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RELATIVISTIC MODELING OF RADIATING FLUID BALL WITH OCCURRENCE OF HORIZON

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Dedicated to Prof. A.M. Mathai on his 80^{th} birth anniversary

Abstract: The objective of this paper is to present a new class of exact solution of the Einstein field equations for a spherically symmetric shear free isotropic fluid collapsing under its own gravity and undergoing radial heat flow due to Tewari [1]. The interior metric fulfilled all the relevant physical and thermodynamic conditions that matched with Vaidya exterior metric over the boundary. At the beginning, the interior solutions represent a static configuration of perfect fluid which later on gradually starts evolving into radiation collapse. Consequently we have obtained the expressions of various physical and thermal parameters and found that they are physically reasonable for a set of model parameters and there are a number of such parameters for which the solution is well behaved. The final fate of our model is formation of black hole.

Keywords: Exact solutions, Gravitational collapse, Radiating star, Black hole, Naked singularity.

1. Introduction

The attractive character of gravitational collapse is the formation of a star from nebulae, galaxies and cluster in astrophysics. The balance of any star is totally depends on the balance of two incompatibility effects. The internal thermal pressure of an element contracting the material of the star in the stellar interior and the opposite gravitational force attracting the same towards the centre. When this reaction end and no other source of pressure acts on this, this balance is broken and a massive star undergoes the continuous catastrophic contraction. Such a phenomenon is known as gravitational collapse. The initiator of gravitational collapse was astrophysicist Chandrasekhar [2]. Hence gravitational collapse of massive stars under various conditions is one of the most interesting phenomenon in relativistic astrophysics and it attracts the researchers also. Gravitational collapse transferred into a black hole but several counter examples shown that naked singularity can also be formed still no established theory can explain the formation of either black hole or naked singularity (Joshi and Malafarina [3]). In order to understand this it is must to form realistic model and to solve non-linear differential equation.

In 1916 Karl Schwarzschild [4] gave an exact solution to the Einstein field equations for a spherically symmetric bounded matter distribution having a vacuum exterior. Gravitational collapse was first taken by Oppenheimer and Snyder [5], in which they studied the dust collapse according them singularity is neither locally nor globally naked, i.e. the final fate of the dust collapses a black hole. In 1951, Vaidya [6] published exact solution to the Einstein field equations describing the metric corresponding to the exterior gravitational field of a radiating star. The Santos [7] junction condition based on Vaidyas outgoing solution has given the way for studying dissipative gravitational collapse. Herrera and Santos [8] and Mitra [9] established the fact that gravitational collapse is a high energy dissipating energy process which plays a dominant role in the formation and evolution of stars. The dissipation of energy from collapsing fluid distribution is described in two limiting cases. The first case describes the free streaming approximation while second one is diffusion approximation. The prominent work in first case is due to Tewari and he solved the Einstein's field equations with a new approach and developed the Quasar models Tewari [10]-[13]. While a number of realistic models in diffusion approximation are due to de Oliveira et al. [14]; Bonnor et al. [15]; Banerjee et al.[16]; Herrera et al.[17, 18]; Ivanov [19]; Sharif and Abbas [20]; Tewari [21]; Tewari and Charan [22-25], Tewari et al.[26, 27].

In this paper we present a class of new solutions of Einsteins equations for shear-free spherically symmetric non adiabatic collapsing fluid with radial heat flow. It is presented with the reference of relevant conditions with separable metric. The interior space time metric is match with Vaidyas exterior metric at the boundary. The final fate of such models is a formation of black hole. The paper is organised as follows: In sec. 2 the field equations and the junction conditions which match the interior metric of the collapsing fluid with the exterior metric are given. In section 3 a new class of exact solutions of the field equations are presented. In section 4 a detailed study of a class of solutions for a collapsing radiating star is given and finally in section 5 some concluding remarks have been made.

2. The interior structure of fluid distribution

The Riemannian space time of radiating star will be separated by its boundary into two distinct regions, the interior space time and the exterior space time for a stellar model. These specific regions is give a detailed account in word by a odd smooth time like three dimensional hyper surface

$$ds_-^2 = -A^2(r,t)dt^2 + B^2(r,t)\{dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\} \tag{1}$$

The energy momentum tensor describe the physical content of the interior space time

$$T_{\mu\nu} = (\epsilon + p)w_\mu w_\nu + pg_{\mu\nu} + q_\mu w_\nu + q_\nu w_\mu \tag{2}$$

where ϵ is the energy density of the fluid, p the isotropic pressure, w_μ is the four velocity and q_μ the radial heat flux vector. Assuming comoving coordinates, we have $w^\mu = \delta_0^\mu$. The heat flow vector q^μ is orthogonal to the velocity vector so that $q^\mu w_\mu = 0$ and takes the form $q^\mu = q\delta_1^\mu$.

The line element (1) corresponds to shear- free spherically symmetric fluid (Glass [28]), as the shear tensor vanishes identically. The fluid collapse rate $\Theta = w^\mu_{;\mu}$ of the fluid distribution (1) is given by

$$\Theta = \frac{3\dot{B}}{AB} \tag{3}$$

Non-trivial Einsteins field equations in view of (1) and (2) are given by following system of equations

$$\kappa\epsilon = -\frac{1}{B^2}\Big(\frac{2B''}{B} - \frac{B'^2}{B^2} + \frac{4B'}{rB}\Big) + \frac{3\dot{B}^2}{A^2B^2} \tag{4}$$

$$\kappa p = \frac{1}{B^2}\Big(\frac{B'^2}{B^2} + \frac{2A'B'}{AB} + \frac{2A'}{AB} + \frac{2A'}{rA} + \frac{2B'}{rB}\Big) + \frac{1}{A^2}\Big(-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB}\Big) \tag{5}$$

$$\kappa p = \frac{1}{B^2}\Big(\frac{B''}{B} - \frac{B'^2}{B^2} + \frac{B'}{rB} + \frac{A''}{A} + \frac{A'}{rA}\Big) + \frac{1}{A^2}\Big(-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB}\Big) \tag{6}$$

$$\kappa q = \frac{-2}{AB^2}\Big(-\frac{\dot{B}'}{B} + \frac{B'\dot{B}}{B^2} + \frac{A'\dot{B}}{AB}\Big) \tag{7}$$

here and hereafter the primes and dots stand for differentiation with respect to r and t respectively. The coupling constant in geometrized units is $\kappa = 8\pi (i.e. G = c = 1)$.

3. The exterior space time and the junction conditions

The exterior space-time is described by Vaidyas exterior metric [6] which represents an outgoing radial flow of radiation

$$ds_+^2 = -\Big(1 - \frac{2M(v)}{R}\Big)dv^2 - 2dRdv + R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{8}$$

where v is the retarded time and M(v) is the exterior Vaidya mass.

The junction conditions for matching two line elements (1) and (8) continuously across a spherically symmetric time-like hyper surface Σ are well known and obtained by Santos [7]

$$(rB)_\Sigma = R_\Sigma(v) \tag{9}$$

$$(p_r)_\Sigma = (qB)_\Sigma \tag{10}$$

$$m_\Sigma(r,t) = M(v) = \left\{\tfrac{r^3B\dot{B}^2}{2A^2} - r^2B' - \tfrac{r^3B'^2}{2B}\right\}_\Sigma \tag{11}$$

where m_Σ is the mass function calculated in the interior at $r = r_\Sigma$ (Cahill et al. [29], Misner and Sharp [30]).

Some other characteristics of the model such as the surface luminosity and the boundary redshift z_Σ observed on Σ are

$$L_\Sigma = \tfrac{\kappa}{2}\{r^2B^3q\}_\Sigma \tag{12}$$

$$z_\Sigma = \left[1 + \tfrac{rB'}{B} + \tfrac{r\dot{B}}{A}\right]_\Sigma^{-1} - 1 \tag{13}$$

The total luminosity for an observer at rest at infinity is

$$L_\infty = -\frac{dM}{dv} = \frac{L_\Sigma}{(1+z_\Sigma)^2} \tag{14}$$

4. Solution of the field equations

In order to solve the field equations we choose a particular form of the metric coefficients given in (1) into functions of r and t coordinates as $A(r,t) = A_0(r)g(t)$ and $B(r,t) = B_0(r)f(t)$.

In view of the above metric coordinates the Einstein's field equations (6)-(9) lead to the following system of equations

$$\kappa\epsilon = \frac{\epsilon_0}{f^2} + \frac{3\dot{f}^2}{A_0^2g^2f^2} \tag{15}$$

$$\kappa p = \frac{p_0}{f^2} + \frac{1}{A_0^2g^2}\Big(-\frac{2\ddot{f}}{f} - \frac{\dot{f}^2}{f^2}\Big) \tag{16}$$

where

$$\epsilon_0 = -\frac{1}{B_0^2}\Big(\frac{2B_0''}{B_0} - \frac{B_0'^2}{B_0^2} + \frac{4B_0'}{rB_0}\Big) \tag{17}$$

$$p_0 = \frac{1}{B_0^2}\Big(\frac{B_0'^2}{B_0^2} + \frac{2B_0'}{rB_0} + \frac{2A_0'B_0'}{A_0B_0} + \frac{2A_0'}{rA_0}\Big) \tag{18}$$

here the quantities with the suffix 0 corresponds to the static star model with metric components $A_0(r),\ B_0(r)$.

In the absence of dissipative forces the equation (10), $(p)_\Sigma = (qB)_\Sigma$, reduces to the condition $[p_0]_\Sigma = 0$ and yields at $r = r_\Sigma = R_\Sigma$

$$\frac{2\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} - \frac{2\dot{g}\dot{f}}{gf} = \frac{2\alpha g\dot{f}}{f^2} \tag{19}$$

where

$$\alpha = \Big(\frac{A_0'}{B_0}\Big)_\Sigma \tag{20}$$

To solve the equation (19), by assuming $g(t) = f(t)$ (Tewari [1]) obtained the following solution

$$\dot{f} = -2\alpha\sqrt{f}(1-\sqrt{f}) \tag{21}$$

$$t = \tfrac{1}{\alpha}ln(1-\sqrt{f}) \tag{22}$$

We observed that the function $f(t)$ decreases monotonically from the value $f(t) = 1$ at $t = -\infty$ to $f(t) = 0$ at $t = 0$.

5. Parametric class of Solutions

The isotropy of pressure would give the equation

$$\frac{A_0''}{A_0} + \frac{B_0''}{B_0} = \left(\frac{2B_0'}{B_0} + \frac{1}{r}\right)\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right) \tag{23}$$

where the quantities with the suffix 0 corresponds to the static star model metric components $A_0(r), B_0(r)$.

The new parametric class of solutions of equation (23) obtained by Tewari [1] is

$$A_0 = D_2(1 + C_1r^2)^{\frac{n}{l+1}} + D_1(1 + C_1r^2)^{\frac{2-n}{l+1}+1} \tag{24}$$

$$B_0 = C_2(1 + C_1r^2)^{\frac{1}{l+1}} \tag{25}$$

$$n = \frac{1}{2}\Big\{(l+3) \pm (l^2 + 10l + 17)^{\frac{1}{2}}\Big\} \tag{26}$$

where n, l, C_1, C_2, D_1 and D_2 are constants and n is real if $l \geq -5+2\sqrt{2}$ or $l \leq -5-2\sqrt{2}$.

One can arrive at a number of solutions for different values of n from above class of solutions. Already a number of solutions have been obtained with the help of this class of solution.

6. Detailed study of specific model

In order to construct the new realistic model we assume $n = -7/5$ and, from (24) and (25) we obtain,

$$A_0 = D_2(1+C_1r^2)^{\frac{7}{47}} + D_1(1+C_1r^2)^{\frac{31}{47}} \tag{27}$$

$$B_0 = C_2(1+C_1r^2)^{\frac{-5}{47}} \tag{28}$$

In view of (27) and (28) the equations (17) and (18) reduces in following expressions

$$\epsilon_0 = \frac{20C_1}{2209C_2^2(1+C_1r^2)^{\frac{84}{87}}}(141+42C_1r^2) \tag{29}$$

$$p_0 = \frac{4C_1}{2209C_2^2(1+C_1r^2)^{\frac{84}{87}}}[(94+49C_1r^2)+ \frac{23D_1(1+C_1r^2)^{\frac{23}{47}}}{\{D_2+D_1(1+C_1r^2)^{\frac{23}{47}}\}}(47+37C_1r^2)] \tag{30}$$

The junction condition $[p_0]_\Sigma = 0$ gives

$$D_2 = -\frac{25D_1(1+C_1r_\Sigma^2)^{\frac{23}{47}}(47+36C_1r_\Sigma^2)}{(94+49C_1r_\Sigma^2)} \tag{31}$$

We observed that $\epsilon_0 > 0$, $p_0 > 0$, $\frac{p_0}{\epsilon_0} < 1$, $\epsilon_0' < 0$, $p_0' < 0$ at the centre are satisfied with suitable choice of constants $C_1 > 0$, $C_2 > 0$, $D_2 > 0$, $D_1 < 0$ and $D_2 > -\frac{25D_1}{2}$.

The total energy inside Σ for the static system

$$m_0 = \frac{10C_1C_2r_\Sigma^3(47+42C_1r_\Sigma^2)}{2209(1+C_1r^2)^{\frac{99}{47}}} \tag{32}$$

Now the explicit expressions for ϵ, p, q, and Θ become

$$\epsilon = \frac{\epsilon_0}{f^2} + \frac{12\alpha^2}{A_0^2}\frac{(1-\sqrt{f})^2}{f^3} \tag{33}$$

$$p = \frac{p_0}{f^2} + \frac{4\alpha^2}{A_0^2}\frac{(1-\sqrt{f})}{f^{\frac{5}{2}}} \tag{34}$$

$$q = \frac{4C_1 r^2\{(\frac{7}{47})D_2 + \frac{30}{47}D_1(1+C_1r^2)^{\frac{23}{47}}\}}{C_2^2(1+C_1r^2)^{44/47}\{D_2 + D_1(1+C_1r^2)^{\frac{23}{47}}\}}\frac{2\alpha(1-\sqrt{f})}{f^{\frac{7}{2}}} \tag{35}$$

$$\Theta = \frac{-6\alpha(1-\sqrt{f})}{(1+C_1r^2)^{\frac{7}{47}}\{D_2 + D_1(1+C_1r^2)^{\frac{23}{47}}\}f^{\frac{3}{2}}} \tag{36}$$

where

$$\alpha = -\frac{230}{47}\frac{D_1C_1r_\Sigma}{C_2(1+C_1r_\Sigma^2)^{\frac{12}{47}}}\frac{(94+49C_1r_\Sigma^2)}{(47+42C_1r_\Sigma^2)} \tag{37}$$

We can see the physical parameters ϵ, p, q are finite, positive monotonically decreasing at any instant with respect to radial coordinate for $0 \le r \le r_\Sigma$. Initially collapse is zero and it becomes infinite at final phase of the configuration.

The total energy entrapped inside Σ is given by

$$M(v) = \left[2\left(\frac{10}{47}\right)^2 \frac{C_2C_1^2r_\Sigma^5(47+42C_1r_\Sigma^2)^2}{(1+C_1r_\Sigma^2)^{\frac{99}{47}}(47+37C_1r_\Sigma^2)^2}(1-\sqrt{f})^2 + m_0 f\right] \tag{38}$$

The luminosity and the red shift observed on Σ and luminosity observed by a distant observer are given by

$$L_\Sigma = 2\left(\frac{10}{47}\right)^2 \frac{C_1^2r_\Sigma^4(47+42C_1r_\Sigma^2)^2}{(1+C_1r_\Sigma^2)^2(47+37C_1r_\Sigma^2)^2}\frac{(1-\sqrt{f})^2}{\sqrt{f}} \tag{39}$$

$$L_\infty = 2\left(\frac{10}{47}\right)^2 \frac{C_1^2r_\Sigma^4(47+42C_1r_\Sigma^2)^2}{(1+C_1r_\Sigma^2)^2(47+37C_1r_\Sigma^2)^2}\frac{(1-\sqrt{f})^2}{\sqrt{f}}\frac{1}{(1+z_\Sigma)^2} \tag{40}$$

$$z_\Sigma = \frac{\left\{\frac{(47+42C_1r_\Sigma^2)^2\sqrt{f} + 20C_1r_\Sigma^2(47+42C_1r_\Sigma^2)(1-\sqrt{f})}{71\sqrt{f}(1+C_1r_\Sigma^2)(47+37C_1r^2\Sigma)}\right\}_\Sigma}{(1-\frac{2M}{rB_0f})_\Sigma} - 1 \tag{41}$$

The above expressions show that L_∞ vanishes in the beginning when$f(t) \to 1$ and at the stage when$z_\Sigma \to \infty$.

We obtain the black hole formation time as

$$\sqrt{f_{BH}} = \frac{20C_1 r_\Sigma^2(47+42C_1 r_\Sigma^2)}{2209(1+C_1 r_\Sigma^2)^2} \tag{42}$$

and

$$t_{BH} = \frac{1}{\alpha} In \frac{(47+37C_1 r_\Sigma^2)^2}{2209(1+C_1 r_\Sigma^2)^2} \tag{43}$$

7. Temperature evaluation

The effective surface temperature observed by external observer can be calculate similar as Tewari [1]

$$T_\Sigma^4 = \frac{200}{2209\pi\delta C_2^2} \frac{C_1^2 r_\Sigma^2 (47+42C_1 r_\Sigma^2)^2}{(1+C_1 r_\Sigma^2)^{\frac{84}{47}}(47+37C_1 r_\Sigma^2)^2} \frac{(1-\sqrt{f})}{f^{\frac{5}{2}}} \frac{1}{(1+z_\Sigma)^2} \tag{44}$$

where the constant δ in Photon is given by

$$\delta = \frac{\pi^2 k^4}{15\hbar^3} \tag{45}$$

where k and $\hbar$ denoting respectively Boltzmann and Plank constants.

The temperature inside the star is given by Tewari [1]

$$T^4 = \left[\frac{T_0(t)}{(1+C_1 r^2)^{\frac{28}{47}}\left\{D_2 + D_1(1+C_1 r^2)^{\frac{23}{47}}\right\}^4}\right] - \left[\frac{16\alpha(1-\sqrt{f})}{3\gamma f^{\frac{3}{2}}(1+C_1 r^2)^{\frac{7}{47}}\left\{D_2 + D_1(1+C_1 r^2)^{\frac{23}{47}}\right\}}\right] \tag{46}$$

where

$$T_0(t) = \left\{\frac{16\alpha}{3k\gamma} \frac{\{D_2 + D_1(1+C_1 r^2)^{\frac{23}{47}}\}(1+C_1 r^2)^{\frac{21}{47}}(1-\sqrt{f})}{f^{\frac{3}{2}}}\right\}_\Sigma + \left\{\frac{2\alpha\{D_2 + D_1(1+C_1 r^2)^{\frac{23}{47}}\}^2(1+C_1 r^2)^{\frac{12}{47}}(1-\sqrt{f})}{\pi\delta r^2 f^{\frac{5}{2}}}\right\}_\Sigma \frac{1}{(1+z_\Sigma)^2} \tag{47}$$

It follows that the surface temperature of the collapsing star tends to zero at the beginning of the collapse $[f \to 1]$ and the stage of formation of black hole $[z_\Sigma \to \infty]$.

8. Conclusion

We here presented a new radiating fluid model collapsing in the influence of its own gravity using Tewari solution [1] as seed solution. The interior fluid is spherically symmetric shear-free isotropic and radiating away its energy in the form of radial heat flow. Keeping in mind pressure isotropy a simple radiating star model for $n = -\frac{7}{5}$ studied in detail. We observed that the function $f(t)$ decreases monotonically from the value $f(t) = 1$ at $t = -\infty$ to $f(t) = 0$ at $t = 0$. The model is physically and thermodynamically sound as it corresponds to well-behaved nature for the fluid density, isotropic pressure and radiation flux density throughout the fluid sphere. Initially the interior solutions represent a static configuration of perfect fluid which then gradually starts evolving into radiating collapse. The apparent luminosity as observed by the distant observer at rest at infinity is zero in remote past at the instance when collapse begins and at the stage when collapsing configuration reaches the horizon of the black hole. The surface temperature and the temperature inside the star of the collapsing body is zero at the beginning and become infinite at the final phase of the configuration. We have a number of applications of our work i.e. one can construct models of Quasars, Supernovae, Shock Waves, Black holes, Warm holes, Gravitational lensing, Quark stars/Strange stars, Boson stars, Gravastars, Eternally Collapsing Objects and various high energy astronomical objects.

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