*J. of Ramanujan Society of Math. and Math. Sc.* **ISSN : 2319-1023**
*Vol.4, No.2 (2015), pp. 51-60*

# On Generating Functions of Hypergeometric Polynomials by Group-theoretic Method

**K. P. Samanta and B. Samanta***
Department of Mathematics,
Indian Institute of Engineering Science and Technology, Shibpur,
Howrah - 711103, W.B., India.
E-mail: kalipadasamanta2010@gmail.com
* Department of Mathematics,
Shibpur Dinobundhoo Institution (College), 412/1,G.T.Road,Shibpur,
Howrah -711 102, W.B., India.
E-mail: bijoysamanta@yahoo.com

***Dedicated to Prof. Hari M. Srivastava on his*** $75^{th}$ ***birth anniversary***

**Abstract:** In this paper we have obtained some novel generating functions of ${}_2F_1(-n, \alpha; \gamma + n; x)$ - the modified form of Hypergeometric polynomials ${}_2F_1(-n, \alpha; \gamma; x)$ by utilizing L. Weisner's group-theoretic method of obtaining generating functions. In section-2, we obtain a set of infinitesimal operators by giving suitable interpretations to both the index $(n)$ and the parameter $(\gamma)$ of the polynomial under consideration, known as raising and the lowering operators has been introduced and on showing that they generate a four dimensional Lie algebra, we have obtained, in section-3, a novel generating functions of the Hypergeometric polynomials which in turn yields a number of new and known results on generating functions.

**Keywords:** Generating functions,Hypergeometric polynomials,Jacobi polynomials.
**AMS-2010 Subject Classification Code:** 33C65.

## 1. Introduction

The Hypergeometric polynomials [5] ${}_2F_1(-n, \alpha; \gamma; x)$ is a solution of the following ordinary differential equation:

$$[x(1-x)\frac{d^2}{dx^2} + \{\gamma + (n - \alpha - 1)x\}\frac{d}{dx} + n\alpha]y = 0. \tag{1.1}$$

In this paper we have encountered a problem on generating functions of ${}_2F_1$(-n, $\alpha; \gamma + n; x)$ - the modified form of ${}_2F_1(-n, \alpha; \gamma; x)$ by employing the method of Weisner [2-4] with the suitable interpretations of $n$, $\gamma$ simultaneously. Weisner's method consists in constructing a partial differential equation from an ordinary differential equation by giving suitable interpretation of $n$, $\gamma$ simultaneously and then on finding a non-trivial continuous transformations group admitted by the differential equation. The method of Weisner is lucidly presented in the monograph "Obtaining Generating Functions" written by E. B. McBride [8]. For previous works on Hypergeometric polynomials one can see the works of [10-17]. The object of the present note is to derive some new and known generating relations involving modified Hypergeometric polynomials ${}_2F_1(-n, \alpha; \gamma + n; x)$,which satisfies the following ordinary differential equation :

$$[x(1-x)\frac{d^2}{dx^2} + \{\gamma + n + (n-\alpha-1)x\}\frac{d}{dx} + n\alpha]u = 0. \quad (1.2)$$

The main results of our investigation are given in section-3.

**2. Group-Theoretic Discussion :**

Replacing $\frac{d}{dx}$ by $\frac{\partial}{\partial x}$,$n$ by $y\frac{\partial}{\partial y}$,$\gamma$ by $z\frac{\partial}{\partial z}$ and $u$ by $v(x,y,z)$ in (1.2) we get the following partial differential equation :

$$x(1-x)\frac{\partial^2 v}{\partial x^2} + (1+x)y\frac{\partial^2 v}{\partial x \partial y} + z\frac{\partial^2 v}{\partial x \partial z} - (1+\alpha)x\frac{\partial v}{\partial x} + \alpha y\frac{\partial v}{\partial y} = 0. \quad (2.1)$$

Thus we see that $v_1(x,y,z) = {}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma$ is a solution of (2.1), since ${}_2F_1(-n, \alpha; \gamma + n; x)$ is a solution of (1.2).

We now define the infinitesimal operators $A_i$ (i= 1,2,3,4) as follows :

$$A_1 = y\frac{\partial}{\partial y}; A_2 = z\frac{\partial}{\partial z}; A_3 = (1-x)\frac{z^2}{y}\frac{\partial}{\partial x} + z^2\frac{\partial}{\partial y};$$

$$A_4 = x(1-x)\frac{y}{z^2}\frac{\partial}{\partial x} + \frac{y^2}{z^2}\frac{\partial}{\partial y} + \frac{y}{z}\frac{\partial}{\partial z} - (1+\alpha x)\frac{y}{z^2},$$

such that

$$A_1({}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma) = n\ {}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma,$$

$$A_2({}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma) = \gamma\ {}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma,$$

$$A_3({}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma) = \frac{n(n+\gamma-\alpha)}{n+\gamma}\ {}_2F_1(-(n-1), \alpha; \gamma + n + 1; x)y^{n-1} z^{\gamma+2},$$

$$A_4({}_2F_1(-n, \alpha; \gamma + n; x)y^n z^\gamma) = (n+\gamma-1)\ {}_2F_1(-(n+1), \alpha; \gamma + n - 1; x)y^{n+1} z^{\gamma-2}.$$

We now proceed to find the commutator relations. Using the notation, $[A, B]u = (AB - BA)u$, we have

$$[A_1, A_2] = 0; [A_1, A_3] = -A_3; [A_1, A_4] = A_4; [A_2, A_3] = 2A_3;$$

$$[A_2, A_4] = -2A_4; [A_3, A_4] = A_2 - (1 + \alpha).$$

From the above commutator relations,we state the following theorem .
**Theorem 1:** The set of operators $\{1, A_i(i = 1, 2, 3, 4)\}$ where 1 stands for the identity operator, generates a Lie algebra $Ł$.
It can be easily shown that the partial differential operator $Ł$ given by

$$Ł = x(1-x)\frac{\partial^2}{\partial x^2} + (1+x)y\frac{\partial^2}{\partial x \partial y} + z\frac{\partial^2}{\partial x \partial z} - (1+\alpha)x\frac{\partial}{\partial x} + \alpha y\frac{\partial}{\partial y}$$

can be expressed as follows :

$$(1-x)Ł = A_4A_3 - {A_1}^2 - A_1A_2 + (2+\alpha)A_1. \tag{2.2}$$

From the above commutator relations,it can be easily verified that $(1 - x)Ł$ commutes with $A_i(i = 1, 2, 3, 4)$,

$$i.e. \qquad [(1-x)Ł, A_i] = 0, i = 1, 2, 3, 4. \tag{2.3}$$

The extended form of the groups generated by $A_i(i = 1, 2, 3, 4)$ are

$$e^{a_1A_1}f(x, y, z) = f(x, e^{a_1}y, z),$$

$$e^{a_2A_2}f(x, y, z) = f(x, y, e^{a_2}z),$$

$$e^{a_3A_3}f(x, y, z) = f\left(\frac{x + a_3y^{-1}z^2}{1 + a_3y^{-1}z^2}, y(1 + a_3y^{-1}z^2), z\right),$$

$$e^{a_4A_4}f(x, y, z) = (1 + a_4xyz^{-2})^{-\alpha}(1 + a_4yz^{-2})^{-1}\times$$
$$\times f\left(x\left(\frac{1 + a_4yz^{-2}}{1 + a_4xyz^{-2}}\right), y(1 + a_4yz^{-2}), z(1 + a_4yz^{-2})\right).$$

From the above we get,

$$e^{a_4A_4}e^{a_3A_3}e^{a_2A_2}e^{a_1A_1}f(x, y, z) = (1 + a_4xyz^{-2})^{-\alpha}(1 + a_4yz^{-2})^{-1}f(\zeta, \eta, \theta), \tag{2.4}$$

where

$$\zeta = \frac{(1+a_4yz^{-2})(x+a_3y^{-1}z^2(1+a_4xyz^{-2}))}{(1+a_4xyz^{-2})\{1+a_3y^{-1}z^2(1+a_4yz^{-2})\}},$$

$$\eta = e^{a_1}y(1+a_4yz^{-2})\{1+a_3y^{-1}z^2(1+a_4yz^{-2})\},$$

$$\theta = e^{a_2}z(1+a_4yz^{-2}).$$

**3. Generating Functions :**

From (2.1), $u(x,y,z) = {}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma$ is a solution of the system

$$\begin{cases} Lu = 0 & Lu = 0 & Lu = 0 \\ (A_1-n)u = 0; & (A_2-\gamma)u = 0; & (A_1+A_2-n-\gamma)u = 0. \end{cases}$$

From (2.3),we easily get

$$S(1-x)L({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma) = (1-x)LS({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma) = 0,$$

where

$$S = e^{a_4A_4}e^{a_3A_3}e^{a_2A_2}e^{a_1A_1}.$$

Therefore the transformation $S({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma)$ is annulled by $(1-x)L$. Now putting $a_1 = a_2 = 0$ and replacing $f(x,y,z)$ by ${}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma$ in (2.4), we get

$$e^{a_4A_4}e^{a_3A_3}({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma) = (1+a_4xyz^{-2})^{-\alpha}(1+a_4yz^{-2})^{n+\gamma-1}$$
$$\times(1+(1+a_4yz^{-2})a_3y^{-1}z^2)^n {}_2F_1(-n,\alpha;\gamma+n;\zeta)y^nz^\gamma, \tag{3.1}$$

where

$$\zeta = \frac{(1+a_4yz^{-2})(x+a_3y^{-1}z^2(1+a_4xyz^{-2}))}{(1+a_4xyz^{-2})(1+a_3y^{-1}z^2(1+a_4yz^{-2}))}.$$

In the other hand we get

$$e^{a_4A_4}e^{a_3A_3}({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma)$$

$$= \sum_{k=0}^{\infty}\sum_{p=0}^{n+k}\frac{(-a_3)^p}{p!}\frac{(-a_4)^k}{k!}\frac{(-n)_p(n+\gamma-\alpha)_p}{(n+\gamma)_p}(-n-\gamma-p+1)_k$$
$$\times{}_2F_1(-(n-p+k),\alpha;\gamma+n+p-k;x)y^{n-p+k}z^{\gamma+2p-2k}. \tag{3.2}$$

Equating (3.1) and (3.2),we get

$$(1+a_4xyz^{-2})^{-\alpha}(1+a_4yz^{-2})^{n+\gamma-1}(1+(1+a_4yz^{-2})a_3y^{-1}z^2)^n$$

$$\times {}_2F_1\left(-n,\alpha;\gamma+n;\frac{(1+a_4yz^{-2})(x+a_3y^{-1}z^2(1+a_4xyz^{-2}))}{(1+a_4xyz^{-2})(1+a_3y^{-1}z^2(1+a_4yz^{-2}))}\right)$$

$$=\sum_{k=0}^{\infty}\sum_{p=0}^{n+k}\frac{(-a_3)^p}{p!}\frac{(-a_4)^k}{k!}\frac{(-n)_p(n+\gamma-\alpha)_p}{(n+\gamma)_p}(-n-\gamma-p+1)_k$$

$$\times {}_2F_1(-(n-p+k),\alpha;\gamma+n+p-k;x)y^{k-p}z^{2p-2k}, \tag{3.3}$$

which seems to be new.
Replacing $\gamma$ by $\gamma-n$ in (3.3), we get the exact relation found derived in [ 14 ]. It is interesting to mention that the results found derived in [16] can be easily obtained by replacing $\gamma$ by $(\gamma-2n)$ in (3.3).

Before discussing particular cases of the result (3.3) we would like to point it out that the operators $A_3$, $A_4$ being non-commutative,as seen from the commutator relation $[A_3,A_4]=A_2-(1+\alpha)$, the relation (3.3) will change if their order be interchanged in $e^{a_4A_4}e^{a_3A_3}$, which is given in section 5. we now consider the following particular cases :

**Case 1 :** Putting $a_4=0$ and replacing $(-\frac{a_3z^2}{y})$ by $t$ in (3.3),we get

$$(1-t)^n{}_2F_1(-n,\alpha;\gamma+n;\frac{x-t}{1-t})$$

$$=\sum_{p=0}^{n}\frac{(-n)_p(n+\gamma-\alpha)_p}{(n+\gamma)_p(p!)}{}_2F_1(-(n-p),\alpha;\gamma+n+p;x)t^p. \tag{3.4}$$

**Case 2 :** Putting $a_3=0$ and replacing $(-\frac{a_4y}{z^2})$ by $t$ in (3.3),we get

$$(1-tx)^{-\alpha}(1-t)^{n+\gamma-1}{}_2F_1(-n,\alpha;\gamma+n;\frac{x-xt}{1-xt})$$

$$=\sum_{k=o}^{\infty}\frac{(-n-\gamma+1)_k}{k!}{}_2F_1(-(n+k),\alpha;\gamma+n-k;x)t^k. \tag{3.5}$$

**Case 3:** Substituting $a_3=-\frac{1}{w}$,$a_4=1$ and $\frac{y}{z^2}=t$ in (3.3),we get

$$(1+xt)^{-\alpha}(1+t)^{n+\gamma-1}(t-\frac{1}{w}(1+t))^n{}_2F_1\left(-n,\alpha;\gamma+n;\frac{(1+t)(x-\frac{1}{wt}(1+xt))}{(1+xt)(1-\frac{1}{wt}(1+t))}\right)$$

$$=\sum_{k=0}^{\infty}\sum_{p=0}^{n+k}\frac{(\frac{1}{w})^p}{p!}\frac{(-1)^k}{k!}\frac{(-n)_p(n+\gamma-\alpha)_p}{(n+\gamma)_p}(-n-\gamma-p+1)_k$$

$$\times {}_2F_1(-(n-p+k), \alpha; \gamma+n+p-k; x)t^{n+k-p}. \tag{3.6}$$

**4. Special cases :** Some special cases of interest are given below :

**Special case 1:** Replacing $\alpha$, $\gamma$ and $x$ by $1+\alpha+\beta+n$, $1+\alpha-n$ and $\dfrac{1-y}{2}$ respectively in (3.4) - (3.6), we get the following results of Jacobi polynomials :

$$(1+t)^n P_n^{(\alpha,\beta)}(\frac{y-t}{1+t}) = \sum_{p=0}^{n} \frac{(-\beta-n)_p}{p!} P_{n-p}^{(\alpha+p,\beta)}(y)t^p, \tag{4.1}$$

$$\{1-\frac{t}{2}(y-1)\}^{-1-\alpha-\beta-n}(1+t)^\alpha P_n^{(\alpha,\beta)}(\frac{y+\frac{t}{2}(y-1)}{1-\frac{t}{2}(y-1)})$$

$$= \sum_{k=o}^{\infty} \frac{(n+1)_k}{k!} P_{n+k}^{(\alpha-k,\beta)}(x)t^k, \tag{4.2}$$

$$\{1-\frac{t}{2}(y-1)\}^{-1-\alpha-\beta-n}(1+t)^\alpha\{t-\frac{1}{w}(1+t)\}^n$$

$$\times P_n^{(\alpha,\beta)}\left(\frac{t(-t+x(2+t))+\frac{(1+t)}{w}(2+t(1-x))}{(t-\frac{1}{w}(1+t))(2+t(1-x))}\right)$$

$$= \sum_{k=0}^{\infty}\sum_{p=0}^{n+k} \frac{(\frac{1}{w})^p}{p!}\frac{(-1)^k}{k!}(-n-\beta)_p(n-p+1)_k P_{n-p+k}^{(\alpha+p-k,\beta)}(y)t^{n+k-p}. \tag{4.3}$$

The generating relations (4.1)-(4.3) are found in[9].

**Subcase :** Putting $n=0$ in (4.2),we get

$$(1+t)^\alpha\{1-\frac{t}{2}(x-1)\}^{-1-\alpha-\beta} = \sum_{k=o}^{\infty} P_k^{(\alpha-k,\beta)} t^k. \qquad (4.4).$$

Finally,using the symmetry relation[5] :

$$P_n^{(\beta,\alpha)}(-x) = (-1)^n P_n^{(\alpha,\beta)}(x),$$

and then simplifying,we get

$$\{1-\frac{t}{2}(x+1)\}^{-1-\alpha-\beta}(1-t)^\beta = \sum_{k=0}^{\infty} P_k^{(\alpha,\beta-k)}(x)t^k. \tag{4.5}$$

Relation (4.5) is worthy of notice and is found derived in [1,6,7] by different methods.

**Special case 2 :** Replacing $\alpha$, $\gamma$ and $x$ by $1+\alpha+\beta+2n$, $1+\alpha$ and $\dfrac{1-y}{2}$ respectively in (3.4)-(3.6), and simplifying, we get the results of Jacobi polynomials found in [18].

**5. Variants of the result(3.3) :**

By interchanging the order of operators $A_3$ and $A_4$ in

$$e^{a_4A_4}e^{a_3A_3}e^{a_2A_2}e^{a_1A_1},$$

we get

$$e^{a_3A_3}e^{a_4A_4}e^{a_2A_2}e^{a_1A_1}f(x,y,z)=(1+a_4xyz^{-2})^{-\alpha}(1+a_4yz^{-2})^{-1}f(\zeta,\eta,\theta), \quad (5.1)$$

where

$$\zeta=\frac{(x+a_3y^{-1}z^2)(1+a_3a_4+a_4yz^{-2})}{(1+a_3y^{-1}z^2)(1+a_3a_4+a_4xyz^{-2})},$$
$$\eta=e^{a_1}y(1+a_3y^{-1}z^2)(1+a_3a_4+a_4yz^{-2}),$$
$$\theta=e^{a_2}z(1+a_3a_4+a_4yz^{-2}).$$

Now putting $a_1=a_2=0$ and replacing $f(x,y,z)$ by ${}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma$ in (5.1),we get

$$\begin{aligned}&e^{a_3A_3}e^{a_4A_4}({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma)\\&=(1+a_4xyz^{-2})^{-\alpha}(1+a_4yz^{-2})^{-1}(1+a_3y^{-1}z^2)^n\\&\times(1+a_3a_4+a_4yz^{-2})^{n+\gamma}{}_2F_1(-n,\alpha;\gamma+n;\zeta)y^nz^\gamma,\end{aligned} \quad (5.2)$$

where

$$\zeta=\frac{(x+a_3y^{-1}z^2)(1+a_3a_4+a_4yz^{-2})}{(1+a_3y^{-1}z^2)(1+a_3a_4+a_4xyz^{-2})}.$$

On the other hand we get,

$$e^{a_3A_3}e^{a_4A_4}({}_2F_1(-n,\alpha;\gamma+n;x)y^nz^\gamma)$$

$$\begin{aligned}&=\sum_{k=o}^{\infty}\sum_{p=o}^{n+k}\frac{(-a_3)^p}{p!}\frac{(-a_4)^k}{k!}(-n-\gamma+1)_k\frac{(n-k)_p(n+\gamma-k-\alpha)_p}{(n+\gamma-k)_p}\\&\times{}_2F_1(-(n-p+k),\alpha;\gamma+n+p-k;x)y^{n-p+k}z^{\gamma+2p-2k}.\end{aligned} \quad (5.3)$$

Equating (5.2) and(5.3),we get

$$(1+a_4xyz^{-2})^{-\alpha}(1+a_4yz^{-2})^{-1}(1+a_3y^{-1}z^2)^n(1+a_3a_4+a_4yz^{-2})^{n+\gamma}$$

$$\times {}_2F_1\left(-n,\alpha;\gamma+n;\frac{(x+a_3y^{-1}z^2)(1+a_3a_4+a_4yz^{-2})}{(1+a_3y^{-1}z^2)(1+a_3a_4+a_4xyz^{-2})}\right)$$

$$=\sum_{k=o}^{\infty}\sum_{p=o}^{n+k}\frac{(-a_3)^p}{p!}\frac{(-a_4)^k}{k!}(-n-\gamma+1)_k\frac{(n-k)_p(n+\gamma-k-\alpha)_p}{(n+\gamma-k)_p}$$

$$\times {}_2F_1(-(n-p+k),\alpha;\gamma+n+p-k;x)y^{k-p}z^{2p-2k}.$$

**6. Application:** We now proceed to derive some novel results on bilateral generating relations by the application of the generating relation (3.5). The main result is stated in the following theorem :

**Theorem 2 :** If there exists a unilateral relation of the form :

$$G(x,t)=\sum_{n=o}^{\infty}a_n{}_2F_1(-n,\alpha;\gamma+n;x)t^n \tag{6.1}$$

then

$$(1-xy)^{-\alpha}(1-y)^{\gamma-1}G\left(\frac{x-xy}{1-xy},ty(1-y)\right)=\sum_{n=o}^{\infty}y^n\sigma_n(x,t), \tag{6.2}$$

where

$$\sigma_n(x,t)=\sum_{k=0}^{n}a_k\frac{(-\gamma-k+1)_{n-k}}{(n-k)!}{}_2F_1(-n,\alpha,\gamma-n+2k;x)t^k.$$

**Proof :** R.H.S.

$$=\sum_{n=o}^{\infty}y^n\sigma_n(x,t)$$

$$=\sum_{n=o}^{\infty}y^n\sum_{k=0}^{n}a_k\frac{(-\gamma-k+1)_{n-k}}{(n-k)!}{}_2F_1(-n,\alpha;\gamma-n+2k;x)t^k$$

$$=\sum_{k=0}^{\infty}a_k(yt)^k\sum_{n=0}^{\infty}\frac{(-\gamma-k+1)_n}{n!}{}_2F_1(-(n+k),\alpha;\gamma-n+k;x)y^n$$

$$=(1-xy)^{-\alpha}(1-y)^{\gamma-1}\sum_{k=o}^{\infty}a_k{}_2F_1\left(-k,\alpha;\gamma+k,\frac{x-xy}{1-xy}\right)(yt(1-y))^k[\text{using (3.5)}]$$

$$=(1-xy)^{-\alpha}(1-y)^{\gamma-1}G\left(\frac{x-xy}{1-xy},ty(1-y)\right)\qquad[\text{using (6.1)}]$$

$$=L.H.S$$

The importance of the above theorem lies in the fact that whenever one knows a generating relation of the form (6.1) then the corresponding bilateral generating function can at once be written down from (6.2). So one can get a large number of bilateral generating functions by attributing different suitable values to $a_n$ in (6.1).

## Acknowledgement

The authors are greatly indebted to Dr. A. K. Chongdar of IIEST, Shibpur for his constant encouragement and guidance.

## References

[1] Feldheim E.: Relation entre les polynomes de Jacobi, Laguerre et, Hermite, Acta Mathematica,75(1943), 117-138.

[2] Weisner L.: Group-theoretic origin of certain generating function, Pacific Jour. Math. 5(1955), 1033-1039.

[3] Weisner L.: Generating functions for Hermite functions, Canad. J. Math. 11 (1959), 141-147.

[4] Weisner L.: Generating functions for Bessel functions, Canad J. Math. 11 (1959), 148-155.

[5] Rainville E.D.: Special functions, Macmillan Co., New York. 1960.

[6] Al-Salam W.A.: Operational reprentation for the Laguerre and other polynomials, Duke Math. Jour. 31(1964), 127-142.

[7] Verma V.K.: Double Hypergeometric functions as generating functions of Jacobi and Laguerre polynomials, J. Indian Math. Soc. 32 (1968), 1-5.

[8] McBride E.B.: Obtaining generating functions,Springer Verlag, New York. 1971.

[9] Chakrabarti A.B.: Group-theoretic study of certain generating functions of Jacobi polynomials, J. Indian Inst. Sci. 64B (1983), 97-103.

[10] Chakrabarti A.B. & Chongdar A.K.: Group-theoretic study of certain generating functions of Hypergeometric polynomials, Tamkang Jour. Math. 16 (3) (1985), 1-10.

[11] Chongdar A.K.: Group-theoretic study of certain generating functions, Bull. Cal. Math. Soc. 77 (1985), 151-157.

[12] Das S.: Group-theoretic study of certain generating functions for Hypergeometric polynomials-I, Pure Math. Manuscript. 5 (1986), 63-69.

[13] Das S.: Group-theoretic study of certain generating functions for Hypergeometric polynomials-II, Pure Math. Manuscript. 5 (1986), 71-76.

[14] Ghosh B.: Some generating functions involving Hypergeometric polynomials by Lie algebraic method, Bull. Inst. Math. Acad. Sinica. 16 (2) (1988), 149-155.

[15] Ghosh B.: Some generating functions of Hypergeometric polynomials by Lie algebraic method, Jour. Ind. Math. Soc. 53 (1988), 195-208.

[16] Chongdar A.K. et al.: Weisner's Methodic study of Hypergeometric polynomials,Quademi No. 40(1999),1-31. Dipartimento di Matematica, Universita di Torino, Italy.

[17] Ghosh B. & Basu D.K.: Origin of certain generating function of $F(-n, \beta; \gamma; x)$ from a view point of Lie algebra, Nepali Math. Sci. Rep. 18(1999/00), No. 1-2, 73-77.

[18] Maiti P.K. & Chondar A.K.: Some generating functions of modified Jacobi polynomials by Lie algebraic method, Ultra Scientist. 24 (1) B( 2012), 1-6.