

AN INTERESTING OBSERVATION ON RAMANUJAN NUMBER 1729

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Abstract: In this paper by taking Ramanujan number 1729 as an illustration an important method of obtaining some prime numbers has been studied satisfying some specific conditions or properties.

Keywords and Phrases: Concatenation, reverse concatenation, key, prime key, Disjoint Prime Concatenation (DPC).

1. Introduction

For a given prime number p , one can get another prime number through a given natural number.

Theorem 1. *Let P be any prime number. Let a be any integer. Then there exists natural number n such that $P + n.a$ is a prime number.*

Example 1. For the given prime number 13 and for the positive integer 5, $13 + 2 \times 5 = 23$ is a prime number. Also, $13 + 6 \times 5 = 43$, a prime.

In mathematics **concatenation** of two positive integers is the joining of two numbers by their numerals. That is, the concatenation of 231 and 795 is 231795.

Theorem 2. *For a given composite number t , and for a given number n which is relatively prime to t , there exists a natural number r such that $t + n \times r$ is prime.*

For example; with a given composite number 24 and 7, $24 + 7 \times 5 = 59$, a prime number.

The concatenation of the first n Fibonacci numbers gives 1, 11, 112, 1123, 11235, ... There are no primes in the first 1580 terms other than $n = 2$ and 4, corresponding to primes 11 and 1123. 1729 is known as Ramanujan number and this is not a prime number as $1729 = 7 \times 13 \times 19$.

This number can be identified as concatenation of two prime numbers 17 and 29. Thus 1729 can be expressed as concatenation of two disjoint prime numbers 17 and 29 that is (17)(29). This can be explained as decomposition of 1729 in to two disjoint prime numbers 17 and 29, that is (17)(29) and this is called “Disjoint Prime Concatetion (or DPC)” of 1729.

The Reverse concatenation of Ramanujan number is (29)(17) which produce the number 2917. Note that 2917 is a prime number.

Let us initiate an interesting recreational mathematics with 1729.

Note that 1729 is not a prime number. The nearest prime numbers of 1729 having DPC are 1723 and 1741. Now by considering the next higher and prime number, it is a task to reach the nearest higher prime number 1741 in terms of a “prime-key”. Let us define a “key” for a natural number N . Take any natural number $k < N$ and k is prime to N . Then k is said to a “prime-key” of N if there exists natural number t such that $N + t.k$ is a prime number. For 1729 let us select a 3 as a “key”. Now, $1729 + 4 \times 3 = 1741$ which is a prime number and $1741 \rightarrow (17)(41)$ is a DPC. Note that the reverse concatenation of $1741 = (17)(41)$ is 4117 and 4117 is not a prime. Using the key 3 to get the nearest prime 4129 having disjoint prime partition (DPC) which is (41)(29). That is $4117 + 4 \times 3 = 4129$ and $4129 \rightarrow (41)(29)$ is a DPC. Note that the reverse concatenation or reversal of 4129 is (29)(41) or 2941 which is not a prime number. By proceeding in the same manner, one can get $2941 + 4 \times 3 = 2953$, a prime and (29)(53) is DPC. The reverse concatenation or the reversal of (29)(53) is 5329 and this is not a prime number. Using key 3 and proceeding as earlier $5329 + 6 \times 3 = 5347 \rightarrow (53)(47)$ is obtained which is a prime number. The reverse concatenation of 5347 is (47)(53) or 4753 which is not a prime. Using key 3, the required prime number $4759 = 4753 + 2 \times 3$ is obtained and (47)(59) is a DPC. Again, 5947 is not a prime. Then $5947 + 2 \times 3 = 5953$ or (59)(53) is a prime and the reverse concatenation 5359 is not a prime. In a similar manner, the next number is obtained as $2359 + 13 \times 3 = 5407$, a prime. Now, (54)(07) is not a prime partition so next consider the next prime 5413, but this number also cannot be granted for same reason. Take the next prime 5419. Here (5)(419) is a DPC. Now (419)(5) = 4195 is not a prime number, then proceeding as earlier, the next number is $4195 + 2 \times 3 = 4201$, a prime. It is to be noted that no DPC can be obtained for numbers less than 4993. As $4201 + 264 \times 3 = 4993$, a prime number and $4993 \rightarrow (499)(3)$ is a DPC and also 3499 is a prime number. So we stop the process.

Therefore, starting with the Ramanujan number 1729 and selecting “prime-key” as

3 one will reach to 4993 so that the reverse DPC of 4993 is 3499 which is a prime number.

Remark 1.

- (i) *From the number 1729, a prime number 4993 which is larger than the original number 1729 is obtained through the above process. In the next example, it will be shown that from a given number, it is possible to obtain a smaller prime number in the same process.*
- (ii) *It must be noted that the nearest prime number of 1729 is 1723 and the reversal of 1723 is 3271 which is a prime number.*
- (iii) *The nearest prime number which is larger than 1729 is 1741 which is obtained by using the key $3(1729 + 4 \times 3 = 1741)$. The reversal of 1741 is 1471 which is a prime number and it is smaller than the given number 1729.*

Example 2. Consider the reverse Ramanujan number 9271 which is not a prime number. To obtain a prime number p from 9271 by using the key 9(9 is taken as key as it is the largest digit) so that either the reversal of p or the reverse concatenation of p is again a prime number. Thus $9271 \rightarrow 9271 + 38 \times 9 = 9613$. Now the reversal of 9613 is 3169 which is a prime number.

Project The no 1729 is not a prime number but the numbers 17291, 17293, 17299 are prime numbers where the Ramanujan number 1729 is embedded. Now the primes 17291 and 17299 have no DPC so let us consider 17293. Now $17293 \rightarrow (17)(29)(3)$ is a DPC. Among the permutations of the members of DPC, $(3)(29)(17) \rightarrow 32917$ is a prime number; $(29)(17)(3) \rightarrow 29173$ is a prime number; but $(29)(3)(17) \rightarrow 29317$ is not a prime. For instance, let us deal with 29317. Take 3 as a key for 29317. Then $29317 + 10 \times 3 = 29347$ is a prime. Also, $29347 \rightarrow (293)(47)$ is a Disjoint Prime Concatenation and the reverse DPC is $(47)(293) \rightarrow 47293$ which is a prime number.

In this paper in view of recreational mathematics, an successful attempt is made to generate prime numbers by taking Ramanujan number which is not prime.

Remark 2. *This study may be helpful for the development of new type of method for exchanging secret information very much equivalent to Coding theory.*

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References

- [1] Smarandache, F. Only Problems, Not Solutions!, 4th ed. Phoenix, AZ: Xi-quan, 1993.
- [2] Smith, S., A Set of Conjectures on Smarandache Sequences, Bull. Pure Appl. Sci. 15E, 101-107, 1996.