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K BANHATTI AND K HYPER-BANHATTI INDICES OF WINDMILL GRAPHS

V. R. Kulli[1], B. Chaluvaraju[2] and H. S. Boregowda[3]

[1] Department of Mathematics,
Gulbarga University, Gulbarga - 585 106, INDIA.
E-mail: vrkulli@gmail.com

[2] Department of Mathematics, Bangalore University,
Jnana Bharathi Campus, Bangalore -560 056, INDIA
E-mail: bchaluvaraju@gmail.com

[3] Department of Studies and Research in Mathematics,
University Constituent College Campus,
Tumkur University, Tumkur - 571 103, INDIA.
E-mail: bgsamarasa@gmail.com

Abstract: Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The first and second K Banhatti indices of G are defined as $B_1(G) = \sum_{ue}[d_G(u) + d_G(e)]$ and $B_2(G) = \sum_{ue}[d_G(u)d_G(e)]$, where ue means that the vertex u and edge e are incident in G. The first and second K hyper-Banhatti indices of G are defined as $HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$ and $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2$, respectively. In this paper, we compute the first and second K Banhatti indices of windmill graphs. In addition, the first and second K hyper-Banhatti indices of dutch and french windmill graphs are determined.

Keywords: K Banhatti indices, K hyper-Banhatti indices, dutch windmill graph and french windmill graph.

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1. Introduction

All graphs considered in this paper are finite, connected, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [5].

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. In Chemical Science, the physico- chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [3].

The first and second K Banhatti indices of G are defined as $B_1(G) = \sum_{ue}[d_G(u)+d_G(e)]$ and $B_2(G) = \sum_{ue}[d_G(u)d_G(e)]$, where ue means that the vertex u and edge e are incident in G. The K Banhatti indices were introduced by Kulli in [6]. Recently many other indices were studied, for example, in [1], [4], [7], [8] and [10].

The first and second K hyper-Banhatti indices of G are defined as $HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$ and $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2$. The K hyper-Banhatti indices were introduced by Kulli in [9].

In this paper, we consider dutch and french windmill graphs and determine their K Banhatti indices and also their K hyper-Banhatti indices.

2. Dutch windmill graph

The dutch windmill graph $D_n^{(m)}$ is the graph obtained by taking $m \geq 1$ copies of the cycle C_n; $n \geq 3$ with a vertex in common. This graph is shown in Figure-1. The dutch windmill graph $D_3^{(m)}$ is called a friendship graph. For more details on windmill graph, see [2].

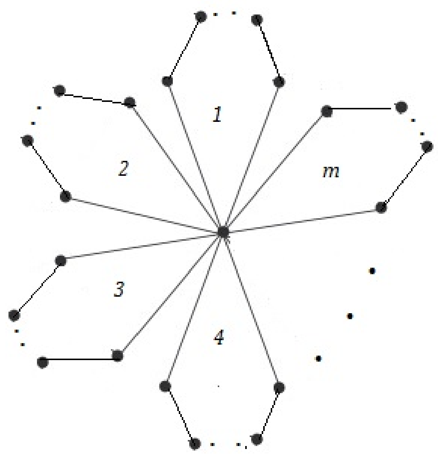

Figure 1: Dutch windmill graph of $D_n^{(m)}$.

2.1. K Banhatti indices of $D_n^{(m)}$

Theorem 2.1. *The first K Banhatti index of a dutch windmill graph is* $4m(3m+2n-3)$.

Proof: Let $G = D_n^{(m)}$, where $D_n^{(m)}$ is a dutch windmill graph. By algebraic method, we get $|V(G)| = m(n-1)+1$ and $|E(G)| = mn$. We have two partitions of the vertex set $V(G)$ as follows:
$V_2 = \{v \in V(G) : d_G(v) = 2\}, |V_2| = m(n-1)$, and
$V_{2m} = \{v \in V(G) : d_G(v) = 2m\}, |V_{2m}| = 1$.
Also we have two partitions of the edge set $E(G)$ as follows:
$E_4 = E_4^* = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 2m\}, |E_4| = |E_4^*| = m(n-2)$, and
$E_{2m+2} = E_{2(2m)}^* = \{uv \in E(G) : d_G(u) = d_G(v) = 2\}, |E_{2m+2}| = |E_{2(2m)}^*| = 2m$.
The edge degree partition of the dutch windmill graph G is given in table-1.

$d_G(u), d_G(v) : e = uv \in E(G)$	$(2,2)$	$(2,2m)$
$d_G(e)$	2	$2m$
Number of edges	$m(n-2)$	$2m$

Table 1: Edge degree partition of $D_n^{(m)}$.

Now $B_1(G) = \sum_{ue}[d_G(u) + d_G(e)] = \sum_{e=uv\in E_4}[(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] + \sum_{e=uv\in E_{2m+2}}[(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))]$
$= m(n-2)[(2+2) + (2+2)] + 2m[(2+2m) + (2m+2m)]$
$= 8m(n-2) + 2m(2+6m)$
$= 4m(3m+2n-3)$.

Corollary 2.1. *The first K Banhatti index of a friendship graph* $D_3^{(m)}$ *is* $12m(m+1)$.

Theorem 2.2. *The second K Banhatti index of a dutch windmill graph is* $8m(m^2+m+n-2)$.

Proof: Let $G = D_n^{(m)}$, where $D_n^{(m)}$ is a dutch windmill graph. Now $B_2(G) = \sum_{ue}[d_G(u)d_G(e)] = \sum_{e=uv\in E_4^*}[(d_G(u)d_G(e)) + (d_G(v)d_G(e))] +$
$\sum_{e=uv\in E_{2(2m)}^*}[(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$
$=m(n-2)[(2\times 2) + (2\times 2)] + 2m[(2\times 2m) + (2m\times 2m)]$
$= 8m(n-2) + 2m(4m+4m^2)$
$= 8m(m^2+m+n-2)$.

Corollary 2.2. *The second K Banhatti index of a friendship graph* $D_3^{(m)}$ *is* $8m(m^2+m+1)$.

2.2. K hyper - Banhatti indices of $D_n^{(m)}$

Theorem 2.3. *The first K hyper-Banhatti index of a dutch windmill graph is* $8m(5m^2+2m+4n-7)$.

Proof: Let $G = D_n^{(m)}$, where $D_n^{(m)}$ is a dutch windmill graph. Now $HB_1(G) = \sum_{ue}[d_G(u)+d_G(e)]^2 = \sum_{e=uv\in E_4}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]+$
$\sum_{e=uv\in E_{2m+2}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]$
$= m(n-2)[(2+2)^2+(2+2)^2]+2m[(2+2m)^2+(2m+2m)^2]$
$= 32m(n-2)+2m(4+8m+4m^2+16m^2)$
$= 8m(5m^2+2m+4n-7)$.

Corollary 2.3. *The first K hyper-Banhatti index of a friendship graph* $D_3^{(m)}$ *is* $8m(5m^2+2m+5)$

Theorem 2.4. *The second K hyper-Banhatti index of a dutch windmill graph is* $32m(m^4+m^2+n-2)$.

Proof: Let $G = D_n^{(m)}$, where $D_n^{(m)}$ is a dutch windmill graph. Now $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2 = \sum_{e=uv\in E_4^*}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2]+$
$\sum_{e=uv\in E^*_{2(2m)}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2]$
$= m(n-2)[(2\times 2)^2+(2\times 2)^2]+2m[(2\times 2m)^2+(2m\times 2m)^2]$
$= 32m(n-2)+2m(16m^2+16m^4)$
$= 32m(m^4+m^2+n-2)$.

Corollary 2.4. *The second K hyper-Banhatti index of a friendship graph* $D_3^{(m)}$ *is* $32m(m^4+m^2+1)$.

3. French windmill graph

The french windmill graph $F_n^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the complete graph $K_n; n \geq 2$ with a vertex in common. This graph is shown in Figure-2. The french windmill graph $F_2^{(m)}$ is called a star graph, the french windmill graph $F_3^{(m)}$ is called a friendship graph and the french windmill graph $F_3^{(2)}$ is called a butterfly graph. Further, note that $F_3^{(m)}$ is same as $D_3^{(m)}$.

3.1. K Banhatti indices of $F_n^{(m)}$

Theorem 3.1. *The first K Banhatti index of a french windmill graph is* $m(n^2-3n+2)(3n-5)+(n-1)m[3(n-1)(m+1)-4]$.

Proof: Let $G = F_n^{(m)}$, where $F_n^{(m)}$ is a french windmill graph.

Clearly, we get $|V(G)| = m(n-1)+1$ and $|E(G)| = \dfrac{mn(n-1)}{2}$.

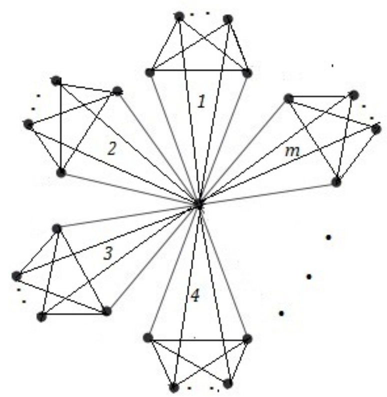

Figure 2: French windmill graph of $F_n^{(m)}$.

We have two partitions of the vertex set $V(G)$ as follows:
$V_{n-1} = \{v \in V(G) : d_G(v) = n-1\}, |V_{n-1}| = m(n-1)$, and
$V_{(n-1)m} = \{v \in V(G) : d_G(v) = (n-1)m\}, |V_{(n-1)m}| = 1$.
Also we have two partitions of the edge set $E(G)$ as follows:
$E_{2(n-1)} = E^*_{2(n-1)} = \{uv \in E(G) : d_G(u) = d_G(v) = n-1\}$,
$|E_{2(n-1)}| = |E^*_{2(n-1)}| = m[\dfrac{n(n-1)}{2} - (n-1)] = \dfrac{m(n^2-3n+2)}{2}$, and
$E_{2(n-1)m} = E^*_{2(n-1)m} = \{uv \in E(G) : d_G(u) = n-1, d_G(v) = (n-1)m\}$,
$|E_{2(n-1)m}| = |E^*_{2(n-1)m}| = (n-1)m$.
The edge degree partition of the french windmill graph G is given in table-2. Now

$d_G(u), d_G(v) : e = uv \in E(G)$	$(n-1, n-1)$	$((n-1), (n-1)m)$
$d_G(e)$	$2n-4$	$(n-1)(m+1)-2$
Number of edges	$\dfrac{m(n^2-3n+2)}{2}$	$(n-1)m$

Table 2: Edge degree partition of $F_n^{(m)}$.

$B_1(G) = \sum_{ue}[d_G(u) + d_G(e)] = \sum_{e=uv \in E_{2(n-1)}}[(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] +$

$\sum_{e=uv\in E_{2(n-1)m}}[(d_G(u)+d_G(e))+(d_G(v)+d_G(e))]$

$= \dfrac{m(n^2-3n+2)}{2}[(n-1)+(2n-4)+(n-1)+(2n-4)]+(n-1)m[(n-1)+ (n-1)(m+1)-2+(n-1)m+(n-1)(m+1)-2]$

$= m(n^2-3n+2)(3n-5)+(n-1)m[3(n-1)(m+1)-4].$

Corollary 3.1. *The first K Banhatti index of a star graph* $F_2^{(m)}$ *is* $m(3m-1)$.

Corollary 3.2. *The first K Banhatti index of a butterfly graph* $F_3^{(2)}$ *is* 72.

Theorem 3.2. *The second K Banhatti index of a french windmill graph is* $m(n-1)^2\{2(n-2)^2+(n-1)(m+1)^2-2(m+1)\}$.

Proof: Let $G = F_n^{(m)}$, where $F_n^{(m)}$ is a french windmill graph. Now $B_2(G) = \sum_{ue}[d_G(u)d_G(e)] = \sum_{e=uv\in E_{2(n-1)}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]$
$+\sum_{e=uv\in E_{2(n-1)m}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]$

$= \dfrac{m(n^2-3n+2)}{2}[(n-1)(2n-4)+(n-1)(2n-4)]$
$+(n-1)m\{(n-1)[(n-1)(m+1)-2]+(n-1)m[(n-1)(m+1)-2]\}$

$= 2m(n^2-3n+2)(n-1)(n-2)$
$+(n-1)m[(n-1)\{(n-1)(m+1)-2+m[(n-1)(m+1)-2]\}]$

$= m(n-1)^2\{2(n-2)^2+(n-1)(m+1)^2-2(m+1)\}.$

Corollary 3.3. *The second K Banhatti index of a star graph* $F_2^{(m)}$ *is* $m(m-1)(m+1)$.

Corollary 3.4. *The second K Banhatti index of a butterfly graph* $F_3^{(2)}$ *is* 112.

3.2. K hyper - Banhatti indices of $F_n^{(m)}$

Theorem 3.3. *The first K hyper-Banhatti index of a french windmill graph is* $(n-1)m[(n-2)(3n-5)^2+(n-1)^2(5m^2+8m+5)+8-12(n-1)(m+1)]$.

Proof: Let $G = F_n^{(m)}$, where $F_n^{(m)}$ is a french windmill graph. Now $HB_1(G) = \sum_{ue}[d_G(u)+d_G(e)]^2 = \sum_{e=uv\in E_{2(n-1)}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]+$
$\sum_{e=uv\in E_{2(n-1)m}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]$

$= \dfrac{m(n^2-3n+2)}{2}\{[(n-1)+(2n-4)]^2+[(n-1)+(2n-4)]^2\}$

$+ (n-1)m\{[(n-1)+(n-1)(m+1)-2]^2+[(n-1)m+(n-1)(m+1)-2]^2\}$

$= m(n-1)(n-2)(3n-5)^2+(n-1)m[(n-1)^2(m+2)^2+4-4(n-1)(m+2)+(n-1)^2(2m+1)^2+4-4(n-1)(2m+1)]$

$= (n-1)m[(n-2)(3n-5)^2+(n-1)^2(5m^2+8m+5)+8-12(n-1)(m+1)].$

Corollary 3.5. *The first K hyper-Banhatti index of a star graph $F_2^{(m)}$ is $m[5m^2-4m+1]$.*

Corollary 3.6. *The first K hyper-Banhatti index of a butterfly graph $F_3^{(2)}$ is 464.*

Theorem 3.4. *The second K hyper-Banhatti index of a french windmill graph is $m(n-1)^3\{4(n-2)^3+(m^2+1)(mn+n-m-3)^2\}$.*

Proof: Let $G = F_n^{(m)}$, where $F_n^{(m)}$ is a french windmill graph. Now $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2 = \sum_{e=uv\in E_{2(n-1)}}[(d_G(u)d_G(e))^2$
$+(d_G(v)d_G(e))^2] + \sum_{e=uv\in E_{2(n-1)m}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2]$

$$= \frac{m(n^2-3n+2)}{2}\{[(n-1)(2n-4)]^2+[(n-1)(2n-4)]^2\}$$
$$+ (n-1)m\{[(n-1)[(n-1)(m+1)-2]^2+[(n-1)m((n-1)(m+1)-2)]^2\}$$

$$= m(n-1)(n-2)4(n-1)^2(n-2)^2+(n-1)m(n-1)^2[(n-1)(m+1)-2]^2(m^2+1)$$

$$= 4m(n-1)^3(n-2)^3+(n-1)^3m(m^2+1)[mn+n-m-3]^2$$

$$= m(n-1)^3\{4(n-2)^3+(m^2+1)(mn+n-m-3)^2\}.$$

Corollary 3.7. *The second K hyper-Banhatti index of a star graph $F_2^{(m)}$ is $m(m^2+1)(m-1)^2$.*

Corollary 3.8. *The second K hyper-Banhatti index of a butterfly graph $F_3^{(2)}$ is 1344.*

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