. The tables and graphs are given as follows :

Table – 1

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\frac{w(x,y,t)}{A}\times 10^5$	4.976	-1.037	-6.391	-5.762	-.272	5.441	6.329	2.194	-4.568	-6.392

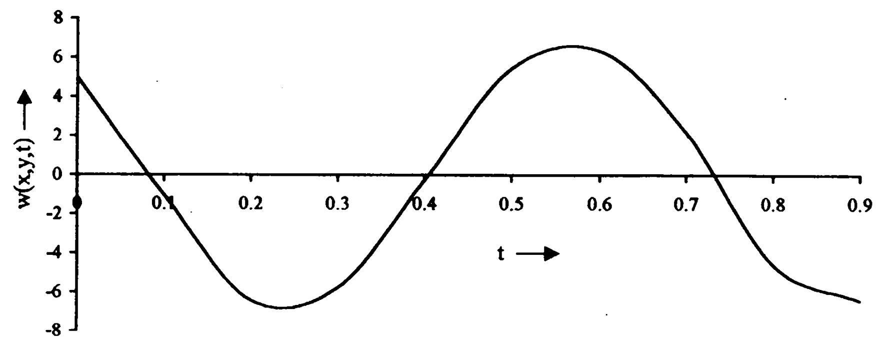

Fig. 1 : Velocity Profile for Periodic Pressure Gradient

Table – 2

t	0	0.5	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$\frac{w(x,y,t)}{A}\times 10^5$	12.246	7.427	4.505	2.732	1.656	1.005	0.609	0.369	0.224	0.135	0.082

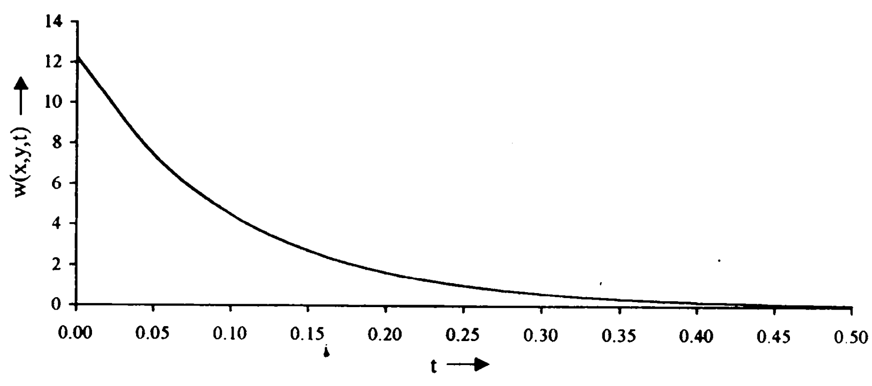

Fig. 2 : Velocity Profile for Transient Pressure Gradient

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