J. of Ramanujan Society of Mathematics and Mathematical Sciences
Vol. 13, No. 2 (2026), pp. 77-88

DOI: 10.56827/JRSMMS.2026.1302.6 **ISSN (Online): 2582-5461**

ISSN (Print): 2319-1023

NEUTROSOPHIC SIMPLY b-SEPARATION AXIOMS IN NEUTROSOPHIC TOPOLOGICAL SPACES

D. M. S. Malkanthi and P. Elango

Department of Mathematics,
Faculty of Science,
Eastern University, SRI LANKA

E-mail : sujeewadissanayakadmsm96@gmail.com, elangop@esn.ac.lk

(**Received:** *Dec. 02, 2025* **Accepted:** *May 02, 2026* **Published:** *Jun. 30, 2026)*

Abstract: In this paper, we define neutrosophic simply $b-T_0$ space, neutrosophic simply $b-T_1$ space and neutrosophic simply $b-T_2$ space based on single-valued neutrosophic topological spaces and studied a few properties and relationships among them. We try to establish some relationships between existing neutrosophic separation axioms and newly defined neutrosophic simply b-separation axioms. we also explore some results implementing neutrosophic simply b-open function, neutrosophic simply b-continuous function based on our defined definitions.

Keywords and Phrases: Neutrosophic topological spaces, Neutrosophic simply b-open sets,Neutrosophic simply $b-T_0$ space, Neutrosophic simply $b-T_1$ space, Neutrosophic simply $b-T_2$ space.

2020 Mathematics Subject Classification: 54A05, 54E55, 54D05.

1. Introduction

The concept of a fuzzy set was first introduced by Zadeh [7] in 1965 . This foundational idea was later extended by Atanassov [1] in 1986 through the development of intuitionistic fuzzy sets, which provide a more generalized framework. Subsequently, Smarandache [5] advanced the theory further by proposing the notion of neutrosophic sets. A neutrosophic set is characterized by three independent membership functions: truth-membership, falsity-membership, and indeterminacy-membership. These three components function independently and allow for a more nuanced modeling of uncertainty and imprecision.

Following Smarandaches [4] introduction of neutrosophy, researchers worldwide have contributed to expanding and refining this theory. Traditional models, whether classical or fuzzy, often fall short when addressing real-world complexities due to limited parameterization capabilities. Neutrosophic theory, with its broader and more flexible framework, offers a more effective approach for tackling such problems.

Building on this foundation, Ebenanjar et al. [3] introduced the concept of neutrosophic b-open sets and explored their fundamental properties. Suman Das et al. [6] introduced neutrosophic simply b-open sets and explored their fundamental properties.

In the present work, we aim to define and analyze separation axioms within the framework of neutrosophic simply b-open sets, which we term neutrosophic simply b-separation axioms. To establish this, we first revisit essential definitions related to neutrosophic interior, closure, and b-open sets, as they are crucial for our development of separation axioms in this context. We proceed by defining and examining the properties of neutrosophic simply $b - T_0$, simply $b - T_1$, and simply $b - T_2$ spaces. Furthermore, we explore the connections between classical neutrosophic separation axioms [2] and the newly introduced neutrosophic simply b-separation axioms.

2. Preliminaries

Definition 2.1. [4] *Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represents the degree of membership function, the degree of indeterminacy function and the degree of non-membership function respectively of each element $x \in X$ to the set A.*

Definition 2.2. [4] *Let X be a non-empty set, and let the neutrosophic sets A and B be $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu B(x), \sigma B(x), \gamma B(x)\rangle : x \in X\}$. Then we may consider two possible definitions for subsets $(A \subseteq B)$. That is, $A \subseteq B$ may be defined as in the following two different ways*

1. *$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu B(x), \sigma_A(x) \leq \sigma B(x)$ and $\gamma_A(x) \geq B(x) \forall x \in X$*
2. *$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu B(x), \sigma_A(x) \geq \sigma B(x)$ and $\gamma_A(x) \geq B(x) \forall x \in X$*

Definition 2.3. [4] *Let X be a non-empty set and let the neutrosophic sets A and B be $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle, B = \langle x, \mu B(x), \sigma B(x), \gamma B(x)\rangle$. Then,*
1. *$A \cap B$ may be defined as in the following in two different ways.*

1. *$A \cap B = \langle x, \mu_A(x) \wedge \mu B(x), \sigma_A(x) \wedge \sigma B(x)$ and $\gamma_A(x) \vee \gamma B(x)\rangle$*

2. $A \cap B = \langle x, \mu_A(x) \wedge \mu B(x), \sigma_A(x) \vee \sigma B(x)$ *and* $\gamma_A(x) \vee \gamma B(x) \rangle$

2. $A \cup B$ *may be defined as in the following in two different ways.*

1. $A \cup B = \langle x, \mu_A(x) \vee \mu B(x), \sigma_A(x) \vee \sigma B(x)$ *and* $\gamma_A(x) \wedge \gamma B(x) \rangle$

2. $A \cup B = \langle x, \mu_A(x) \vee \mu B(x), \sigma_A(x) \wedge \sigma B(x)$ *and* $\gamma_A(x) \wedge \gamma B(x) \rangle$

Here the notations $\wedge$ *and* $\vee$ *means minimum and maximum respectively.*

Definition 2.4. [4] *A neutrosophic topology for a non-empty set* X *is a family* τ *of neutrosophic subsets in* X *satisfying the following axioms :*

1. $0_N, 1_N \in \tau$,

2. $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

3. $\cup G_i \in \tau$ *for every* $\{G_i : i \in J\} \subseteq \tau$

The pair (X, τ) *is called a neutrosophic topological space.*

The elements of τ *are called neutrosophic open sets. The complement of a neutrosophic open set is called a neutrosophic closed set.*

Definition 2.5. [4] *Let* (X, τ) *be neutrosophic topological space and* $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ *be a neutrosophic set in* X*. Then the neutrosophic closure and neutrosophic interior of* A *are defined by*

1. $Ncl(A) = \cap\{K : K$ *is a neutrosophic closed set in* X *and* $A \subseteq K$ $\}$

2. $Nint(A) = \cup\{G : G$ *is a neutrosophic open set in* X *and* $G \subseteq A$ $\}$.

It can be also shown that $Ncl(A)$ *is neutrosophic closed set and* $Nint(A)$ *is a neutrosophic open set in* X*. Further,*
a) A *is a neutrosophic open set if and only if* $A = Nint(A)$.
b) A *is a neutrosophic closed set if and only if* $A = Ncl(A)$.

Definition 2.6. [3] *A neutrosophic set* U *in a neutrosophic topological space* X *is called a neutrosophic b-open set if* $U \subseteq Nint(Ncl(U)) \cup Ncl(Nint(U))$.

Definition 2.7. [6] *A neutrosophic set* U *in a neutrosophic topological space* X *is called a neutrosophic simply b-open set iff it is a neutrosophic b-open set and* $NintNcl(U) \subseteq NclNint(U)$.

Definition 2.8. [6] *Let* f *be a function from a neutrosophic topological space* (X, τ)

to a neutrosophic topological space (Y, σ). Then f is said to be a neutrosophic simply b-continuous function if $f^{-1}(B)$ is a neutrosophic simply b-open set in X for every neutrosophic open set B in Y.

Definition 2.9. [6] *Let f be a function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). Then f is said to be a neutrosophic simply b-open function if $f(B)$ is a neutrosophic simply b-open set in Y, whenever B is a neutrosophic open set in X.*

Definition 2.10. [2] *A neutrosophic topological space (X, τ) is called a neutrosophic T_0-space (NT_0-space, for short) if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, there exists neutrosophic open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \notin U$ or $x_{\alpha,\beta,\gamma} \notin V, y_{\alpha',\beta',\gamma'} \in V$.*

Definition 2.11. [2] *A neutrosophic topological space (X, τ) is called a neutrosophic T_1-space (NT_1-space, for short) if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, there exist neutrosophic open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \notin U$ and $x_{\alpha,\beta,\gamma} \notin V, y_{\alpha',\beta',\gamma'} \in V$.*

Definition 2.12. [2] *A neutrosophic topological space (X, τ) is called a neutrosophic T_2-space or neutrosophic Hausdorff space(NT_2-space or N-Hausdorff space, for short) if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, there exist two neutrosophic open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'}$ in V and $U \cap V = 0_N$.*

3. Main Results

Definition 3.1. *A neutrosophic topological space (X, τ) is called a neutrosophic simply $b-T_0$ space if for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, there exist neutrosophic simply b-open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \notin U$ or $x_{\alpha,\beta,\gamma} \notin V, y_{\alpha',\beta',\gamma'} \in V$.*

Example 3.2. Let $X = \{a, b\}$ and $\tau = \{\phi, X, A, B\}$ where $A = \{< a, 1, 0, 0 >, < b, 0, 1, 1 >\}$ and $B = \{< a, 0, 1, 1 >, < b, 1, 0, 0 >\}$. Clearly the neutrosophic topological space (X, τ) is a neutrosophic simply $b - T_0$ space.

Proposition 3.3. *Let τ and τ' be two neutrosophic topological spaces on a set X such that τ' is simply b-finer than τ. If (X, τ) is a neutrosophic simply $b-T_0$ space then (X, τ') is also a neutrosophic simply $b - T_0$ space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ be two neutrosophic points in X. Since (X, τ) is a neutrosophic simply $b-T_0$ space, there exist τ-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \notin G$ or $x_{\alpha,\beta,\gamma} \notin H, y_{\alpha',\beta',\gamma'} \in H$. Since τ' is simply b-finer than τ, every τ-neutrosophic simply b-open set in X is a τ'-

neutrosophic simply b-open set in X. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $X \neq Y$, there exist τ'-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \notin G$ or $x_{\alpha,\beta,\gamma} \notin H, y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ') is a neutrosophic simply $b - T_0$ space.

We state the following result without proof, which is obvious in view of the Definitions.

Proposition 3.4. *Let (X, τ) be a neutrosophic topological space. If X is neutrosophic T_0 space then X is neutrosophic simply $b - T_0$ space.*

The converse of the above proposition is not true in general.

Example 3.5. Let $X = \{a, b\}$ and $\tau = \{\phi, X\}$. Clearly (X, τ) is not a neutrosophic T_0 space. We show that (X, τ) is a neutrosophic simply $b - T_0$ space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be two neutrosophic points in X,$a \neq b$. Also let $A = < a, 1, 0, 0 >, < b, 0, 1, 1 >$. Obviously A is a neutrosophic set, $a_{\alpha,\beta,\gamma} \in A$ and $b_{\alpha',\beta',\gamma'} \notin A$. Now $Nint(Ncl(A)) \cup Ncl(Nint(A)) \supseteq Nint(Ncl(A)) = int(\overline{X}) = \overline{X} \supseteq A$ and $NintNcl(U) \subseteq NclNint(U)$. Therefore A is a neutrosophic simply b-open set in X. Thus for any two neutrosophic points $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$,$a \neq b$,there exists a neutrosophic simply b-open set A in X such that $a_{\alpha,\beta,\gamma} \in A$ and $b_{\alpha',\beta',\gamma'} \notin A$. Therefore X is a neutrosophic simply $b - T_0$ space.

Proposition 3.6. *Let f be a bijective neutrosophic simply b-open function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). If (X, τ) is neutrosophic T_0 space then (Y, σ) is neutrosophic simply $b - T_0$ space.*

Proof. Let $y_{p,q,r}$ and $y'_{p',q',r'}$ be any two neutrosophic points in Y such that $y \neq y'$. Since f is bijective, there exists two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$, in X such that $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$ and $f(x'_{\alpha',\beta',\gamma'}) = y'_{p',q',r'}$. Since X is neutrosophic T_0 space, there exist τ-open neutrosophic sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, x'_{\alpha',\beta',\gamma'} \notin G$ or $x_{\alpha,\beta,\gamma} \notin H, x'_{\alpha',\beta',\gamma'} \in H$. Suppose G exists such that $x_{\alpha,\beta,\gamma} \in G, x'_{\alpha',\beta',\gamma'} \notin G$. Since f is neutrosophic simply b-open function, $f(G)$ is a σ-neutrosophic simply b-open set such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma}) \in f(G)$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly, $f(H)$ is a σ-neutrosophic simply b-open set such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma}) \notin f(H)$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}$ in Y such that $y \neq y'$, there exist σ-neutrosophic simply b-open sets $f(G), f(H)$ such that $y_{p,q,r} \in f(G)$,$y'_{p',q',r'} \notin f(G)$ or $y_{p,q,r} \notin f(H)$ and $y'_{p',q',r'} \in f(H)$. Therefore (Y, σ) is a neutrosophic simply $b - T_0$ space.

Proposition 3.7. *Let f be a one to one neutrosophic simply b-continuous function from a neutrosophic topological space (X, τ) to a neutrosophic topological space*

(Y, σ). *If* (Y, σ) *is neutrosophic* T_0 *space then* (X, τ) *is a neutrosophic simply* $b-T_0$ *space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$ be any two neutrosophic points in X. Since f is one to one, there exist two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}, y \neq y'$ in Y such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma})$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'})$ i.e.$x_{\alpha,\beta,\gamma} = f^{-1}(y_{p,q,r})$ and $x'_{\alpha',\beta',\gamma'} = f^{-1}(y'_{p',q',r'})$.Since Y is neutrosophic T_0 space, there exist σ-open neutrosophic sets G, H such that $y_{p,q,r} \in G$,$y'_{p',q',r'} \notin G$ or $y_{p,q,r} \notin H$ and $y'_{p',q',r'} \in H$. Since f is neutrosophic simply b-continuous function, $f^{-1}(G)$ is a τ-neutrosophic simply b-open set in X. Also $y_{p,q,r} \in G \Rightarrow f^{-1}(y_{p,q,r}) \in f^{-1}(G) \Rightarrow x_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y'_{p',q',r'} \notin G \Rightarrow f^{-1}(y'_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x'_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-neutrosophic simply b-open set in X such that $x'_{\alpha',\beta',\gamma'} \in f^{-1}(H), x_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$ in X, there exist τ-neutrosophic simply b-open sets $f^{-1}(G), f^{-1}(H)$ in X such that $x_{\alpha,\beta,\gamma} \in f^{-1}(G), x'_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or $x_{\alpha,\beta,\gamma} \notin f^{-1}(H), x'_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X, τ) is a neutrosophic simply $b - T_0$ space.

Definition 3.8. *A neutrosophic topological space* (X, τ) *is called a neutrosophic simply* $b-T_1$ *space if for any two neutrosophic Points* $x_{\alpha,\beta,\gamma}$ *and* $y_{\alpha',\beta',\gamma'}, x \neq y$, *there exist neutrosophic simply b-open sets* U, V *in* X *such that* $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \notin U$ *and* $x_{\alpha,\beta,\gamma} \notin V, y_{\alpha',\beta',\gamma'} \in V$.

Example 3.9. Let $X = \{a, b\}$ and $\tau = \{\phi, X, A, B\}$ where $A =< a, 1, 0, 0 >$, $< b, 0, 1, 1 >$ and $B = < a, 0, 1, 1 >, < b, 1, 0, 0 >$. Clearly the neutrosophic topological space (X, τ) is a neutrosophic simply $b - T_1$ space.

Proposition 3.10. *Let* τ *and* τ' *be two neutrosophic topological spaces on a set* X *such that* τ' *is simply b-finer than* τ. *If* (X, τ) *is a neutrosophic simply* $b - T_1$ *space then* (X, τ') *is also a neutrosophic simply* $b - T_1$ *space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ be two neutrosophic points in X. Since (X, τ) is a neutrosophic simply $b-T_1$ space, there exist τ-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \notin G$ and $x_{\alpha,\beta,\gamma} \notin H, y_{\alpha',\beta',\gamma'} \in H$. Since τ' is simply b-finer than τ, every τ-neutrosophic simply b-open set in X is a τ'-neutrosophic simply b-open set in X. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $X \neq Y$, there exists τ'-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \notin G$ and $x_{\alpha,\beta,\gamma} \notin H, y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ') is a neutrosophic simply $b - T_1$ space.

We state the following result without proof, which is obvious in view of the Definitions.

Proposition 3.11. *Let* (X, τ) *be a neutrosophic topological space. If* X *is neutro-*

sophic T_1 space then X is neutrosophic simply $b-T_1$ space.

The converse of the above proposition is not true in general.

Example 3.12. Let $X = \{a, b\}$ and $\tau = \{\phi, X\}$. Clearly (X, τ) is not a neutrosophic T_1 space. We show that (X, τ) is a neutrosophic simply $b-T_1$ space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be two neutrosophic points in X,$a \neq b$. Also let $A = < a, 1, 0, 0 >, < b, 0, 1, 1 >$ and $B = < a, 0, 1, 1 >, < b, 1, 0, 0 >$. Clearly A and B are two neutrosophic simply b-open sets in X. Thus for any two neutrosophic points $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$,$a \neq b$,there exist neutrosophic simply b-open sets A, B in X such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \notin A$ and $a_{\alpha,\beta,\gamma} \notin B$, $b_{\alpha',\beta',\gamma'} \in B$. Therefore X is a neutrosophic simply $b-T_1$ space.

We state the following result without proof, which is obvious in view of the Definitions.

Proposition 3.13. *Let (X, τ) be a neutrosophic topological space. If (X, τ) is a neutrosophic simply $b-T_1$ space then it is a neutrosophic simply $b-T_0$ space.*

The converse of the above proposition is not true in general.

Example 3.14. Let $X = \{a, b\}$ and $\tau = \{\phi, X, A\}$ where $A =< a, 1, 0, 0 >$, $< b, 0, 1, 1 >$ is neutrosophic simply b-open set. Clearly (X, τ) is a neutrosophic simply $b-T_0$ space but not a neutrosophic simply $b-T_1$ space.

Proposition 3.15. *Let f be a bijective neutrosophic simply b-open function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). If (X, τ) is neutrosophic T_1 space then (Y, σ) is neutrosophic simply $b-T_1$ space.*

Proof. Let $y_{p,q,r}$ and $y'_{p',q',r'}$ be any two neutrosophic points in Y such that $y \neq y'$. Since f is bijective, there exists two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$, in X such that $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$ and $f(x'_{\alpha',\beta',\gamma'}) = y'_{p',q',r'}$. Since X is neutrosophic T_1 space, there exist τ-open neutrosophic set G, H in X such that $x_{\alpha,\beta,\gamma} \in G, x'_{\alpha',\beta',\gamma'} \notin G$ and $x_{\alpha,\beta,\gamma} \notin H, x'_{\alpha',\beta',\gamma'} \in H$. Since f is neutrosophic simply b-open function, $f(G)$ is a σ-neutrosophic simply b-open set such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma}) \in f(G)$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly, $f(H)$ is a σ-neutrosophic simply b-open set such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma}) \notin f(H)$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}$ in Y such that $y \neq y'$, there exist σ-neutrosophic simply b-open sets $f(G), f(H)$ such that $y_{p,q,r} \in f(G)$,$y'_{p',q',r'} \notin f(G)$ and $y_{p,q,r} \notin f(H)$ and $y'_{p',q',r'} \in f(H)$. Therefore (Y, σ) is a neutrosophic simply $b-T_1$ space.

Proposition 3.16. *Let f be a one to one neutrosophic simply b-continuous function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). If (Y, σ) is neutrosophic T_1 space then (X, τ) is a neutrosophic simply $b-T_1$*

space.

Proof. Let $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$ be any two neutrosophic points in X. Since f is one to one, there exist two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}, y \neq y'$ in Y such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma})$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'})$ i.e.$x_{\alpha,\beta,\gamma} = f^{-1}(y_{p,q,r})$ and $x'_{\alpha',\beta',\gamma'} = f^{-1}(y'_{p',q',r'})$.Since Y is neutrosophic T_1 space, there exist σ-open neutrosophic sets G, H such that $y_{p,q,r} \in G, y'_{p',q',r'} \notin G$ and $y_{p,q,r} \notin H$ and $y'_{p',q',r'} \in H$. Since f is neutrosophic simply b-continuous function, $f^{-1}(G)$ and $f^{-1}(H)$ are τ-neutrosophic simply b-open sets in X. Also $y_{p,q,r} \in G \Rightarrow f^{-1}(y_{p,q,r}) \in f^{-1}(G) \Rightarrow x_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y'_{p',q',r'} \notin G \Rightarrow f^{-1}(y'_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x'_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-neutrosophic simply b-open set in X such that $x'_{\alpha',\beta',\gamma'} \in f^{-1}(H), x_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$ in X, there exist τ-neutrosophic simply b-open sets $f^{-1}(G), f^{-1}(H)$ in X such that $x_{\alpha,\beta,\gamma} \in f^{-1}(G), x'_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and $x_{\alpha,\beta,\gamma} \notin f^{-1}(H), x'_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X, τ) is a neutrosophic simply $b - T_1$ space.

Definition 3.17. *A neutrosophic topological space (X, τ) is called a neutrosophic simply $b - T_2$ space or neutrosophic Hausdorff space if for any two neutrosophic Points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, there exist two neutrosophic simply b-open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \phi$.*

Example 3.18. Let $X = \{a, b\}$ and $\tau = \{\phi, X, A, B\}$ where $A =< a, 1, 0, 0 >$, $< b, 0, 1, 1 >$ and $B = < a, 0, 1, 1 >, < b, 1, 0, 0 >$. Clearly the neutrosophic topological space (X, τ) is a neutrosophic simply $b - T_2$ space.

Proposition 3.19. *Let τ and τ' be two neutrosophic topological spaces on a set X such that τ' is simply b-finer than τ. If (X, τ) is a neutrosophic simply $b - T_2$ space then (X, τ') is also a neutrosophic simply $b - T_2$ space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ be two neutrosophic points in X. Since (X, τ) is a neutrosophic simply $b-T_2$ space, there exists two τ-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \phi$. Since τ' is simply b-finer than τ, every τ-neutrosophic simply b-open set in X is a τ'-neutrosophic simply b-open set in X. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $X \neq Y$, there exists τ'-neutrosophic simply b-open sets G, H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \phi$. Hence (X, τ') is a neutrosophic simply $b - T_2$ space.

Proposition 3.20. *Let (X, τ) be a neutrosophic topological space. If X is neutrosophic T_2 space then X is neutrosophic simply $b - T_2$ space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ be two neutrosophic points in X. Since (X, τ) is a neutrosophic T_2 space, there exists τ-open neutrosophic sets G and H in X

such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \phi$. Since every neutrosophic open set is a neutrosophic simply b-open set, for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ there exists τ-neutrosophic simply b-open sets G and H in X such that $x_{\alpha,\beta,\gamma} \in G, y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \phi$. Hence (X, τ) is a neutrosophic simply $b - T_2$ space.

The converse of the above proposition is not true in general.

Example 3.21. Let $X = \{a, b\}$ and $\tau = \{\phi, X\}$. Clearly (X, τ) is not a neutrosophic T_2 space. We show that (X, τ) is a neutrosophic simply $b - T_2$ space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be two neutrosophic points in X,$a \neq b$. Also let $A = < a, 1, 0, 0 >, < b, 0, 1, 1 >$ and $B = < a, 0, 1, 1 >, < b, 1, 0, 0 >$. Clearly A and B are two neutrosophic simply b-open sets in X such that $A \cap B = \phi$. Thus for any two neutrosophic points $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$,$a \neq b$,there exist neutrosophic simply b-open sets A and B in X such that $a_{\alpha,\beta,\gamma} \in A$ and $b_{\alpha',\beta',\gamma'} \in B$. Therefore X is a neutrosophic simply $b - T_2$ space.

Proposition 3.22. *Let (X, τ) be a neutrosophic topological space. If (X, τ) is a neutrosophic simply $b - T_2$ space then it is a neutrosophic simply $b - T_1$ space.*
Proof. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$, be two neutrosophic points in X. Since X is neutrosophic simply $b - T_2$ space, there exists τ-neutrosophic simply b-open sets U and V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \phi$. Since $x_{\alpha,\beta,\gamma} \in U$ and $U \cap V = \phi, x_{\alpha,\beta,\gamma} \notin V$. Similarly $y_{\alpha',\beta',\gamma'} \notin U$. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}, x \neq y$ there exists a τ-neutrosophic simply b-open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U, y_{\alpha',\beta',\gamma'} \notin U$ and $x_{\alpha,\beta,\gamma} \notin V, y_{\alpha',\beta',\gamma'} \in V$. Hence (X, τ) is a neutrosophic simply $b - T_1$ space.

Proposition 3.23. *Let f be a bijective neutrosophic simply b-open function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). If (X, τ) is neutrosophic T_2 space then (Y, σ) is neutrosophic simply $b - T_2$ space.*
Proof. Let $y_{p,q,r}$ and $y'_{p',q',r'}$ be any two neutrosophic points in Y such that $y \neq y'$. Since f is bijective, there exists two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$, in X such that $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$ and $f(x'_{\alpha',\beta',\gamma'}) = y'_{p',q',r'}$. Since X is neutrosophic T_2 space, there exists a τ-open neutrosophic sets G and H in X such that $x_{\alpha,\beta,\gamma} \in G, x'_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \phi$. Since f is neutrosophic simply b-open function, $f(G)$ is a σ-neutrosophic simply b-open set such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma}) \in f(G)$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'}) \in f(H)$. Again since f is bijective, $f(G) \cap f(H) = f(G \cap H) = f(\phi) = \phi$. Thus for any two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}$ in Y such that $y \neq y'$, there exists σ-neutrosophic simply b-open sets $f(G)$ and $f(H)$ such that $y_{p,q,r} \in f(G), y'_{p',q',r'} \in f(H)$ and $f(G) \cap f(H) = \phi$. Therefore (Y, σ) is a neutrosophic simply $b - T_2$ space.

Proposition 3.24. *Let f be a one to one neutrosophic simply b-continuous function from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). If (Y, σ) is neutrosophic T_2 space then (X, τ) is a neutrosophic simply $b-T_2$ space.*

Proof. Let $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}, x \neq x'$ be any two neutrosophic points in X. Since f is one to one, there exist two neutrosophic points $y_{p,q,r}$ and $y'_{p',q',r'}, y \neq y'$ in Y such that $y_{p,q,r} = f(x_{\alpha,\beta,\gamma})$ and $y'_{p',q',r'} = f(x'_{\alpha',\beta',\gamma'})$ i.e.$x_{\alpha,\beta,\gamma} = f^{-1}(y_{p,q,r})$ and $x'_{\alpha',\beta',\gamma'} = f^{-1}(y'_{p',q',r'})$.Since Y is neutrosophic T_2 space, there exists σ-open neutrosophic sets G and H such that $y_{p,q,r} \in G, y'_{p',q',r'} \in H$ and $G \cap H = \phi$. Since f is neutrosophic simply b-continuous function, $f^{-1}(G)$ and $f^{-1}(H)$ are τ-neutrosophic simply b-open sets in X. Now $y_{p,q,r} \in G \Rightarrow f^{-1}(y_{p,q,r}) \in f^{-1}(G) \Rightarrow x_{\alpha,\beta,\gamma} \in f^{-1}(G)$. Similarly, $y'_{p',q',r'} \in H$. Thus for any two neutrosophic points $x_{\alpha,\beta,\gamma}$ and $x'_{\alpha',\beta',\gamma'}$ in X such that $x \neq x'$, there exists τ-neutrosophic simply b-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_{\alpha,\beta,\gamma} \in f^{-1}(G), x'_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore (X, τ) is a neutrosophic simply $b - T_2$ space.

4. Conclusion

In this paper, we defined and studied neutrosophic simply b-T_0, neutrosophic simply b-T_1, and neutrosophic simply b-T_2 spaces in neutrosophic topological spaces. We discussed several properties and relationships between these newly introduced separation axioms and the existing neutrosophic separation axioms. Further, we investigated some properties of neutrosophic simply b-open functions and neutrosophic simply b-continuous functions. Thus, this work extends the study of neutrosophic topology and provides a basis for further research in generalized neutrosophic topological structures.

Acknowledgement

The authors are highly thankful to the editor and referees for the valuable comments and suggestions for improving the quality of the paper.

References

[1] Atanassov, K., Intuitionstic fuzzy sets, Fuzzy sets and systems, 20 (1986), 87-96.

[2] Dey, S. and Ray, G. C., Separation Axioms in Neutrosophic Topological Spaces, Neutrosophic Systems with Applications(2023), 2, 38-54. `https://doi.org/10.5281/zenodo.8195851`

[3] Evanzalin, E. P., Jude, I. H. and Bazil, W. C., On Neutrosophic b-open sets in Neutrosophic topological space, International Conference on Applied and Computational Mathematics, J. Phys.: Conf. Ser., 1139 (2018), 012062.

[4] Salama, A. A. and Alblowi, S. A., Neutrosophic set and neutrosophic topological space, ISOR J. mathematics, 3(4) (2012), 31-35.

[5] Smarandache, F., Neutrosophic set- a generalization of the intuitionstic fuzzy set, International Journal of pure and applied mathematics, 24(3) (2005), 287-294.

[6] Suman, D. and Binod, C.T., Neutrosophic Simply *b*-Open Set in Neutrosophic Topological Spaces, Iraqi Journal of Science, Vol. 62, No. 12 (2021), 4830-4838

[7] Zadeh, L.A., Fuzzy sets, Information and Control, 8(3) (1965), 338-353.

This page intentionally left blank.