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CALCULATION OF TRANSMISSION SPECTRA OF APERIODIC PHOTONIC CRYSTALS

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Abstract: In recent years, photonic quasicrystals (PQs) with aperiodic structures have attracted many interests for their amusing photonic band gap (PBG) properties analogical to those of periodic photonic crystals. The diversity of the PBGs of PQs is magnetic both theoretically and experimentally for potential applications in novel optical and optoelectronic devices. Among various PQs, the properties of one-dimensional PQs could be simulated more precisely. One-dimensional PQs, including Fibonacci and Thue- Morse (TM) sequences have been constructed experimentally. Here, the transmission spectra of aperiodic photonic crystals are calculated by transfer matrix method in order to discuss the PBG.

Keywords and Phrases: Transmission spectra, aperiodic photonic crystals, transfer matrix method.

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1. Introduction

There is currently a great interest in the physics and applications of one-dimensional spatially periodic, quasiperiodic and random photonic bandgap (PBG) structures [1, 2]. Quasi-crystals can be considered as suitable models to describe the transition from the perfect periodic structure [3] to the random structure [4, 5]. PQs can be generated by stacking together layers of different dielectric materials according to a simple deterministic generation rule [6]. Thue-Morse structure [7], Fibonacci sequence [8-10], Cantor layer etc. are some examples of the one dimensional quasiperiodic structures.

Quasicrystals represent an intermediate organization stage between periodic dielectric materials and random media and have fascinating properties like the formation of multiple frequency band gap regions, transmission resonances and the occurrence of critically localized states.

In this paper, the mathematical formulation of the optical spectra of Fibonacci and Thue-Morse aperiodic structures is discussed.

Theory

Fibonacci sequences are multilayer structures consisting of two different materials as building blocks. Two materials are labeled as H and L, where H represents the material with high refractive index and L represents the material with low refractive index. The number of layers in a structure depends on the order of the Fibonacci sequence. Fibonacci sequence can be generated by the recursive relation,

$$S_{j+2} = \{S_j, S_{j+1}\}, \qquad j \geq 0 \tag{1}$$

with $S_0 = H$ and $S_1 = L$, where $S_j(J > 1)$ is the j^{th} generation of the Fibonacci structure. H and L are material with refractive index nH and nL and thicknesses dH and dL respectively.

The number of layers in a sequence is given by F_j, where F_j is a Fibonacci number obtained from the recursive law $F_{j+1} = F_j + F_{j-1}$, with $F_0 = F_1 = 1$. For $j \geq 2$, the systems S_j are known as quasiperiodic. First eight Fibonacci sequences are given in Table 1.

Simple transfer matrix method used to study the quasiperiodic Fibonacci structures [11, 12]. The transfer matrix for Fibonacci system can be written as,

$$M_j = M_{j-2}M_{j-1}, \qquad j \geq 2, \tag{2}$$

with $M_0 = M_H$ and $M_1 = M_L$.

Table 1: Definition of first eight Fibonacci sequences

Fibonacci sequence	Sequence definition	Number of layers
S_0	H	1
S_1	L	1
S_2	HL	2
S_3	LHL	3
S_4	HLLHL	5
S_5	LHLHLLHL	8
S_6	HLLHLLHLHLLHL	13
S_7	LHLHLLHLHLLHLLHLHLLHL	21
S_8	HLLHLLHLHLLHLLHLHLLHLHLLHLLHLHLLHL	34

The transfer matrices for the single layer H and L are given by

$$M_H = \begin{bmatrix} \cos\beta_H & -\frac{i}{q_H}\sin\beta_H \\ -iq_H\sin\beta_H & \cos\beta_H \end{bmatrix} \tag{3}$$

and

$$M_L = \begin{bmatrix} \cos\beta_L & -\frac{i}{q_L}\sin\beta_L \\ -iq_L\sin\beta_L & \cos\beta_L \end{bmatrix} \tag{4}$$

where $\beta_H = \frac{2\pi}{\lambda} n_H d_H \cos\theta_H$ and $\beta_L = \frac{2\pi}{\lambda} n_L d_L \cos\theta_L$ are the layer phase thicknesses. θ_H and θ_L are the angle of refractions in layers H and L respectively which are determined by the Snell's law and λ is the wavelength of incident wave. Parameters q_H and q_L are given by, $q_H = n_H\cos\theta_H$ and $q_L = n_L\cos\theta_L$ for TE polarization $q_H = \frac{\cos\theta_H}{n_H}$ and $q_L = \frac{\cos\theta_L}{n_L}$ for TM polarization. Thus the transfer matrices M_j are $M_2 = M_H M_L$, $M_3 = M_L M_H M_L$ and $M_4 = M_h M_L M_L M_H M_L$ for S_2, S_3 and S_4 respectively.

Let us consider an N-period finite structure whose basic cell is the Fibonacci structure S_j. The overall transfer matrix M of the system is obtained to be

$$M = (M_j)^N = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{5}$$

The reflection coefficient is given by

$$r = \frac{(M_{11} + q_t M_{12})q_i - (M_{21} + q_t M_{22})}{(M_{11} + q_t M_{12})q_i + (M_{21} + q_t M_{22})}, \tag{6}$$

where $q_{i,t} = n_{i,t}\cos\theta_{i,t}$ for TE wave and $q_{i,t} = \frac{\cos\theta_{i,t}}{n_{i,t}}$ for TM wave, where i and t represent incident medium and substrate respectively. The reflectivity is given by,

$$R = |r|^2. \tag{7}$$

The above theoretical analysis can be applied for optical transmission measurements on the symmetric Fibonacci films. It is well known that the Fibonacci sequence, which contains two units H and L, can be produced by repeated application of the substitution rules $H \to HL$ and $L \to H$. Since Merlin et al. first reported the realization of Fibonacci superlattices in 1985; much attention has been paid to the exotic wave phenomena of Fibonacci systems without the mirror symmetry. But symmetric Fibonacci sequence opens a way for technological applications in several fields. The symmetric Fibonacci sequence can be generated in the following way. The j-th generation of the sequence can be expressed as,

$S_j = \{G_j, T_j\}$ where G_j and T_j are Fibonacci sequences. G_j and T_j obey the recursion relations,
$G_j = G_{j-1}G_{j-2}$, and $T_j = T_{j-2}T_{j-1}$, with $G_0 = T_0 = L$ and $G_1 = T_1 = H$.
Therefore

$$S_j = G_{j-1}G_{j-2}T_{j-2}T_{j-1} \tag{8}$$

Considering the symmetry in the structure as shown in the above equation and using the unitary condition $\det|M_j| = 1$, the transmission coefficient of the light wave through the multilayers with internal symmetry can be written as,

$$T(S_j) = \frac{4}{|M_j|^2 + 4} = \frac{4}{[M_{12} + M_{21}]^2 + 4} \tag{9}$$

As can be seen from the above equation, if the condition $M_{12} + M_{21} = 0$ is satisfied, perfect transmission peaks are indeed obtained. Thus, resonant transmissions can be obtained in the dielectric multilayers with mirror symmetry.

Thue-Morse Structure

The Thue-Morse sequence is one of the well known examples in one-dimensional aperiodic structure. The T-M 1-D structure is constituted by the sequence of two layers A and B with refractive indices n_A and n_B, and thicknesses d_A and d_B respectively. It can be produced by repeating application of the substitution rules $A \to AB$ and $B \to BA$. For example, the first few generations S_n of Thue-Morse sequence are as follows,

$S_0 = [A]$
$S_1 = [AB]$
$S_2 = [ABBA]$
$S_3 = [ABBABAAB]$
$S_4 = [ABBABAABBAABABBA]$
....

The reflectivity of optical waves through a T-M dielectric multilayer for both transverse electric (TE) and transverse magnetic (TM) polarizations and for different incident angles can be calculated by using the transfer matrix method as in the above case of Fibonacci sequence. Also, the even-order T-M multilayer has the characteristic of mirror symmetry.

As in the case of Fibonacci sequence, if the conditions $M_{12} + M_{21} = 0$, for even order $T - M$ and, $M_{12} - M_{21} = 0$, for odd order $T - M$

are satisfied, a perfect transmission of light will definitely occur in the Thue-Morse dielectric multilayers.

Conclusion

The Transfer Matrix Method (TMM) is used to calculate the photonic band structure for dispersive materials in 1-D, 2-D and 3-D photonic structures, which are frequency dependent [13]. The TMM consists on writing the Maxwells equations in the k-space and rewriting them on a mesh. This method is capable of handling PBG materials of finite thickness with layer by layer calculations. Structures with defects can be dealt only by considering a super-cell. The band structures, reflectivity and transmission coefficients can be found by this method easily. Many researchers have used this method [14-15]. It has also proved very useful and accurate when comparisons with experimental structures are undertaken. The limitations of this method are the memory storage but also it is difficult to deal with geometry different from the cubic geometry.

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