J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.5, No.2 (2016), pp. 89-98

MULTIVARIABLE I-FUNCTION OF RELATING SOME MULTIPLES INTEGRALS

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Abstract: Recently Kushwah et al [1] have provided closed-form expressions for a number of general integrals involving the I-function of two variables. Motivated by this recent work, we establish several multiples integrals involving the products of the generalized multivariable I-function in terms of multiple Mellin-Barnes type contour integral. Some attractives integrals involving the product of orthogonal polynomials and generalized hypergeometric function have also been obtained as particular cases of the main results.

Keywords and Phrases: Generalized multivariable I-function, multiple integral, Jacobi polynomial, Legendre function, Hermite polynomial , Generalized hypergeometric function.

2000 AMS Subject Classification: 33C99, 33C60, 44A20.

1. Introduction and preliminaries

The object of this document is to study a number of a general integrals involving the generalized multivariable I-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [4]. The generalized multivariable I-function is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the generalized multiple I-function throughout our present study and will be defined and represented as follows.

For convenience, we will ask.

$$R_i = p_i, q_i; R; R_{i'} = p_{i'}, q_{i'}; R : ... : R_{i^{(r)}} = p_{i^{(r)}}, q_{i^{(r)}}; R \tag{1.1}$$

$$A(r) = \{(a_j; \alpha_j^{(1)}, ..., \alpha_j^{(r)})_{1,n}\}, \{(a_{ji}; \alpha_{ji}^{(1)}, ..., \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.2}$$

$$B(r) = \{(b_j; \beta_j^{(1)}, ..., \beta_j^{(r)})_{1,m}\}, \{(b_{ji}; \beta_{ji}^{(1)}, ..., \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.3}$$

$$C^{(1)} = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \{(c_{ji}^{(1)}; \gamma_{ji}^{(1)})_{n_1+1,p_{i'}}\} \tag{1.4}$$

$$C^{(r)} = \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \{(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.5}$$

$$D^{(1)} = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \{(d_{ji'}^{(1)}; \delta_{ji'}^{(1)})_{m_1+1,q_{i'}}\} \tag{1.6}$$

$$D^{(r)} = \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \{(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.7}$$

$$dx_1...dx_r = dx_j \quad \text{and} \quad ds_1...ds_r = ds_j \tag{1.8}$$

We have

$$I(x_1, ..., x_r) = I_{R_i:R_{i'};...;R_{i^{(r)}}}^{m,n:m_1,n_1;...;m_r,n_r}\left(\begin{array}{c|c} x_1 & \\ . & A(r) : C^{(1)}; ...; C^{(r)} \\ . & \\ . & B(r) : D^{(1)}; ...; D^{(r)} \\ x_r & \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\psi(s_1, ..., s_r)\prod_{j=1}^{r}\phi(s_j)x_j^{s_j}ds_1...ds_r \tag{1.9}$$

where

$$\psi(s_1, ..., s_r) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \sum_{k=1}^{r}\beta_j^{(k)}s_k)\prod_{j=1}^{n}\Gamma(1 - a_j + \sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}\left[\prod_{j=n+1}^{p_i}\Gamma(a_{ji} - \sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=m+1}^{q_i}\Gamma(1 - b_{ji} + \sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)\right]} \tag{1.10}$$

and

$$\phi_j(s_j) = \frac{\prod_{l=1}^{m_j}\Gamma(d_l^{(j)} - \delta_l^{(j)}s_j)\prod_{l=1}^{n_j}\Gamma(1 - c_l^{(j)} + \gamma_l^{(j)}s_j)}{\sum_{i=1}^{r}\left[\prod_{l=m_j+1}^{q_i}\Gamma(1 - d_{lj}^{(j)} + \delta_{lj}^{(j)}s_j)\prod_{l=n_j+1}^{p_i}\Gamma(c_{lj}^{(j)} - \gamma_{li}^{(j)}s_j)\right]} \tag{1.11}$$

where $j = 1$ to r and k=1 to r.
Suppose, as usual, that the parameters $a_i, j = 1, ..., n$; $a_ji, j = n+1, ..., p_i$; $b_{ji}, j = 1, ..., q_i$; $c_j^{(k)}, j = 1, ..., n$ $c_{ji^{(k)}}, j = n_k + 1, ..., q_{i^{(k)}}$; $d_j^{(k)}, j = 1, ..., m_k$; $d_{ji^{(k)}}^{(k)}, j = m_k + 1, ..., q_{i^{(k)}}$; with k=1 to r, i=1 to R, $i^{(k)} = 1$ to R are complex numbers, and

the $\alpha's, \beta's, \gamma's$ and are assumed to be positive real numbers for standardization purpose such that

$$U_k = \sum_{j=1}^{n} \alpha_j^{(k)} + \sum_{j=1}^{m} \beta_j^{(k)} + \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_i^{(k)}} \gamma_{ji^{(k)}}^{(k)}$$

$$- \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} \leq 0 \tag{1.12}$$

The contour L_k is in the s_k-plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j - \sum_{i=1}^{r} \beta_j^{(k)} s_k)$ with $j = 1$ to m, $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k lie to the right and the poles of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n, $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with to $j = 1$ to n_k the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as: $|\arg x_k| < \frac{1}{2} A_{i^{(k)}} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} - \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{m} \beta_j^{(k)} - \sum_{j=m+1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)}$$

$$- \sum_{j=n_k+1}^{p_i^{(k)}} \gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with k=1 to r, i=1 to } R, i^{(k)} = 1 \text{ to } R \tag{1.13}$$

Throughout this document, we assume the existence and absolute convergence conditions of the generalized multivariable I-function.

2. Multiples integrals with algebraic functions

(i)

$$\int_0^1 \cdots \int_0^1 \prod_{j=1}^{r} x_j^{\rho_j - 1} (1 - x_j)^{\sigma_j - 1} I(\alpha_1 x_1, ..., \alpha_r x_r) dx_1 ... dx_r = \int_0^1 \cdots \int_0^1 \times$$

$$\times \prod_{j=1}^{r} x_j^{\rho_j - 1} (1 - x_j)^{\sigma_j - 1} \left(\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, ..., s_r) \prod_{j=1}^{r} \phi(s_j) (\alpha_j x_j)^{s_j} ds_j \right) dx_j$$

Now using the definition of beta function.

$$\int_0^1 x^{\rho+s-1}(1-x)^{\sigma-1}dx = \frac{\Gamma(\rho+s)\Gamma(\sigma)}{\Gamma(\rho+\sigma+s)} \tag{2.1}$$

Putting value of (2.1) in equation (1.1) , we get the following result :

$$\int_0^1 ... \int_0^1 \prod_{j=1}^r x_j^{\rho_j-1}(1-x_j)^{\sigma_j-1} I(\alpha_1 x_1, ..., \alpha_r x_r) dx_1...dx_r = \prod_{j=1}^r \frac{\Gamma(\rho_j)\Gamma(\sigma_j)}{\Gamma(\rho_j+\sigma_j)} \times$$

$$I^{m,n:m_1,n_1+1;...;m_r,n_r+1}_{R_i:R^{11}_{i'};...;R^{11}_{i(r)}} \left(\begin{array}{c|c} \alpha_1 & \\ . & A(r) : (1-\rho_1;1), C^{(1)}; ...; (1-\rho_r;1), C^{(r)} \\ . & \\ . & B(r) : (1-\rho_1-\sigma_1;1), D^{(1)}; ...; (1-\rho_r-\sigma_r;1), D^{(r)} \\ \alpha_r & \end{array} \right) \tag{2.2}$$

with $R^{11}_{i'} = p_{i'} + 1,\ q_{i'} + 1;\ R, ..., R^{22}_{i(r)} = p_{i(r)} + 1,\ q_{i(r)} + 1;\ R$

(ii)

$$\int_0^1 ... \int_0^1 \prod_{j=1}^r x_j^{\rho_j-1}(1-x_j)^{\sigma_j-1} I(\alpha_1 x_1(1-x_1), ..., \alpha_r x_r(1-x_r)) dx_1...dx_r$$

$$= \prod_{j=1}^r \frac{\Gamma(\rho_j)\Gamma(\sigma_j)}{\Gamma(\rho_j+\sigma_j)} \quad I^{m,n:m_1,n_1+2;...;m_r,n_r+2}_{R_i:R^{11}_{i'};...;R^{11}_{i(r)}} \times$$

$$\times \left(\begin{array}{c|c} \alpha_1 & \\ . & A(r) : (1-\rho_1;1), (1-\sigma_1;1), C^{(1)}; ...; (1-\rho_r;1), (1-\sigma_r;1), C^{(r)} \\ . & \\ . & B(r) : (1-\rho_1-\sigma_1;2), D^{(1)}; ...; (1-\rho_r-\sigma_r;2), D^{(r)} \\ \alpha_r & \end{array} \right) \tag{2.3}$$

with $R^{11}_{i'} = p_{i'} + 1,\ q_{i'} + 1;\ R, ..., R^{22}_{i(r)} = p_{i(r)} + 1,\ q_{i(r)} + 1;\ R$ because

$$\int_0^1 x^{\rho+s-1}(1-x)^{\sigma+\rho-1}dx = \frac{\Gamma(\rho+s)\Gamma(\sigma+s)}{\Gamma(\rho+\sigma+2s)} \tag{2.4}$$

3. Simple integrals of multivariable I-function with orthogonals polynomials

(a) Jacobi polynomial

(i)

$$\int_{-1}^{1}...\int_{-1}^{1}\prod_{j=1}^{r}x_j^{\lambda_j}(1+x_j)^{\delta_j}(1-x_j)^{\alpha_j}P_{n_j}^{(\alpha_j,\beta_j)}(x_j)I(z_1(1+x_1)^{h_1},...,z_r(1+x_r)^{h_r})dx_j$$

$$=\int_{-1}^{1}...\int_{-1}^{1}\prod_{j=1}^{r}x_j^{\lambda_j}(1+x_j)^{\delta_j}(1-x_j)^{\alpha_j}P_{n_j}^{(\alpha_j,\beta_j)}(x_j)\left(\frac{1}{(2\pi\omega)}\int_{L_1}...\int_{L_r}\right.$$

$$\left.\psi(s_1,...,s_r)\prod_{j=1}^{r}\phi(s_j)z_j^{s_j}(1+x_j)^{s_jh_j}ds_j\right)dx_j$$

$$=\frac{1}{(2\pi\omega)}\int_{L_1}...\int_{L_r}\psi(s_1,...,s_r)\prod_{j=1}^{r}\phi(s_j)z_j^{s_j}\left(\int_{-1}^{1}x_j^{\lambda_j}(1-x_j)^{\alpha_j}\right.$$

$$\left.(1+x_j)^{\delta_j+h_js_j}P_{n_j}^{\alpha_j,\beta_j}(x_j)dx_j\right)ds_j$$

Use the following formula : see V.P. Saxena , page 52 [3].

$$\int_{-1}^{1}x^{\lambda}(1-x)^{\alpha}(1+x)^{\delta}P_n^{(\alpha,\beta)}(x)dx=(-1)^n2^{\alpha+\delta+1}\times$$

$$\times\frac{\Gamma(\delta+1)\Gamma(n+\alpha+1)\Gamma(\delta+\beta+1)}{n!\Gamma(\delta+\beta+n+1)\Gamma(\delta+\alpha+n+2)}\,{}_3F_2\left(\begin{array}{c}-\lambda,\delta+\beta+1,\delta+1\\ \delta+\beta+n+1,\delta+\alpha+n+2\end{array}\right) \tag{3.1}$$

Provided $\alpha>1$ and $\beta>1$.

Further simplifying the hypergeometric function in terms of serie summation and applied the definition of the Aleph function of two variables , we get the following result.

$$\int_{-1}^{1}...\int_{-1}^{1}\prod_{j=1}^{r}x_j^{\lambda_j}(1+x_j)^{\delta_j}(1-x_j)^{\alpha_j}P_{n_j}^{(\alpha_j,\beta_j)}(x_j)I(z_1(1+x_1)^{h_1},...,z_r(1+x_r)^{h_r})dx_j$$

$$=\prod_{j=1}^{r}(-)^{n_j}\frac{2^{\alpha_j+\delta_j+1}\Gamma(\alpha_j+n_j+1)}{n_j!}\sum_{k_1=1}^{\infty}...\sum_{k_r=1}^{\infty}\prod j=1^r\frac{(-\lambda_j)_{k_j}2^{k_j}}{k_j!}\times$$

$$\times I^{m,n:m_1,n_1+2;...;m_r,n_r+2}_{R_i:R^{22}_{i'};...;R^{22}_{i^{(r)}}}\left(\begin{array}{c} z_1 \\ . \\ . \\ . \\ z_r \end{array}\middle| \begin{array}{l} A(r):(\beta_1-\delta_1-k_1;h_1),(-\delta_1-k_1;h_1), \\ \\ B(r):(\beta_1-\delta_1+n_1-k_1;h_1),(-\delta_1-\alpha_1-n_1 \end{array}\right.$$

$$\left.\begin{array}{l} C^{(1)};...;(\beta_r-\delta_r-k_r;h_r),(-\delta_r-k_r;h_r),C^{(r)} \\ -k_1-1;h_1)D^{(1)};...;(\beta_r-\delta_r+n_r-k_r;h_r),(-\delta_r-\alpha_r-n_r-k_r-1;h_r),D^{(r)} \end{array}\right) \tag{3.2}$$

with $R^{22}_{i'}=p_{i'}+2,\ q_{i'}+2;\ R,...,R^{22}_{i^{(r)}}=p_{i^{(r)}}+2,\ q_{i^{(r)}}+2;\ R$

(ii)

$$\int_{-1}^{1}...\int_{-1}^{1}\prod_{j=1}^{r}(1-x_j)^{\rho_j}(1+x_j)^{\sigma_j}I(z_1(1-x_1)^{h_1}(1+x_1)^{k_1},...,z_r(1-x_r)^{h_r}(1+x_r)^{k_r})$$

$$\times P^{\alpha_j,\beta_j}_{nj}(x_j)dx_1...dx_r=2^{\sum_{j=1}^{r}(\rho_j+\sigma_j)+r}\sum_{l_1=0}^{\infty}...\sum_{l_r=0}^{\infty}\prod_{j=1}^{r}\frac{(-n_j)_{l_j}(\alpha_j+\beta_j+n_j+1)_{l_j}}{(\alpha_j+1)_{l_j}l_j!}\times$$

$$\times I^{m,n:m_1,n_1+2;...;m_r,n_r+2}_{R_i:R^{21}_{i'};...;R^{21}_{i^{(r)}}}\left(\begin{array}{c} 2^{h_1+k_1}z_1 \\ . \\ . \\ . \\ 2^{h_r+k_r}z_r \end{array}\middle| \begin{array}{l} A(r):(-\sigma_1,k_1),(-\rho_1-l_1;h_1),C^{(1)} \\ \\ B(r):(-1-\rho_1-\sigma_1-l_1;h_1+k_1),D^{(1)} \end{array}\right.$$

$$\left.\begin{array}{l} ;...;(\sigma_r;k_r),(-\rho_r-l_r;h_r),C^{(r)} \\ ;...;(-1-\rho_r-\sigma_r-l_r;h_r+k_r),D^{(r)} \end{array}\right) \tag{3.3}$$

with $R^{21}_{i'}=p_{i'}+2,\ q_{i'}+1;\ R,...,R^{21}_{i^{(r)}}=p_{i^{(r)}}+2,\ q_{i^{(r)}}+1;\ R$

(iii)

$$\int_{-1}^{1}...\int_{-1}^{1}\prod_{j=1}^{r}(1-x_j)^{\delta_j}(1+x_j)^{\beta_j}P^{\alpha_j,\beta_j}_{n_j}(x_j)P^{\rho_j,\sigma_j}_{m_j}(x_j)I(z_1(1-x_1)^{h_1},...,z_r(1-x_r)^{h_r})\times$$

$$\times dx_1...dx_r = \prod_{j=1}^{r} \frac{2^{\beta_j+\delta_j+1}\Gamma(\beta_j+n_j+1)\Gamma(\rho_j+m_j+1)}{m_j!n_j!}\times$$

$$\times \sum_{k_1=1}^{\infty} ... \sum_{k_r=1}^{\infty} \prod_{j=1}^{r} \frac{(-m_j)_{k_j}\Gamma(\rho_j+\sigma_j+m_j+1)}{k_j!\Gamma(\rho_j+k_j+1)}\times$$

$$I^{m,n:m_1+1,n_1+3;...;m_r+1,n_r+3}_{R_i:R^{44}_{i'};...;R^{44}_{i(r)}} \left(\begin{array}{c|c} 2^{h_1}z_1 & \\ . & A(r):(\alpha_1-\delta_1-k_1;h_1),(-\delta_1-k_1;h_1),C^{(1)} \\ . & \\ . & B(r):(-\beta_1-\delta_1-k_1-n_1-1;h_1),D^{(1)} \\ 2^{h_r}z_r & \end{array} \right.$$

$$,(\alpha_1+n_1-\delta_1;h_1),(\alpha_1-\delta_1;h_1);...;C^{(r)},(-\delta_r-k_r;h_r),$$

$$,(\alpha_1-\delta_1;h_1),(\alpha_1+n_1-\delta_1-k_1;h_1),(\alpha_1-\delta_1+n_1;h_1);...;D^{(r)},(\alpha_r-\delta_r$$

$$\left. \begin{array}{l} ,(\alpha_r+n_r-\delta_r;h_r),(\alpha_r-\delta_r-k_r;h_r),(\alpha_r-\delta_r;h_r) \\ +n_r;h_r),(\alpha_r-\delta_r;h_r),(\alpha_r+n_r-\delta_r-k_r;h_r),(-\beta_r-\delta_r-k_r-n_r-1;h_r) \end{array} \right) \tag{3.4}$$

with $R^{44}_{i'} = p_{i'}+4,\ q_{i'}+4;\ R, ..., R^{44}_{i(r)} = p_{i(r)}+4,\ q_{i(r)}+4;\ R$

(b) Legendre Polynomial

We have

$$\int_{-1}^{1} ... \int_{-1}^{1} \prod_{j=1}^{r} (1+x_j)^{\rho_j+1} P_{\nu_j}(x_j) I(\alpha_1(1+x_1)^{\gamma_1}, ..., \alpha_r(1+x_r)^{\gamma_r}) dx_1...dx_r$$

$$= 2^{\sum_{j=1}^{r}\rho_j} \times I^{m,n:m_1,n_1+2;...;m_r,n_r+2}_{R_i:R^{22}_{i'};...;R^{22}_{i(r)}} \left(\begin{array}{c|c} \alpha_1 2^{\gamma_1} & \\ . & A(r):(1-\rho_1;\gamma_1),(1-\rho_1;\gamma_1), \\ . & \\ . & B(r):(\rho_1-\nu_1;\gamma_1),(1-\rho_1+\nu_1;\gamma_1), \\ \alpha_r 2^{\gamma_r} & \end{array} \right.$$

$$\left. \begin{array}{l} , C^{(1)}; ...; (1-\rho_r;\gamma_r), (1-\rho_r;\gamma_r), C^{(r)} \\ \\ , D^{(1)}; ...; (\rho_r-\nu_r;\gamma_r), (1-\rho_r+\nu_r;\gamma_r), D^{(r)} \end{array} \right) \tag{3.5}$$

with $\Re(\rho) > 0,\ R_{i'}^{22} = p_{i'}+2,\ q_{i'}+2;\ R, ..., R_{i^{(r)}}^{22} = p_{i^{(r)}}+2,\ q_{i^{(r)}}+2;\ R$

(c) Hermite Polynomial

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{j=1}^{r} e^{-x_j^2} x_j^{2\rho_j} H_{2\mu_j}(x_j) I(\alpha_1 x_1^{2a_1}, ..., \alpha_r x_r^{2a_r}) dx_1 ... dx_r = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{j=1}^{r}$$

$$\times e^{-x_j^2} x_j^{2\rho_j} H_{2\mu_j}(x_j) \left(\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \prod_{j=1}^{r} \psi(s_1, ..., s_r) \phi(s_j) [\alpha_j x_j^{2a_j}]^{s_j} ds_j \right) dx_j$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, ..., s_r) \prod_{j=1}^{r} \phi(s_j) \alpha_j^{s_j} \left(\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{j=1}^{r} e^{-x_j^2} \times \right.$$

$$\left. \times x^{2(\rho_j + a_j s_j)} H_{2\mu_j}(x_j) dx_j \right) ds_j$$

Using the following formula

$$\int_{-\infty}^{\infty} x^{2\rho} e^{-x^2} H_{2\mu}(x) dx = \frac{\pi^{\frac{1}{2}} 2^{\mu-\rho} \Gamma(2\rho+1)}{\Gamma(\rho-\mu+1)} \tag{3.6}$$

We have

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{j=1}^{r} e^{-x_j^2} x_j^{2\rho_j} H_{2\mu_j}(x_j) I(\alpha_1 x_1^{2a_1}, ..., \alpha_r x_r^{2a_r}) dx_1 ... dx_r$$

$$= \pi^{\frac{1}{2}} 2^{2\sum_{j=1}^{r}(\mu_j-\rho_j)} \times I_{R_i : R_{i'}^{11}; ...; R_{i^{(r)}}^{11}}^{m,n:m_1+1,n_1;...;m_r+1,n_r} \left(\begin{array}{c} \alpha_1 2^{-2a_1} \\ . \\ . \\ . \\ \alpha_r 2^{-2a_r} \end{array} \middle| \begin{array}{l} A(r) : (-2\rho_1; 2a_1), \\ \\ B(r) : (\mu_1-\rho_1; a_1), \end{array} \right.$$

$$\left. \begin{array}{l} , C^{(1)}; ...; (-2\rho_r; a_r), C^{(r)} \\ \\ , D^{(1)}; ...; (\mu_r-\rho_r; a_r), D^{(r)} \end{array} \right) \tag{3.7}$$

with $R_{i'}^{11} = p_{i'} + 1,\ q_{i'} + 1;\ R, ..., R_{i^{(r)}}^{11} = p_{i^{(r)}} + 1,\ q_{i^{(r)}} + 1;\ R$

(4) Generalized Hypergeometric Function

We note $f^{(j)}(k_j) = \dfrac{\prod_{l=1}^{p_j}(g_l)_{k_j}}{\prod_{l=1}^{q_j}(h_l)_{k_j}} \times \dfrac{a_j^{k_j}}{k_j!}$ for $j = 1$ to r

$$\int_0^{t_1} ... \int_0^{t_r} \prod_{j=1}^{r} x_j^{\rho_j - 1}(t_j - x_j)^{\sigma_j - 1}\ {}_{p_j}F_{q_j}[(g_{p_j}); (h_{q_j}); \alpha_j x_j^{\mu_j}(t_j - x_j)^{\eta_j}]\times$$

$$\times I(\alpha_1 x_1^{\mu_1}(t_1 - x_1)^{\nu_1}, ..., \alpha_r x_r^{\mu_r}(t_r - x_r)^{\nu_r})dx_1...dx_r = \prod_{j=1}^{r} t_j^{\rho_j + \sigma_j - 1}\times$$

$$\times \sum_{k_1=1}^{\infty} \cdots \sum_{k_r=1}^{\infty} \prod_{j=1}^{r} f^{(j)}(k_j) t_j^{(\mu_j + \eta_j)k_j} I^{m,n:m_1,n_1+2;...;m_r,n_r+2}_{R_i:R_{i'}^{11};...;R_{i^{(r)}}^{11}} \left(\begin{array}{c} \alpha_1 t_1^{\mu_1+\nu_1} \\ . \\ . \\ . \\ \alpha_r t_r^{\mu_r+\nu_r} \end{array} \middle| \begin{array}{c} A(r) : C^{(1)}, \\ \\ B(r) : D^{(1)}, \end{array} \right.$$

$$, (1 - \rho_1 - \mu_1 k_1; \mu_1), (1 - \sigma_1 - \nu_1 k_1; \nu_1); ...; (1 - \rho_r - \mu_r k_r; \mu_r),$$

$$, (1 - \rho_1 - \sigma_1 - \mu_1 k_1 - \nu_1 k_1; \mu_1 + \nu_1); ...; (1 - \rho_r - \sigma_r - \mu_r k_r - \nu_r k_r; \mu_r + \nu_r)$$

$$\left. \begin{array}{c} , (1 - \sigma_r - \nu_r k_r; \nu_r), C^{(r)} \\ \\ , D^{(r)} \end{array} \right) \qquad (4.1)$$

with $R_{i'}^{11} = p_{i'} + 1,\ q_{i'} + 1;\ R, ..., R_{i^{(r)}}^{11} = p_{i^{(r)}} + 1,\ q_{i^{(r)}} + 1;\ R$

5. conclusion

On specializing the parameters , the generalized multivariable I-function may be reduced to I-function of two variables , see C.K.Sharma et al [5] , multivariable H-function and generalized multivariable hypergeometric function and several other higher transcendental functions .

References

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