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CERTAIN CLASS OF EULERIAN INTEGRALS WITH THE MULTIVARIABLE I-FUNCTION DEFINED BY PRASAD

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Abstract: In this paper, first we evaluate a class of MacRobert's integral associated with the multivariable I-function defined by Prasad [2], secondly we evaluate a class of MacRobert's integral with. a extension of Hurwitz-Lerch Zeta-function, a general class of polynomials and the multivariable I-function defined by Prasad [2]. We will study several particular cases.

Keywords and Phrases: General class of polynomials, a extension of Hurwitz-Lerch Zeta function, multivariable I-function, Srivastava-Daoust function, multivariable H-function.

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1. Introduction and preliminaries

In this document, we derive an integral involving a extension of Hurwitz-Lerch Zeta-function, a class of multivariable polynomials and the multivariable I-function. For this multivariable I-function, we adopt the contracted notations. The multivariable I-function defined by Prasad [2] is an extension of the multivariable H-function defined by Srivastava et al [5].

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral

$$I(z_1,...,z_r) = I^{0,n_2;0,n_3;...;0,n_r:m',n';...;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;...;p_r,q_r:p',q';...;p^{(r)},q^{(r)}} \left(\begin{array}{c|c} z_1 & (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};...; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};...; \end{array} \right.$$

$$\left.\begin{matrix}(a_{rj};\alpha'_{rj},...,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};...;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\ \\ (b_{rj};\beta'_{rj},...,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};...;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\psi(t_1,...,t_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}ds_1...ds_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y.N. Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as:
$|\arg z_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} a_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} a_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) +$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right) - \left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)} + \sum_{k=1}^{q_3}\beta_{3k}^{(i)} + ... + \sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.3}$$

where $i = 1, ..., r$.
The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form:

$$I(z_1,...,z_r) = 0(|z_1|^{\gamma'_1},...,|z_r|^{\gamma'_r}), \max(|z_1|,...,|z_r|)\to 0$$
$$I(z_1,...,z_r) = 0(|z_1|^{\beta'_1},...,|z_r|^{\beta'_s}), \min(|z_1|,...,|z_r|)\to \infty$$

where $k = 1, ..., z$; $\alpha'_k = \min[\Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, ..., m_k$ and $\beta'_k = \max[\Re((a_j^{(k)} - 1)/a_j^{(k)})]$, $j = 1, ..., n_k$. We will use these following notations in this paper:

$$U = p_2, q_2; p_3, q_3; ...; p_{r-1}, q_{r-1};\ V = 0, n_2; 0, n_3; ...; 0, n_{s-1} \tag{1.4}$$

$$W = (p', q'); ...; (p^{(r)}, q^{(r)});\ X = (m', n'); ...; (m^{(r)}, n^{(r)}) \tag{1.5}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); ...; (a_{(r-1)k}, \alpha'_{(r-1)k}, \alpha''_{(r-1)k}, ..., \alpha^{(r-1)}_{(r-1)k}) \tag{1.6}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); ...; (b_{(r-1)k}, \beta'_{(r-1)k}, \beta''_{(r-1)k}, ..., \beta^{(r-1)}_{(r-1)k}) \tag{1.7}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha''_{rk}, ..., \alpha^{r}_{rk}) : \ \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta''_{rk}, ..., \beta^{r}_{rk}) \tag{1.8}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; ...; (a^{(r)}_k, \alpha^{(r)}_k)_{1,p^{(r)}}; \ B' = (b'_k, \beta'_k)_{1,q'}; ...; (b^{(r)}_k, \beta^{(r)}_k)_{1,q^{(r)}} \tag{1.9}$$

The contracted form is:

$$I(z_1, ..., z_r) = I^{V;0,n_r;X}_{U;p_r,q_r;W} \left(\begin{array}{c|c} z_1 & A; \mathfrak{A} : A' \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & B; \mathfrak{B} : B' \end{array} \right) \tag{1.10}$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S^{h_1,...,h_u}_L[z_1, ..., z_u] = \sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u \leq L} (-L)_{h_1R_1+...+h_uR_u} B(E; R_1, ..., R_u) \frac{z_1^{R_1}...z_u^{R_u}}{R_1!...R_u!} \tag{1.11}$$

The coefficients $B(E; R_1, ..., R_u)$ are arbitrary constants, real or complex.
We will note;

$$B_u = \frac{(-L)_{h_1R_1+...+h_uR_u} B(E; R_1, ..., R_u)}{R_1!...R_u!} \tag{1.12}$$

2. Extension of the Hurwitz-Lerch Zeta function

The extension of the Hurwitz-Lerch Zeta function $\phi(z, \mathfrak{s}, a)$ is introduced by Srivastava et al ([6], eq.(6.2), page 503) as follows

$$\phi^{(\rho_1,...,\rho_p,\sigma_1,...,\sigma_q)}_{(\lambda_1,...,\lambda_p,\mu_1,...,\mu_q)}(z, \mathfrak{s}, a) = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{(a+n)^{\mathfrak{s}} \prod_{j=1}^{q}(\mu_j)_{n\sigma_j}} \times \frac{z^n}{n!} \tag{2.1}$$

with $p, q \in \mathbb{N}_0, \lambda_j \in \mathbb{C}(j = 1, ..., p),\ a, \mu_j \in \mathbb{C}/\mathbb{Z}_0^*\ (j = 1, ..., q), \rho_j, \sigma_k \in R^+$ $j = 1, ..., p;\ k = 1, ..., q.$
where $\Delta > -1$ when $\mathfrak{s}, z \in \mathbb{C}; \Delta = -1$ and $\mathfrak{s} \in \mathbb{C}$, when $|z| < \nabla^*$, $\Delta = -1$ and $Re(x) > \frac{1}{2}$ when $|z| = \nabla^*$

$$\nabla^* = \prod_{j=1}^{p} \rho_j^{\rho_j} \prod_{j=1}^{q} \sigma_j^{\sigma_j}; \quad \Delta = \sum_{j=1}^{q} \sigma_j - \sum_{j=1}^{p} \rho_{ji\chi} = \mathfrak{s} + \sum_{j=1}^{q} \mu_j - \sum_{j=1}^{p} \lambda_j + \frac{p-q}{2}$$

We denote these conditions the conditions (f).

3. Required integral

We have the following integral, see Mac Robert's [1]

Lemma 3.1.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]dt$$

$$=\frac{(1+c)^{-\lambda}(1+d)^{-\mu}\Gamma(\lambda)\Gamma(\mu)\Gamma(\lambda+\mu-\alpha-\beta)}{(b-a)\Gamma(\lambda+\mu-\alpha)\Gamma(\lambda+\mu-\beta)} \tag{3.1}$$

valid for $\Re(\mu)>0$, $\Re(\lambda)>0$, $\Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$ and $b-a+c(t-a)+d(b-t)\neq 0$.

4. Eulerian integral involving the multivariable I-function

Let $G=\Gamma(\mu)(b-a)^{-1}(1+c)^{-\lambda}(1+d)^{-\mu}$, $X=\dfrac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}$ and $b_n=\dfrac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{(a+n)^{\mathfrak{s}}\prod_{j=1}^{q}(\mu_j)_{n\sigma_j}}$

The main result to be established here is

Theorem 4.1.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$I(z_1X^{v_1},\dots,z_rX^{v_r})dt = I^{V;0,n_r+2;X}_{U:p_r+2,q_r+2;W}\left(\begin{array}{c|l} z_1 & A;(1-\lambda;v_1,\dots,v_r), \\ \cdots & \cdots \\ \cdots & B;(1-\lambda-\mu+\alpha;v_1,\dots,v_r), \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{l}(1-\lambda-\mu+\alpha+\beta;v_1,\dots,v_r),\mathfrak{A}:A' \\ \cdots \\ (1-\lambda-\mu+\beta;v_1,\dots,v_r),\mathfrak{B}:B'\end{array}\right) \tag{4.1}$$

provided that $\Re(\mu)>0$, $\Re(\lambda)>0$, $\Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$; $v_i>0$, $i=1,\dots,r$ $b-a+c(t-a)+d(b-t)\neq 0$, the conditions (f) are verified and the conditions of existence of the multivariable function are satisfied.

Proof. Let

$$M\{\} = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)\{\}$$

We first replace the multivariable I-function by its Mellin-Barnes contour integral with the help of (1.1), we get

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$M\left\{\left[\frac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}\right]^{\sum_{j=1}^{r}v_js_j} z_1^{s_1}...z_r^{s_r}ds_1...ds_r\right\}dt \tag{4.2}$$

Assuming the inversion of order of integrations in (4.2) to be permissible by absolute (and uniform) convergence of integrals involved above, we have

$$M\left\{(1+c)^{\sum_{j=1}^{r}s_jv_j}z_1^{s_1}...z_r^{s_r}\int_a^b(t-a)^{\lambda+\sum_{j=1}^{r}v_js_j-1}(b-t)^{\mu-1}\,[b-a+c(t-a)+\right.$$

$$\left. d(b-t)]^{-\lambda-\mu-\sum_{j=1}^{r}v_js_j}\;{}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]dt\right\}ds_1...ds_r \tag{4.3}$$

Now evaluating the inner integral with the help of Lemme (3.1) valid for $\Re(\mu)>0$, $\Re(\lambda)>0$, $\Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$ and finally interpreting the resulting Mellin-Barnes contour integrals as a multivariable I-function, we obtain the desired result.

5. Particular cases

The integral formula (4.1) has manifold generality. By specializing the various parameters and variables involved, the formula can suitable applied to derive the corresponding results involving remarkably wide range of useful functions.

(i) Putting $\beta=\mu$ in (4.1), provided that $\Re(\lambda)>0$, $\Re(\mu)>0$ and $b-a+c(t-a)+d(b-t)\neq 0$; $b\neq a$; $t\in[a,b]$, $\left|\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right|<1$ and using the transformation

$${}_2F_1(a,b;b;z)=(1-z)^{-a} \tag{5.1}$$

we get
Corollary 5.1.

$$\int_a^b (t-a)^{\lambda-\alpha-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda+\alpha-\mu} I(z_1 X^{v_1},...,z_r X^{v_r})dt$$

$$= G_1 I^{V;0,n_r+1;X}_{U:p_r+1,q_r+1;W}\left(\begin{array}{c|l} z_1 & A;(1-\lambda+\alpha;v_1,...,v_r),\mathfrak{A}:A' \\ \cdots & \cdots \\ \cdots & \cdots \\ z_r & B;(1-\lambda-\mu+\alpha;v_1,...,v_r),\mathfrak{B}:B' \end{array}\right) \tag{5.2}$$

where $G_1 = \dfrac{\Gamma(\mu)(1+c)^{-\lambda+\alpha}(1+d)^{-\mu}}{b-a}$.

(ii) Putting $c=d=0$ and $v_1=...=v_r=1$, we get
Corollary 5.2.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}\ {}_2F_1\left[\alpha,\beta;\mu;\frac{b-t}{b-a}\right] I(z_1 Y^{v_1},...,z_r Y^{v_r})dt = \Gamma(\mu)(b-a)^{\lambda+\mu-1}$$

$$I^{V;0,n_r+2;X}_{U:p_r+2,q_r+2;W}\left(\begin{array}{c|l} z_1 & A;(1-\lambda;1,...,1),(1-\lambda-\mu+\alpha+\beta;1,...,1),\mathfrak{A}:A' \\ \cdots & \cdots \\ \cdots & \cdots \\ z_r & B;(1-\lambda-\mu+\alpha;1,...,1),(1-\lambda-\mu+\beta;1,...,1),\mathfrak{B}:B' \end{array}\right) \tag{5.3}$$

where $Y = \dfrac{b-t}{b-a}$

(iii) Putting $\lambda=\mu=\frac{1}{2}$; $c=d=1$; $v_1=...=v_r=1$ and $\alpha=0$, we get
Corollary 5.3.

$$\int_a^b [(t-a)(b-t)]^{-\frac{1}{2}}\ {}_2F_1\left[\alpha,\beta;\mu;\frac{b-t}{b-a}\right] I(z_1 Y^{v_1},...,z_r Y^{v_r})dt$$

$$= \sqrt{\pi} I^{V;0,n_r+1;X}_{U:p_r+1,q_r+1;W}\left(\begin{array}{c|l} z_1 & A;(\frac{1}{2};1,...,1),\mathfrak{A}:A' \\ \cdots & \cdots \\ \cdots & \cdots \\ z_r & B;(0;1,...,1),\mathfrak{B}:B' \end{array}\right) \tag{5.4}$$

6. Multivariable I-function with the extension of Hurwitz-Lerch Zeta-function and a class of polynomials

In this section, we evaluate a class of MacRobert's integral associated with the extension of Hurwitz-Lerch Zeta function, a general class of polynomials and the multivariable I-function defined by Prasad [2], we have

Theorem 6.1.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$${}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]\phi^{(\rho_1,...,\rho_p,\sigma_1,...,\sigma_q)}_{(\lambda_1,...,\lambda_p,\mu_1,...,\mu_q)} S_L^{h_1,...,h_u}(y_1X^{\gamma_1},...,y_uX^{\gamma_u})$$

$$I(z_1X^{v_1},...,z_rX^{v_r})dt = G\sum_{k=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L}\frac{b_kz^k}{k!}B_uy_1^{R_1}...y_u^{R_u}$$

$$I_{U:p_r+2,q_r+2;W}^{V;0,n_r+2;X}\left(\begin{array}{c|l} z_1 & A;(1-\lambda-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r), \\ \cdots & \cdots \\ \cdots & B;(1-\lambda-\mu+\alpha-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r), \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{l}(1-\lambda-\mu+\alpha+\beta-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\mathfrak{A}:A' \\ \cdots \\ (1-\lambda-\mu+\beta-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\mathfrak{B}:B'\end{array}\right) \quad (6.1)$$

provided that $\Re(\mu)>0$, $\Re(\lambda)>0$, $\Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$; $v_i>0$, $i=1,...,r$; $\gamma_j>0$, $j=1,...,u$, $\xi>0$, $b-a+c(t-a)+d(b-t)\neq 0$, the conditions (f) are verified and the conditions of existence of the multivariable function are satisfied.

Proof. Let

$$M\{\} = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)\{\}$$

To prove (6.1), expressing the general class of polynomials of several variables $S_L^{h_1,...,h_u}[.]$ in series with the help of (1.11), a extension of the Hurwitz-Lerch Zeta-function $\phi^{\rho_1,...,\rho_p,\sigma_1,...,\sigma_q}_{(\lambda_1,...,\lambda_p,\mu_1,...,\mu_q)}(.;\mathfrak{s},a)$ in series with the help of (2.1) and the I-function of r variables in Mellin-Barnes contour integral with the help of (1.1), changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we get

$$\sum_{k=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L}B_uy_1^{R_1}...y_u^{R_u}\frac{b_kz^k}{k!}\int_a^b(t-a)^{\lambda-1}(b-t)^{\mu-1}$$

$$[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$M\left\{\left[\frac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}\right]^{k\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}v_js_j} z_1^{s_1}...z_rs^{s_r}ds_1...ds_r\right\}dt \tag{6.2}$$

Assuming the inversion of order of integrations in (6.2) to be permissible by absolute (and uniform) convergence of integrals involved above, we have

$$\sum_{k=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} B_u y_1^{R_1}...y_u^{R_u}\frac{b_kz^k}{k!}M\left\{(1+c)^{k\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}v_js_j} z_1^{s_1}...z_rs^{s_r}\right.$$

$$\int_a^b (t-a)^{\lambda+k\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}v_js_j-1}(b-t)^{\mu-1}\,{}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$\left.[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu-k\xi-\sum_{j=1}^{u}R_j\gamma_j-\sum_{j=1}^{r}v_js_j}dt\right\}ds_1...ds_r \tag{6.3}$$

To evaluate the inner integral, use the Lemme valid for $\Re(\mu)>0$, $\Re(\lambda)>0$, $\Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$, $b-a+c(t-a)+d(b-t)\neq 0$, the conditions (f) are verified and finally interpreting the resulting Mellin-Barnes contour integrals as a multivariable I-function, we obtain the desired result.

7. Multivariable H-function

If $U=V=A=B=0$, the multivariable I-function defined by Prasad [2] reduces to the multivariable H-function defined by Srivastava et al [5]. We have the following results.

Corollary 7.1.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$${}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]\phi^{(\rho_1,...,\rho_p,\sigma_1,...,\sigma_q)}_{(\lambda_1,...,\lambda_p,\mu_1,...,\mu_q)}(zX^{\xi};\mathfrak{s},a)S_L^{h_1,...,h_u}$$

$$(y_1X^{\gamma_1},...,y_uX^{\gamma_u})H(z_1X^{v_1},...,z_rX^{v_r})dt = G\sum_{k=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L}\frac{b_kz^k}{k!}B_uy_1^{R_1}...y_u^{R_u}$$

$$H^{0,n_r+2;X}_{p_r+2,q_r+2;W}\left(\begin{array}{c|l} z_1 & (1-\lambda-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\\ \cdots & \cdots\\ \cdots & (1-\lambda-\mu+\alpha-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\\ z_r & \end{array}\right.$$

$$\left.\begin{array}{l}(1-\lambda-\mu+\alpha+\beta-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\mathfrak{A}:A'\\ \cdots\\ (1-\lambda-\mu+\beta-k\xi-\sum_{i=1}^{u}R_i\gamma_i;v_1,...,v_r),\mathfrak{B}:B'\end{array}\right) \tag{7.1}$$

under the same conditions and notations that (6.1) with $U = V = A = B = 0$.

8. Srivastava-Daoust function

If

$$B(L; R_1, ..., R_u) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1\theta'_j+...+R_u\theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{R_1\psi'_j+...+R_u\psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u\delta_j^{(u)}}} \quad (8.1)$$

then the general class of multivariable polynomial $S_L^{h_1,...,h_u}[z_1, ..., z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [3].

$$F^{1+\bar{A};B';...;B^{(u)}}_{\bar{C}:D';...;D^{(u)}} \left(\begin{array}{c|l} z_1 & [(-L); R_1, ..., R_u][(a); \theta', ..., \theta^{(u)}] : [(b'); \phi']; ...; [(b^{(u)}); \phi^{(u)}] \\ \cdots & \cdots \\ \cdots & \cdots \\ z_u & [(c); \psi', ..., \psi^{(u)}] : [(d'); \delta']; ...; [(d^{(u)}); \delta^{(u)}] \end{array} \right) \quad (8.2)$$

and we have the following formula

Corollary 8.1.

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$${}_2F_1\left[\alpha, \beta; \mu; \frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right] \phi^{(\rho_1,...,\rho_p,\sigma_1,...,\sigma_q)}_{(\lambda_1,...,\lambda_p,\mu_1,...,\mu_q)}(zX^{\xi}; \mathfrak{s}, a) F^{1+\bar{A};B';...;B^{(u)}}_{\bar{C}:D';...;D^{(u)}}$$

$$\left(\begin{array}{c|l} y_1 X^{\gamma_1} & [(-L); R_1, ..., R_u][(a); \theta', ..., \theta^{(u)}] : [(b'); \phi']; ...; [(b^{(u)}); \phi^{(u)}] \\ \cdots & \cdots \\ \cdots & \cdots \\ y_u X^{\gamma^{(u)}} & [(c); \psi', ..., \psi^{(u)}] : [(d'); \delta']; ...; [(d^{(u)}); \delta^{(u)}] \end{array} \right)$$

$$I(z_1 X^{v_1}, ..., z_r X^{v_r})dt = G \sum_{k=0}^{\infty} \sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u \leq L} \frac{b_k z^k}{k!} y_1^{R_1} ... y_u^{R_u} B'_u$$

$$I^{V;0,n_r+2;X}_{U:p_r+2,q_r+2;W} \left(\begin{array}{c|l} z_1 & A; (1-\lambda-k\xi-\sum_{i=1}^u R_i\gamma_i; v_1, ..., v_r), \\ \cdots & \cdots \\ \cdots & B; (1-\lambda-\mu+\alpha-k\xi-\sum_{i=1}^u R_i\gamma_i; v_1, ..., v_r), \\ z_r & \end{array} \right.$$

$$\left. \begin{array}{l} (1-\lambda-\mu+\alpha+\beta-k\xi-\sum_{i=1}^u R_i\gamma_i; v_1, ..., v_r), \mathfrak{A} : A' \\ \cdots \\ (1-\lambda-\mu+\beta-k\xi-\sum_{i=1}^u R_i\gamma_i; v_1, ..., v_r), \mathfrak{B} : B' \end{array} \right) \quad (8.3)$$

under the same conditions and notations that (6.1) and

$B'_u = \frac{(-L)_{h_1R_1+\ldots+h_uR_u}B(E;R_1,\ldots,R_u)}{R_1!\ldots R_u!}$; $B(L;R_1,\ldots,R_u)$ is defined by (8.1).

9. Conclusion

In this paper we have evaluated a general Eulerian integral involving the multivariable I-function defined by Prasad [2], a class of polynomials of several variables and a extension of the Hurwitz-Lerch Zeta function. The integral established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

References

[1] MacRobert T.M., Functions of complex variable, 5^{th} edition, Macmillan and Co, New York (1962).

[2] Y.N. Prasad, Multivariable I-function, Vijnana Parishad Anusandhan Patrika 29 (1986), page 231-237.

[3] Srivastava H.M. and Daoust M.C., Certain generalized Neumann expansions associated with Kampéde Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[4] Srivastava H.M. And Garg M., Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[5] H.M. Srivastava And R.Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

[6] H.M. Srivastava, R.K. Saxena, T.K. Pogány, R. Saxena, Integral and computational representations of the extended HurwitzLerch zeta function, Integr.Transf. Spec. Funct. 22 (2011) 487506

[7] F.Y. Ayant, Multivariable I-function of relating some multiple integrals, Journal of Ramanujan Society of Mathematics and Mathematical Sciences, Vol. 5, No.2, pp. 89-98, 2016.