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ON THE VISCOUS FLUID FLOW THROUGH CYLINDRICAL TUBE OF ELLIPTICAL CROSS-SECTION FILLED WITH POROUS MEDIUM

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Abstract: This paper concerns with the study of viscous fluid flow through cylindrical tube of elliptical cross section filled with porous medium with impermeable core. Velocity profiles and flow rate are obtained analytically. Also, for different value of parameter the behavior of velocity profiles and flow rate are presented graphically.

1. Introduction

The problem of fluid flow through cylinders of elliptical cross sections filled porous medium has been longstanding interest for many researchers because of their applications in physical sciences, chemical engineering and industries. Various aspects of the problem have been investigated by many authors. Chandna [3] give a theoretical investigation of the unsteady two dimensional flow of a viscous, incompressible fluid normal to a thin elliptical cylinder and find the flow solution for small values of time. Choi and Lee [4] investigated turbulent boundary layer on elliptical cylinder. Khan [5] et al. solved the elliptical cylinder problem by using analytical approach. In this paper the problem is solved using semi- analytical approach.

Further EL-Bashir [6] study on Creeping Flow Past an Elliptical Cylinder in the Presence of a Vortex and presented the fluid flow passing through an elliptical cylinder in the presence of a vortex and numerical results are investigated. A calculation of potential flow around an elliptic cylinder using boundary element method is reported by Mushtaq [7] et al. In 2003 DAlessio and Young [8] study the steady flow past an elliptic cylinder inclined to the stream. Rao [9] et al. reported an extensive numerical result on the critical Reynolds numbers denoting wake formation and the onset of vortex shedding for the flow of Newtonian and power-law fluids have been obtained over wide ranges of conditions. Mohammad [10] et al. studied the separation times analysis of unsteady boundary layer flow past an elliptic cylinder near rear stagnation point and by using Keller Box method solve the system of non linear equations.

The present work is related to the study of fluid flow through cylinder of elliptical cross section filled with porous medium. Also velocity profiles and flow rate are obtained analytically and for different value of parameter the behavior of velocity profiles and flow rate are presented graphically.

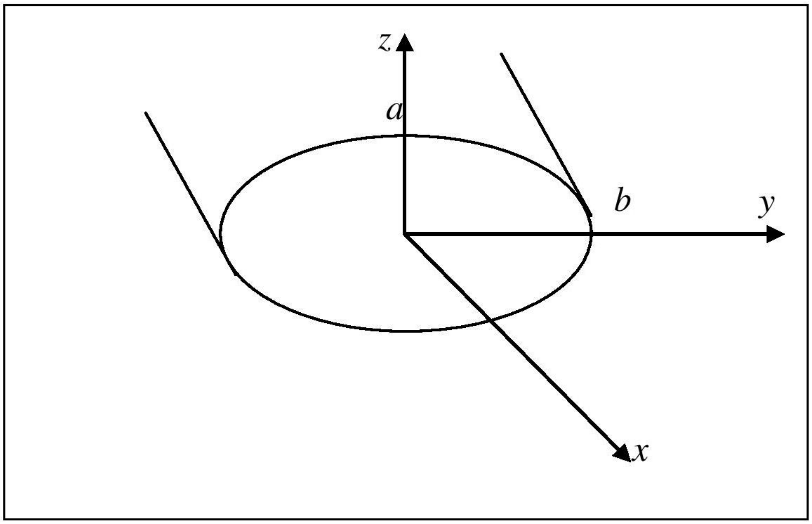

Figure 1: - Two- dimensional Brinkman flow in a tube of elliptical cross-section.

Formulation of the Problem and solution:

In this mathematical problem, consider the two-dimensional steady incompressible Newtonian flow through a porous medium which is filled in a long tube of elliptical cross-section. The flow through porous medium is governed by the Brinkman equation which given below as

$$\widetilde{\mu}_e \widetilde{\nabla}^2 \widetilde{v} - \frac{\widetilde{\mu}}{\widetilde{k}} \widetilde{v} = \widetilde{\nabla} \widetilde{p}. \tag{1}$$

with the equation of continuity

$$\widetilde{\nabla}.\widetilde{v}^{(i)} = 0. \tag{2}$$

Let us introduced the dimensionless variables and constants as follows:

$$r = \frac{\widetilde{r}}{\widetilde{b}},\ \nabla = \widetilde{\nabla}\widetilde{b},\ v = \frac{\widetilde{v}}{U},\ p = \frac{\widetilde{p}}{\widetilde{p}_0},\ \widetilde{p}_0 = \frac{\widetilde{U}\widetilde{\mu}_1}{\widetilde{b}},\ \lambda = \sqrt{\frac{\widetilde{\mu}_2}{\widetilde{\mu}_1}},\ \alpha = \frac{\widetilde{b}}{\sqrt{\widetilde{k}}},\ \sigma = \frac{\alpha}{\lambda}.$$

Then we have the Brinkman equation (1) and the equation of continuity (2) in the dimensionless form as

$$-\sigma^2 v + \nabla^2 v = \nabla p. \tag{3}$$

with the equation of continuity

$$\nabla.v = 0 \tag{4}$$

Let us consider the Cartesian coordinate system to describe the flow, with the x-axis being parallel to the flow direction. Therefore, v_x is the only non-zero velocity constant and

$$v_y = v_z = 0. \tag{5}$$

From the equation of continuity for incompressible flow,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

we find that

$$\frac{\partial v_x}{\partial x} = 0. \tag{6}$$

Equation (6), indicates that v_x does not change in the flow direction i.e. v_x independent of x i.e.

$$v_x = v_x(y, z). \tag{7}$$

From equation (3), the x-component can be written as

$$-\sigma^2 v_x + \left(\frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) = \frac{\partial p}{\partial x} \quad \text{in} \quad \frac{y^2}{a^2} + \frac{z^2}{b^2} \leq 1. \tag{8}$$

The velocity is zero at the wall of tube, and thus the boundary condition can be taken as

$$v_x = 0 \quad \text{on} \quad \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1. \tag{9}$$

Now introducing a new dependent variable v'_x, such that

$$v_x(y,z) = v'_x(y,z) + c_1y^2 + c_2z^2. \tag{10}$$

Where c_1 and c_2 are the non-zero constant to be determined such that v'_x satisfied the Laplace equation and is constant. Substituting equation (10) into (8) we have

$$-\sigma^2(v'_x + c_1y^2 + c_2z^2) + \left(\frac{\partial^2 v'_x}{\partial y^2} + \frac{\partial^2 v'_x}{\partial z^2} + 2c_1 + 2c_2\right) = \frac{\partial p}{\partial x}. \tag{11}$$

Thus, v'_x satisfy the Laplace equation

$$\left(\frac{\partial^2 v'_x}{\partial y^2} + \frac{\partial^2 v'_x}{\partial z^2}\right) = 0. \tag{12}$$

If

$$-\sigma^2(v'_x + c_1y^2 + c_2z^2) + 2c_1 + 2c_2 = \frac{\partial p}{\partial x}. \tag{13}$$

Then, from boundary condition (9) and equation (13) we have

$$2c_1 + 2c_2 = \frac{\partial p}{\partial x}. \tag{14}$$

From boundary condition (9) and equation (10) we have

$$v'_x(y,z) = -c_1y^2 - c_2z^2 = -c_1\left(y^2 + \frac{c_2}{c_1}z^2\right) \quad \text{on} \quad \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1.$$

Setting

$$\frac{c_2}{c_1} = \frac{a^2}{b^2}. \tag{15}$$

Then

$$v'_x(y,z) = -c_1a^2 \quad \text{on} \quad \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1. \tag{16}$$

The maximum principle for the Laplace equation states that v'_x has both the minimum and maximum values on the boundary of the domain [12]. Therefore, v'_x is constant over the whole domain.

$$v'_x(y,z) = -c_1a^2. \tag{17}$$

By using equation (15) and (17) in equation (10) we have

$$v_x(y,z) = -c_1a^2\left[1 - \frac{y^2}{a^2} - \frac{z^2}{b^2}\right]. \tag{18}$$

Then from equation (14) and (15), we have

$$c_1 = \frac{1}{2}\frac{\partial p}{\partial x}\frac{b^2}{(a^2+b^2)}. \tag{19}$$

Hence

$$v_x(y,z) = -\frac{1}{2}\frac{\partial p}{\partial x}\frac{a^2b^2}{a^2+b^2}\left[1-\frac{y^2}{a^2}-\frac{z^2}{b^2}\right]. \tag{20}$$

Now on integration of the velocity over the elliptical cross-section, we have the volumetric flow rate as

$$Q = -\frac{1}{4}\frac{\partial p}{\partial x}\frac{a^3b^3}{a^2+b^2} \tag{21}$$

Consider the case when a = b = R then the velocity profile becomes

$$v_x(y,z) = -\frac{1}{4}\frac{\partial p}{\partial x}R^2\left[1-\frac{y^2+z^2}{R^2}\right]. \tag{22}$$

If we take $r^2 = y^2 + z^2$, then it reduces in cylindrical coordinate as

$$v_x(r) = -\frac{1}{4}\frac{\partial p}{\partial x}[R^2 - r^2]. \tag{23}$$

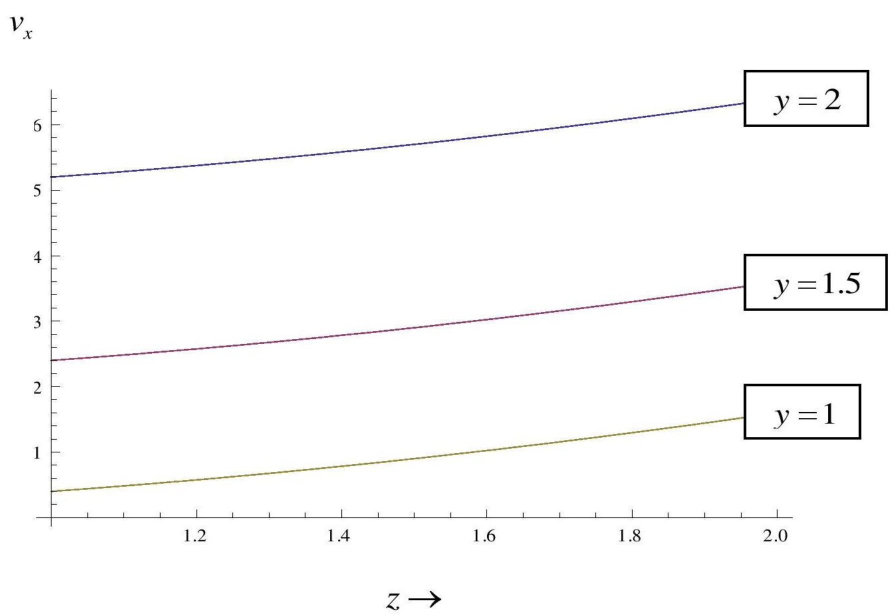

Figure 2: Velocity profile v_x for different value of parameters a = 1, b = 2, $\lambda = 2$.

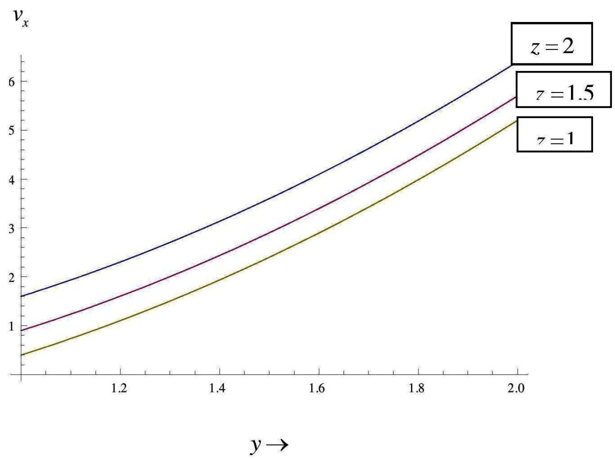

Figure 3: Velocity profile v_x for different value of parameters a = 1, b = 2, $\lambda = 2$.

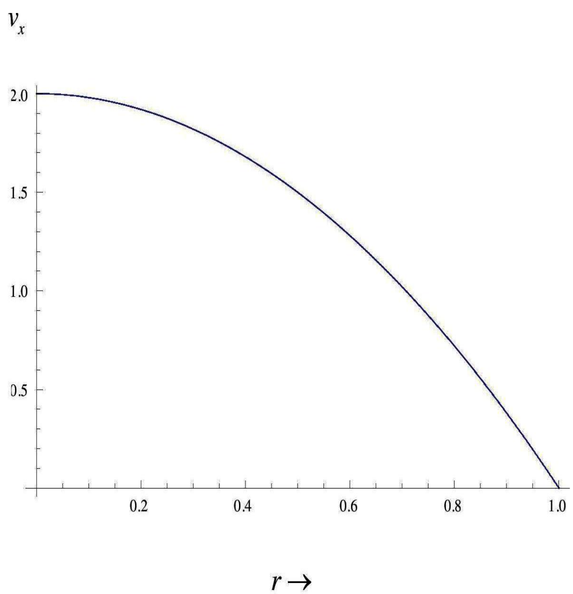

Figure 4: Velocity profile v_x for different value of parameters R = 1, $\lambda = 2$.

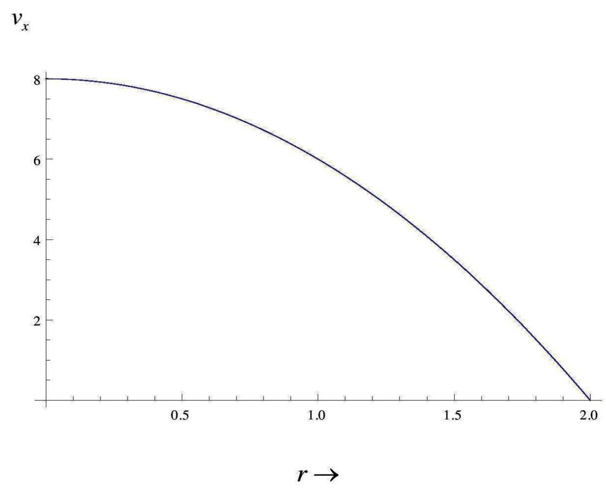

Figure 5: Velocity profile v_x for different value of parameters R = 2, $\lambda = 2$.

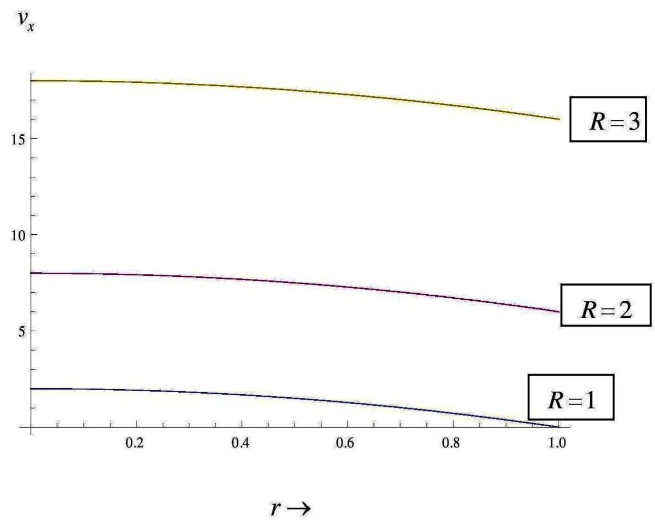

Figure 6: Velocity profile v_x for different value of parameters $\lambda = 2$.

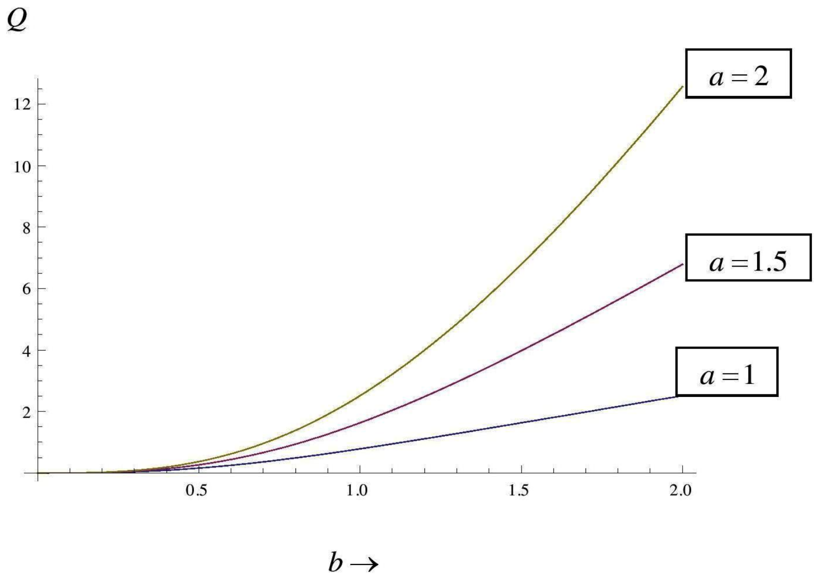

Figure 7: Flow rate Q for different value of parameters $\lambda = 2$.

Results and Discussion:

The discussion of the results is depends on the behavior of graphs which are plotted above. From figure-3 it shows that the for a fix value of z the velocity increases with increase the value of y. It is, Also, shows that for higher value of z velocity is higher.

Figure-4 and 5 shows that flow velocity is maximum at the center and it vanishes at the wall surface of the tube. The velocity is increases with increase in the value of R and for higher value of R it also higher which is shown by figure-6. Figure-7 is related to the behavior of flow rate by it we observe that for different value of a = 1, 1.5, 2 flow rate is increases with increase in the value of b and it also shows that for higher value of a flow rate is higher. Physically it can be observed that the area of the cross-section of a tube is increases flow rate of a fluid passing through the tube increases.

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