

**BILATERAL GENERATING RELATION ASSOCIATED WITH
MULTIPLE GAUSSIAN HYPERGEOMETRIC FUNCTIONS OF
SRIVASTAVA AND EXTON**

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Dedicated to Prof. M.A. Pathan on his 75th birth anniversary

Abstract: In this paper, we obtain an interesting finite combinations of Srivastava's general triple hypergeometric function $F^{(3)}$ as a bilateral generating function for Gauss's ordinary hypergeometric function of one variable ${}_2F_1$ and Exton's double hypergeometric polynomial X , by series rearrangement technique.

Keywords and Phrases: Pochhammer symbols; Bilateral generating relation; Multiple Gaussian hypergeometric functions; Series iteration technique.

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1. Introduction

In 1967, Srivastava[17,p.428] defined the general triple hypergeometric function $F^{(3)}$ in the following form:

$$F^{(3)} \left[\begin{array}{l} (a_A) :: (b_B); (d_D); (e_E) : (g_G); (h_H); (\ell_L); \\ (m_M) :: (n_N); (p_P); (q_Q) : (r_R); (s_S); (t_T); \end{array} \middle| x, y, z \right]$$

$$= \sum_{i,j,k=0}^{\infty} \frac{[(a_A)]_{i+j+k} [(b_B)]_{i+j} [(d_D)]_{j+k} [(e_E)]_{k+i} [(g_G)]_i [(h_H)]_j [(\ell_L)]_k x^i y^j z^k}{[(m_M)]_{i+j+k} [(n_N)]_{i+j} [(p_P)]_{j+k} [(q_Q)]_{k+i} [(r_R)]_i [(s_S)]_j [(t_T)]_k i! j! k!} \quad (1.1)$$

The triple power series (1.1) is the unification and generalization of Lauricella's fourteen complete triple hypergeometric functions of second order $F_1, F_2, F_3, \dots, F_{14}$ [6, pp. 113-114] including Saran's ten triple hypergeometric functions $F_E, F_F, F_G, F_K, F_M, F_N, F_P, F_R, F_S, F_T$ [8;9], extended triple hypergeometric function F_K of Sharma [10, p. 613(2)] and three additional triple hypergeometric functions H_A, H_B, H_C of Srivastava [16, pp. 99-100; see also 13; 14; 15; 18].

In 1982, Exton [3,p.137(1.2)] defined the general double hypergeometric function in the following form:

$$X_{E;G;H}^{A:B;D} \left[\begin{array}{l} (a_A) : (b_B) ; (d_D) \quad ; \\ (e_E) : (g_G) ; (h_H) \quad ; \end{array} \quad x, y \right] = \sum_{m,n=0}^{\infty} \frac{[(a_A)]_{2m+n} [(b_B)]_m [(d_D)]_n x^m y^n}{[(e_E)]_{2m+n} [(g_G)]_m [(h_H)]_n m! n!} \quad (1.2)$$

The double power series (1.2) is the generalization and unification of Horn's non-confluent double hypergeometric function H_4 [1, p. 225 (16)], Horn's confluent double hypergeometric function H_7 [4; 5; see also 1, p. 226(35)]. In another notation of Exton [2, p. 339(13)], (1.2) is also denoted by $X_{E;G;H}^{A:B;D} \equiv \mathcal{H}_{E:G;H}^{A:0;B;D}$.

For the sake of convenience the symbol (a_A) denotes the array of A parameters given by $a_1, a_2, a_3, \dots, a_A$ in the contracted notation of Slater [11, p.54; see also 12, p.41]. The symbol $\Delta(N; b)$ denotes the array of N parameters ($N \geq 1$) given by $(b)/N, (b+1)/N, (b+2)/N, \dots, (b+N-1)/N$. The symbol $\Delta[N; (a_A)]$ denotes the array of NA parameters given by $\Delta(N; a_1), \Delta(N; a_2), \dots, \Delta(N; a_A)$ i.e. $\Delta[N; (a_A)]$ represents the array of NA parameters given by $(a_1)/N, (a_1+1)/N, (a_1+2)/N, \dots, (a_1+N-1)/N, (a_2)/N, (a_2+1)/N, (a_2+2)/N, \dots, (a_2+N-1)/N, \dots, (a_A)/N, (a_A+1)/N, (a_A+2)/N, \dots, (a_A+N-1)/N$.

The asterisk in $\Delta^*(N; j+1)$ represents the fact that the (denominator) parameter N/N obtained from $\Delta(N; j+1)$ is always omitted if $0 \leq j \leq (N-1)$ so that the set $\Delta^*(N; j+1)$ obviously contains only $(N-1)$ parameters.

The Pochhammer's symbol $[(a_A)]_u$ is defined by:

$$[(a_A)]_u = \prod_{m=1}^A \{(a_m)_u\} = \begin{cases} \prod_{m=1}^A \left\{ \frac{\Gamma(a_m+u)}{\Gamma(a_m)} \right\}; & \text{if } a_m \neq 0, -1, -2, \dots \\ \prod_{m=1}^A \{(a_m)(a_m+1) \cdots (a_m+u-1)\}; & \text{if } u = 1, 2, 3, \dots \end{cases} \quad (1.3)$$

with similar interpretation for others and the symbol Γ stands for Gamma function.

$$[\Delta(N; a)]_j = \left(\frac{a}{N}\right)_j \left(\frac{a+1}{N}\right)_j \cdots \left(\frac{a+N-1}{N}\right)_j = \prod_{k=1}^N \left\{ \left(\frac{a+k-1}{N}\right)_j \right\} = \frac{(a)_{Nj}}{N^{Nj}} \tag{1.4}$$

$$[\Delta[N; (b_B)]]_j = \prod_{m=1}^N \left\{ \left[\frac{(b_B) + m - 1}{N}\right]_j \right\} = \frac{(b_1)_{Nj} (b_2)_{Nj} \cdots (b_B)_{Nj}}{N^{BNj}} = \frac{[(b_B)]_{Nj}}{N^{BNj}} \tag{1.5}$$

The identities (1.4) and (1.5) can be verified in view of the definition of Pochhammer’s symbol [20, p. 21(14)] and Gauss’s multiplication theorem [20, p. 23(26); see also 7].

In our investigations, we shall use the following results

$${}_2F_1 \left[\begin{matrix} A, B & ; \\ & z \end{matrix} \right] = (1-z)^{-A} {}_2F_1 \left[\begin{matrix} A, C-B & ; \\ C & ; \end{matrix} -\frac{z}{1-z} \right] \tag{1.6}$$

$$(c \notin \{0, -1, -2, \dots\} \text{ and } |\arg(1-z)| < \pi)$$

$$\sum_{n=0}^{\infty} \sum_{\substack{2j+k \leq n \\ j, k=0}} \Phi(n, j, k) = \sum_{n=0}^{\infty} \sum_{j, k=0}^{\infty} \Phi(n+2j+k, j, k) \tag{1.7}$$

$$(-n-2j-k)_{2j+k} = \frac{(-1)^k (n+2j+k)!}{n!} \tag{1.8}$$

$$\sum_{n=0}^{\infty} \frac{(a)_n t^n}{n!} = (1-t)^{-a} \quad , \quad |t| < 1 \tag{1.9}$$

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \Psi(i, j, k) = \sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \Psi(2Wi+p, Wj+q, 2Wk+u) \tag{1.10}$$

The set $\Delta(0; a)$ is empty.

In the literature, the equation (1.6) is known as Euler’s first linear transformation [20, p. 33 (19, 20); see also 7, p. 60(4)], the equation (1.9) is known as Binomial theorem. The triple series identity (1.7) is due to Srivastava and Manocha [20, p. 102 (17)]. The triple series identity (1.10) is also due to Srivastava [19, pp. 196-197; see also 20, p. 217 (12)].

2. Bilateral Generating Relation

Since Pochhammer’s symbol is associated with Gamma function and Gamma function is undefined for zero and negative integers therefore arguments, numerator and denominator parameters are adjusted in such a way that each term of the following series is completely defined and meaningful, then without any loss of convergence, we obtain

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(a)_n t^n}{n!} {}_2F_1 \left[\begin{matrix} b-n, c & ; \\ a+b & ; \end{matrix} \middle| x \right] X_{E:M;S}^{D+1:L;R} \left[\begin{matrix} -n, (d_D):(l_L);(r_R); & & \\ & & y, z \end{matrix} \right] \\ & = (1-t)^{-a}(1-x)^{-c} \times \\ & \sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \frac{(a)_{p+2q+u}(c)_p[(d_D)]_{2q+u}[(l_L)]_q[(r_R)]_u \left(\frac{x}{(x-1)(1-t)}\right)^p \left(\frac{y t^2}{(1-t)^2}\right)^q \left(\frac{z t}{(t-1)}\right)^u}{p!q!u!(a+b)_p[(e_E)]_{2q+u}[(m_M)]_q[(s_S)]_u} \times \\ & \times F^{(3)} \left[\begin{matrix} \Delta(2W; a+p+2q+u)::-; \Delta[2W; (d_D)+2q+u];-; \\ -::-; \Delta[2W; (e_E)+2q+u];-; \Delta^*(2W; p+1), \Delta(2W; a+b+p); \\ \Delta(2W; c+p) & ; & \Delta[W; (l_L)+q] & ; & \Delta[2W; (r_R)+u] & ; \\ & ; & \Delta^*(W; q+1), \Delta[W; (m_M)+q] & ; & \Delta^*(2W; u+1), \Delta[2W; (s_S)+u] & ; \end{matrix} \right] \\ & \left(\frac{x}{(x-1)(1-t)} \right)^{2W}, \frac{(2W)^{2W(1+D-E)}}{W^{W(1+M-L)}} \left(\frac{y t^2}{(1-t)^2} \right)^W, (2W)^{2W(D-E+R-S)} \left(\frac{z t}{t-1} \right)^{2W} \end{aligned} \tag{2.1}$$

3. Proof of (2.1)

Suppose the left hand side of (2.1) is denoted by S , then it can be represented as

$$\begin{aligned} S & = (1-x)^{-c} \sum_{n=0}^{\infty} \frac{(a)_n t^n}{n!} {}_2F_1 \left[\begin{matrix} a+n, c & ; \\ a+b & ; \end{matrix} \middle| \frac{x}{x-1} \right] \times \\ & X_{E:M;S}^{D+1:L;R} \left[\begin{matrix} -n, (d_D) : (l_L) ; (r_R) ; & & \\ & & y, z \end{matrix} \right] \\ & = (1-x)^{-c} \sum_{n=0}^{\infty} \frac{(a)_n t^n}{n!} \sum_{i=0}^{\infty} \frac{(c)_i (a+n)_i \left(\frac{x}{x-1}\right)^i}{i! (a+b)_i} \times \end{aligned}$$

$$\begin{aligned}
 & \sum_{j,k=0}^{2j+k \leq n} \frac{(-n)_{2j+k} [(d_D)]_{2j+k} [(\ell_L)]_j [(r_R)]_k y^j z^k}{[(e_E)]_{2j+k} [(m_M)]_j [(s_S)]_k j! k!} \\
 = & (1-x)^{-c} \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(a)_{n+2j+k} t^{n+2j+k}}{(n+2j+k)!} \sum_{i=0}^{\infty} \frac{(c)_i (a+n+2j+k)_i \left(\frac{x}{x-1}\right)^i}{(a+b)_i i!} \times \\
 & \times \frac{(-n-2j-k)_{2j+k} [(d_D)]_{2j+k} [(\ell_L)]_j [(r_R)]_k y^j z^k}{j! k! [(e_E)]_{2j+k} [(m_M)]_j [(s_S)]_k} \\
 = & (1-x)^{-c} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(a)_{i+2j+k} (c)_i [(d_D)]_{2j+k} [(\ell_L)]_j [(r_R)]_k \left(\frac{x}{x-1}\right)^i (yt^2)^j (-zt)^k}{i! j! k! (a+b)_i [(e_E)]_{2j+k} [(m_M)]_j [(s_S)]_k} \\
 & \times \sum_{n=0}^{\infty} \frac{(a+i+2j+k)_n t^n}{n!} \\
 = & (1-t)^{-a} (1-x)^{-c} \times \\
 & \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(a)_{i+2j+k} (c)_i [(d_D)]_{2j+k} [(\ell_L)]_j [(r_R)]_k \left(\frac{x}{(x-1)(1-t)}\right)^i \left(\frac{yt^2}{(1-t)^2}\right)^j \left(\frac{zt}{t-1}\right)^k}{i! j! k! (a+b)_i [(e_E)]_{2j+k} [(m_M)]_j [(s_S)]_k} \\
 = & (1-t)^{-a} (1-x)^{-c} \times \\
 & \sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(c)_{2Wi+p} (a)_{2Wi+p+2Wj+2q+2Wk+u} [(\ell_L)]_{Wj+q}}{(a+b)_{2Wi+p} (2Wi+p)! (Wj+q)! (2Wk+u)!} \times \\
 & \times \frac{[(d_D)]_{2Wj+2q+2Wk+u} [(r_R)]_{2Wk+u} \left(\frac{x}{(x-1)(1-t)}\right)^{2Wi+p} \left(\frac{yt^2}{(1-t)^2}\right)^{Wj+q} \left(\frac{zt}{t-1}\right)^{2Wk+u}}{[(e_E)]_{2Wj+2q+2Wk+u} [(m_M)]_{Wj+q} [(s_S)]_{2Wk+u}} \\
 = & (1-t)^{-a} (1-x)^{-c} \times \\
 & \sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \frac{(a)_{p+2q+u} (c)_p [(d_D)]_{2q+u} [(\ell_L)]_q [(r_R)]_u \left(\frac{x}{(x-1)(1-t)}\right)^p \left(\frac{y t^2}{(1-t)^2}\right)^q \left(\frac{z t}{t-1}\right)^u}{(a+b)_p p! q! u! [(e_E)]_{2q+u} [(m_M)]_q [(s_S)]_u} \times \\
 & \times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(a+p+2q+u)_{2W(i+j+k)} (c+p)_{2Wi} [(d_D)+2q+u]_{2W(j+k)}}{(a+b+p)_{2Wi} i! j! k! [(e_E)+2q+u]_{2W(j+k)} [(m_M)+q]_{Wj}} \times \\
 & \times \frac{[(\ell_L)+q]_{Wj} [(r_R)+u]_{2Wk} (1)_i (1)_j (1)_k \left(\frac{x}{(x-1)(1-t)}\right)^{2Wi} \left(\frac{y t^2}{(1-t)^2}\right)^{Wj} \left(\frac{z t}{t-1}\right)^{2Wk}}{[(s_S)+u]_{2Wk} (1+p)_{2Wi} (1+q)_{Wj} (1+u)_{2Wk}}
 \end{aligned}$$

$$\begin{aligned}
&= (1-t)^{-a} (1-x)^{-c} \times \\
&\sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \frac{(a)_{p+2q+u} (c)_p [(d_D)]_{2q+u} [(\ell_L)]_q [(r_R)]_u \left(\frac{x}{(x-1)(1-t)}\right)^p \left(\frac{yt^2}{(1-t)^2}\right)^q \left(\frac{zt}{t-1}\right)^u}{(a+b)_p p! q! u! [(e_E)]_{2q+u} [(m_M)]_q [(s_S)]_u} \times \\
&\quad \times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(2W)^{2W(i+j+k)} \prod_{v=1}^{2W} \left\{ \left(\frac{a+p+2q+u+v-1}{2W} \right)_{i+j+k} \right\}}{i! j! k! \prod_{v=1}^{2W} \left\{ \left(\frac{a+b+p+v-1}{2W} \right)_i \right\}} \times \\
&\quad \times \frac{\prod_{v=1}^{2W} \left\{ \left(\frac{c+p+v-1}{2W} \right)_i \right\} (2W)^{2(D-E)(j+k)W} \prod_{v=1}^{2W} \left\{ \left[\frac{(d_D)+2q+u+v-1}{2W} \right]_{j+k} \right\} W^{(L-M)Wj}}{\prod_{v=1}^{2W} \left\{ \left[\frac{(e_E)+2q+u+v-1}{2W} \right]_{j+k} \right\}} \\
&\quad \times \frac{\prod_{v=1}^W \left\{ \left[\frac{(\ell_L)+q+v-1}{W} \right]_j \right\} (2W)^{2(R-S)Wk} \prod_{v=1}^{2W} \left\{ \left[\frac{(r_R)+u+v-1}{2W} \right]_k \right\}}{\prod_{v=1}^W \left\{ \left[\frac{(m_M)+q+v-1}{W} \right]_j \right\} \prod_{v=1}^{2W} \left\{ \left[\frac{(s_S)+u+v-1}{2W} \right]_k \right\} (2W)^{2Wi}} \times \\
&\quad \times \frac{(1)_i (1)_j (1)_k \left(\frac{x}{(x-1)(1-t)}\right)^{2Wi} \left(\frac{yt^2}{(1-t)^2}\right)^{Wj} \left(\frac{zt}{t-1}\right)^{2Wk}}{\prod_{v=1}^{2W} \left\{ \left(\frac{1+p+v-1}{2W} \right)_i \right\} W^{Wj} \prod_{v=1}^W \left\{ \left(\frac{1+q+v-1}{W} \right)_j \right\} (2W)^{2Wk} \prod_{v=1}^{2W} \left\{ \left(\frac{1+u+v-1}{2W} \right)_k \right\}} \\
&= (1-t)^{-a} (1-x)^{-c} \times \\
&\sum_{p=0}^{2W-1} \sum_{q=0}^{W-1} \sum_{u=0}^{2W-1} \frac{(a)_{p+2q+u} (c)_p [(d_D)]_{2q+u} [(\ell_L)]_q [(r_R)]_u \left(\frac{x}{(x-1)(1-t)}\right)^p \left(\frac{yt^2}{(1-t)^2}\right)^q \left(\frac{zt}{t-1}\right)^u}{(a+b)_p p! q! u! [(e_E)]_{2q+u} [(m_M)]_q [(s_S)]_u} \times \\
&\quad \times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{\prod_{v=1}^{2W} \left\{ \left(\frac{a+p+2q+u+v-1}{2W} \right)_{i+j+k} \right\} \prod_{v=1}^{2W} \left\{ \left[\frac{(d_D)+2q+u+v-1}{2W} \right]_{j+k} \right\} (1)_i}{i! j! k! \prod_{v=1}^{2W} \left\{ \left[\frac{(e_E)+2q+u+v-1}{2W} \right]_{j+k} \right\} \prod_{v=1}^{2W} \left\{ \left(\frac{1+p+v-1}{2W} \right)_i \right\}} \times \\
&\quad \times \frac{\prod_{v=1}^{2W} \left\{ \left(\frac{c+p+v-1}{2W} \right)_i \right\} (1)_j \prod_{v=1}^W \left\{ \left[\frac{(\ell_L)+q+v-1}{W} \right]_j \right\} (1)_k \prod_{v=1}^{2W} \left\{ \left[\frac{(r_R)+u+v-1}{2W} \right]_k \right\}}{\prod_{v=1}^{2W} \left\{ \left(\frac{a+b+p+v-1}{2W} \right)_i \right\} \prod_{v=1}^W \left\{ \left(\frac{1+q+v-1}{W} \right)_j \right\} \prod_{v=1}^W \left\{ \left[\frac{(m_M)+q+v-1}{W} \right]_j \right\} \prod_{v=1}^{2W} \left\{ \left(\frac{1+u+v-1}{2W} \right)_k \right\}} \times
\end{aligned}$$

$$\times \frac{\left(\frac{x}{(x-1)(1-t)}\right)^{2Wi} (2W)^{2W(1+D-E)j} \left(\frac{yt^2}{(1-t)^2}\right)^{Wj} (2W)^{2W(D-E+R-S)k} \left(\frac{zt}{t-1}\right)^{2Wk}}{\prod_{v=1}^{2W} \left\{ \left[\frac{(s_s)+u+v-1}{2W} \right]_k \right\}} W^{W(1+M-L)j}$$

Now interpreting the definition of Srivastava's general triple series into hypergeometric form, we get the right hand side of (2.1).

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