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A STUDY OF $\bar{H}$- FUNCTION

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Abstract: The subject of Fourier series of the generalized hypergeometric functions occupies an important place in the field of special functions. Certain Fourier series of the generalized hypergeometric function play an important role in the development of the theory of special functions and certain Fourier series of the generalized hypergeometric functions enable us to obtain general solutions of some boundary value problems. The Fourier series of the generalized hypergeometric functions were given from time to time by various mathematics with certain restrictions in parameters. An adequate list of reference given here together with sources indicated in these references provide a good converge of the subject. In this paper we have defined Fourier series for $\bar{H}$-function and also we derived integral involving sine function, exponential function, the product of Kampé de Fériet functions and the $\bar{H}$-function to evaluate three Fourier series.

Keywords and Phrases: Kampé de Fériet functions, $\bar{H}$-function and Fourier Series.

2010 Mathematics Subject Classification: 33C05.

1. Introduction

The $\bar{H}$-function is defined by Inayat and Hussain, [5,6]. For the convergence and other details of the $\bar{H}$-function, we refer the original paper of Inayat and Hussain, [5,6].

We now mention in brief some interesting work on this subject. MacRobert [7,8] established a cosine and a sine Fourier series of MacRobert's E-function. Bajpai, S.D. [3] obtained some Fourier series of the G-function. MacRobert [8] have obtained the fourier series for G-function and E-function respectively. Anandani [2], Chandel, Agrawal and Kumar [4] established some Fourier series of Fox's H-function. Recently, Singh and Khan [10] established some Fourier series of generalized hypergeometric function. In the present study we have deduced Fourier series for $\bar{H}$-function. Some interesting results can be seen in [11-13].

2. Formulae Required

In this paper, we will be use the following formulae

(i) Kampé de Fériet hypergeometric function will be represented as follows.

$$F\left(\begin{array}{c|c|}p & a_1,...,a_p \\ \mu & b_1,b_1,...,b_\mu,b_\mu \\ q & c_1,...,c_q \\ \sigma & d_1,d_1,...,d_\sigma,d_\sigma\end{array}\, xy\right)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(a_j)_{m+n}\prod_{j=1}^{\mu}\{(b_j)_m(b'_j)_n\}}{\prod_{j=M+1}^{q}(c_j)_{m+n}\prod_{j=1}^{\sigma}\{(d_j)_m(d'_j)_n\}}\frac{x^m y^n}{m!n!} \tag{2.1}$$

$$(p+v<q+\alpha+1 \;\text{ or }\; p+v=q+\alpha+1 \;\text{ and }\; |x|+|y|<mn(1,2^{q-p+1}));$$

$$=-\frac{1}{4\pi^2 K}\int_{-i\infty}^{+i\infty}\psi(s,t)\Gamma(-s)\Gamma(-t)(-x)^s(-y)^t dsdt,$$

where,

$$K=\frac{\prod_{j=1}^{p}\Gamma(a_j)\prod_{j=1}^{\mu}\{\Gamma(b_j)\Gamma(b'_j)\}}{\prod_{j=M+1}^{q}\Gamma(c_j)\prod_{j=1}^{\sigma}\{\Gamma(d_j)\Gamma(d'_j)_n\}}, \tag{2.2}$$

$$F\left(\begin{array}{c|c|}p & a_1,...,a_p \\ \mu & ------ \\ q & c_1,...,c_q \\ \sigma & ------\end{array}\, xy\right)={}_pF_q\left(\begin{array}{c}a_1,...,a_p \\ \\ c_1,...,c_q\end{array}\; x+y\right). \tag{2.3}$$

For further detail one can refer the monography by Appell and Kampé de Fériet [1], Mishra [9] has evaluated

$$\int_0^{\pi}(\sin x)^{w-1}e^{imx}\,{}_pF_q\left[\begin{array}{l}\alpha_p; \\ \beta_q;\end{array}\; C(\sin x)^{2h}\right]dx=\frac{\pi e^{im\pi/2}}{2^{w-1}}\sum_{r=0}^{\infty}\frac{(\alpha_p)C^r\Gamma(w+2hr)}{(\beta_q)_r r!4^{hr}\Gamma\left(\frac{\omega+2hr\pm M+1}{2}\right)}, \tag{2.4}$$

where $(\alpha)_p$ denotes $a_1, ..., a_p; \Gamma(a \pm b)$ represents $\Gamma(a+b),\ \Gamma(a-b); h$ is a positive integer; $p < q$ and $\Re(w) > 0$. Recall the following elementary integrals,

$$\int_0^{\pi} e^{i(m-n)x} dx = \begin{cases} \pi, & m = n; \\ \\ 0, & m \neq n; \end{cases} \tag{2.5}$$

$$\int_0^{\pi} e^{imx} \cos nx dx = \begin{cases} \frac{\pi}{2}, & m = n \neq 0; \\ \pi, & m = n = 0; \\ 0, & m = n; \end{cases} \tag{2.6}$$

$$\int_0^{\pi} e^{imx} \sin nx dx = \begin{cases} i\frac{\pi}{2}, & m = n; \\ \\ 0, & m \neq n; \end{cases} \tag{2.7}$$

provided either m or n are odd or both m and n are even integers. For brevity, we shall use the following notations.

$$\frac{\prod_{k=1}^{E}(e_k)_{r+t} \prod_{k=1}^{F}(f_k)_r \prod_{k=1}^{F'}(f'_k)_t}{\prod_{k=1}^{G}(g_k)_{r+t} \prod_{k=1}^{H}(h_k)_r \prod_{k=1}^{H'}(h'_k)_t} = \varepsilon.$$

$$\frac{\prod_{k_1=1}^{E_1}(e_{nk_1})_{r_1+t_1} \prod_{k_1=1}^{F_1}(f_{1k_1})_{r_1} \prod_{k_1=1}^{F'_1}(f'_{1j_1})_{t_1}}{\prod_{k_1=1}^{G_1}(e_{nk_1})_{r_1+t_1} \prod_{k_1=1}^{H_1}(h_{1k_1})_{r_1} \prod_{k_1=1}^{H'_1}(h'_{1j_1})_{t_1}} = \varepsilon_1.$$

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$$\frac{\prod_{k_n=1}^{E_n}(e_{nk_n})_{r_n+t_n} \prod_{k_n=1}^{F_n}(f_{nk_n})_{r_n} \prod_{k_n=1}^{F'_n}(f'_{nj_n})_{t_n}}{\prod_{k_n=1}^{G_n}(e_{nk_n})_{r_n+t_n} \prod_{k_n=1}^{H_n}(h_{nk_n})_{r_n} \prod_{k_n=1}^{H'_n}(h'_{nj_n})_{t_n}} = \varepsilon_n.$$

(ii) The integrals to be evaluated which will be using in further investigation

$$\int_0^{\pi} (\sin x)^{w-1} e^{imx} F_{G;H;H'}^{E;F;F'} \left[\begin{array}{c} (e);(f);(f');\alpha(\sin x)^{2\rho} \\ (g);(h);(h');\beta(\sin x)^{2\gamma} \end{array} \right]$$

$$\times \bar{H}_{p,q}^{m,n} \left[z(\sin x)^{2\sigma} \left| \begin{array}{c} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{array} \right. \right] dx$$

$$= \frac{\sqrt{(x)} e^{im\pi/2}}{2^{w-1}} \sum_{r,t=0}^{\infty} \varepsilon \frac{(\alpha/4^{\rho})^{\gamma}(\beta/4^{\gamma})^{t}}{r!t!}$$

$$\times \bar{H}_{p+1,q+1}^{m,n+1} \left[\frac{z}{4^{\sigma}} \left| \begin{array}{c} (1-\omega-2\rho r-2\gamma t, 2\sigma;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \left(\dfrac{1-\omega-2\rho r-2\gamma t \pm m}{2}, \sigma;1 \right) \end{array} \right. \right], \tag{2.8}$$

provided that $|\arg z| < 1/2\pi\Omega$, and $\Re(w) > 0$; $\alpha, \beta, \rho, \gamma, \sigma, z$ are positive integers, where $\Omega = \sum_{j=1}^{m} |B_j| + \sum_{j=1}^{q} |b_j B_j| - \sum_{j=1}^{n} |a_j A_j| - \sum_{j=n+1}^{q} |A_j| > 0,\ 0 < |z| < \infty.$

$$\int_0^{\pi} \ldots \int_0^{\pi} (\sin x)^{w_1-1} \ldots (\sin x)^{w_n-1} e^{i(m_1x_1+\ldots+m_nx_n)}$$

$$\times F_{G_1;H_1;H_1'}^{E_1;F_1;F_1'} \left[\begin{array}{c} (e_1);(f_1);(f_1');\alpha_1(\sin x_1)^{2\rho_1} \\ (g_1);(h_1);(h_1');\beta_1(\sin x_1)^{2\gamma_1} \end{array} \right] \ldots$$

$$F_{G_n;H_n;H_n'}^{E_n;F_n;F_n'} \left[\begin{array}{c} (e_n);(f_n);(f_n');\alpha(\sin x_n)^{2\rho_n} \\ (g_n);(h_n);(h_n');\beta_n(\sin x_n)^{2\gamma_n} \end{array} \right] \bar{H}_{p,q}^{m,n}[z(\sin x_1)^{2\sigma_1} \ldots (\sin x_n)^{2\sigma_n}] dx_1 \ldots dx_n$$

$$= \frac{(\pi)^n e^{i(m_1+\ldots+m_n)\pi/2}}{2(\omega_1+\ldots+\omega_n)-n} \sum_{r_1,t_1=0}^{\infty} \ldots \sum_{r_1,t_1=0}^{\infty} (\varepsilon_1 \ldots \varepsilon_n) \frac{(\alpha_1/4^{\rho_1})^{r_1}(\beta_1/4^{\gamma_1})^{t_1}}{r!t!} \ldots$$

$$\frac{(\alpha_1/4^{\rho_1})^{r_1}(\beta_1/4^{\gamma_1})^{t_1}}{r!t!} \bar{H}_{p+1,q+2}^{m,n+1} \left[\frac{z}{4^{\sigma_1+\ldots+\sigma_n}} \left| \begin{array}{c} (1-\omega_1-2\rho_1 r_1-2\gamma_1 t_1, 2\sigma_1;1) \ldots \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}, \end{array} \right. \right.$$

$$\left. \begin{array}{c} (1-\omega_n-2\rho_n r_n-2\gamma_n t_n, 2\sigma_n;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ \left(\dfrac{1-\omega_n-2\rho_n r_n-2\gamma_n t_n \pm m_1}{2}, \sigma_1;1 \right) \ldots \left(\dfrac{1-\omega_n-2\rho_n r_n-2\gamma_n t_n \pm m_n}{2}, \sigma_n;1 \right) \end{array} \right], \tag{2.9}$$

provided that all the conditions of (2.8) are satisfied and $\Re(wi) > 0$; σ_i, α_i, β_i, ρ_i, γ_i, z_i are positive integers $(i = 1, , n)$.

3. Exponential Fourier Series

Let

$$f(x)=(\sin x)^{w-1}F^{E_1;F;F'}_{G;H;H'}\left[\begin{array}{l}(e);(f);(f');\alpha(\sin x)^{2\rho}\\(g);(h);(h');\beta(\sin x)^{2\gamma}\end{array}\right]$$

$$\times\bar{H}^{m,n}_{p,q}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{array}\right.\right]dx=\sum_{p=-\infty}^{\infty}A_pe^{-ipx}. \tag{3.1}$$

which is valid due to $f(x)$ is continuous and of bounded variation with interval $(0,\pi)$. Now, multiplying by e^{imx} both sides in (3.1) and integrating it with respect to x from 0 to π, and then making an appeal to (2.5) and (2.8), we get

$$A_p=\frac{e^{im\pi/2}}{2^{\omega-1}}\sum_{r,t=0}^{\infty}\varepsilon\frac{(\alpha/4^{\rho})^r}{r!}\frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times\bar{H}^{m,n+1}_{p+1,q+2}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right], \tag{3.2}$$

An application to (3.1) and (3.2) gives the required exponential Fourier series

$$(2\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{l}(e);(f);(f');\alpha(\sin x)^{2\rho}\\(g);(h);(h');\beta(\sin x)^{2\gamma}\end{array}\right]$$

$$\times\bar{H}^{m,n}_{p,q}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{array}\right.\right]dx$$

$$=\sum_{p=-\infty}^{\infty}\sum_{r,t=0}^{\infty}e^{-ip(\pi/2-x)}\varepsilon\frac{(\alpha/4^{\rho})^r}{r!}\frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times\bar{H}^{m,n+1}_{p+1,q+2}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right], \tag{3.3}$$

4. Cosine Fourier Series

Let

$$f(x)=(\sin x)^{w-1}F^{E_1;F;F'}_{G;H;H'}\left[\begin{array}{l}(e);(f);(f');\alpha(\sin x)^{2\rho}\\(g);(h);(h');\beta(\sin x)^{2\gamma}\end{array}\right]$$

$$\times\bar{H}^{m,n}_{p,q}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{array}\right.\right]dx=\frac{B_0}{2}+\sum_{p=1}^{\infty}B_p\cos px. \tag{4.1}$$

Integrating both sides with respect to x from 0 to π, we get

$$\frac{B_0}{2} = \frac{1}{\sqrt{\pi}} \sum \epsilon \frac{(\alpha)^r}{r!} \frac{(\beta)^t}{t!}$$

$$\times \bar{H}^{m,n+1}_{p+1,q+2} \left[z \left| \begin{matrix} \left(\frac{2-w}{2} - \rho r - \gamma t, 2\sigma, 1\right) (a_j, \alpha_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega}{2} - 2\rho r - 2\gamma t, \sigma; 1\right) \end{matrix} \right. \right], \quad (4.2)$$

now multiplying e^{imx} both sides in (4.1) and integrating it with respect to form 0 to π and finally, we derive

$$B_p = \frac{e^{ip\pi/2}}{2^{w-1}} \sum_{r,t=0}^{\infty} \epsilon \frac{(\alpha/4^\rho)^r}{r!} \frac{(\beta/4^\gamma)^t}{t!}$$

$$\times \tilde{H}^{m,n+1}_{p+1,q+2} \left[\frac{z}{4^\sigma} \left| \begin{matrix} (1-\omega-2\rho r - 2\gamma t, 2\sigma; 1), (a_j, \alpha_j; A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; A_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega-2\rho r - 2\gamma t \pm m}{2}, \sigma; 1\right) \end{matrix} \right. \right]. \quad (4.3)$$

using (4.3) from (4.1), we get required Cosine Fourier series.

$$(\sin x)^{w-1} F^{E;F;F'}_{G;H;H'} \left[\begin{matrix} (e); (f); (f'); \alpha (\sin x)^{2\rho} \\ (g); (h); (h'); \beta (\sin x)^{2\gamma} \end{matrix} \right]$$

$$\times \bar{H}^{m,n}_{p,q} \left[z (\sin x)^{2\sigma} \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \end{matrix} \right. \right] dx = \frac{1}{\sqrt{(\pi)}} \sum_{r,t=0}^{\infty} \epsilon \frac{(\alpha)^r}{r!} \frac{(\beta)^t}{t!}$$

$$\times \bar{H}^{m,n+1}_{p+1,q+2} \left[z \left| \begin{matrix} \left(\frac{2-w}{2} - \rho r - \gamma t, 2\sigma, 1\right) (a_j, \alpha_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega}{2} - 2\rho r - 2\gamma t, \sigma; 1\right) \end{matrix} \right. \right]$$

$$+ \sum_{p=-\infty}^{\infty} \sum_{r,t=0}^{\infty} \epsilon e^{ip\pi/2} \cos px \frac{(\alpha/4^\rho)^r}{r!} \frac{(\beta/4^\gamma)^t}{t!} \frac{1}{2^{w-2}}$$

$$\times \bar{H}^{m,n+1}_{p+1,q+2} \left[\frac{z}{4^\sigma} \left| \begin{matrix} (1-\omega-2\rho r - 2\gamma t, 2\sigma; 1), (a_j, \alpha_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega-2\rho r - 2\gamma t \pm m}{2}, \sigma; 1\right) \end{matrix} \right. \right] \quad (4.4)$$

5. Sine Fourier Series

Let

$$f(x) = (\sin x)^{w-1} F^{E;F;F'}_{G;H;H'} \left[\begin{array}{c} (e); (f); (f'); \alpha(\sin x)^{2\rho} \\ (g); (h); (h'); \beta(\sin x)^{2\gamma} \end{array} \right]$$

$$\times \bar{H}^{m,n}_{p,q} \left[z(\sin x)^{2\sigma} \left| \begin{array}{c} (a_j, \alpha_j; A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \end{array} \right. \right] dx = \sum_{p=-\infty}^{\infty} C_p \sin px. \tag{5.1}$$

Multiplying by e^{imx} both sides in (5.1) and the integrating it with respect to x from 0 to π, we obtain

$$C_p = \frac{e^{ip\pi/2}}{2^{w-1}} \sum_{r,t=0}^{\infty} \epsilon \frac{(\alpha/4^{\rho})^r}{r!} \frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times \bar{H}^{m,n+1}_{p+1,q+2} \left[\frac{z}{4^{\sigma}} \left| \begin{array}{c} (1-\omega-2\rho r-2\gamma t, 2\sigma; 1), (a_j, \alpha_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega-2\rho r-2\gamma t \pm m}{2}, \sigma; 1 \right) \end{array} \right. \right]. \tag{5.2}$$

Now making an application of (3.3) and (3.6), we get required sine Fourier Series.

$$(2\sin x)^{w-1} F^{E;F;F'}_{G;H;H'} \left[\begin{array}{c} (e); (f); (f'); \alpha(\sin x)^{2\rho} \\ (g); (h); (h'); \beta(\sin x)^{2\gamma} \end{array} \right]$$

$$\times \bar{H}^{m,n}_{p,q} \left[z(\sin x)^{2\sigma} \left| \begin{array}{c} (a_j, \alpha_j; A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \end{array} \right. \right]$$

$$= \sum_{p=-\infty}^{\infty} \sum_{r,t=0}^{\infty} \frac{2\epsilon e^{ip\pi/2}}{i} \sin px \frac{(\alpha/4^{\rho})^r}{r!} \frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times \bar{H}^{m,n+1}_{p+1,q+2} \left[\frac{z}{4^{\sigma}} \left| \begin{array}{c} (1-\omega-2\rho r-2\gamma t, 2\sigma; 1), (a_j, \alpha_j; A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j; B_j)_{1,m}, (b_j, \beta_j)_{m+1,q} \left(\frac{1-\omega-2\rho r-2\gamma t \pm m}{2}, \sigma; 1 \right) \end{array} \right. \right]. \tag{5.3}$$

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