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On Bailey's Transform and Expansion of Basic Hypergeometric Functions-II

S.N. Singh, Satya Prakash Singh and *Vijay Yadav
Department of Mathematics,
T.D.P.G. College, Jaunpur-222002 India
E-mail:- snsp39@yahoo.com; snsp39@gmail.com
Department of Mathematics and Statistics,
SPDT College, Andheri (East), Mumbai-400059, Maharastra India

Abstract: In a recent communication we dealt with a new technique to establish expansions of basic hypergeometric functions with the help of Bailey's transform and certain known transformations of truncated hypergeometric series. These results do not look possible directly with the help of the traditional method. This is the continuation of the above study. Certain interesting special cases have also been deduced.

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1. Introduction, Notations and Definitions

For $|q| < 1$ and α, real or complex, let

$$[\alpha;q]_n \equiv [\alpha]_n = \begin{cases} (1-\alpha)(1-\alpha q)(1-\alpha q^2)...(1-\alpha q^{n-1}); & n>0 \\ 1 & ;\quad n=0 \end{cases} \tag{1.1}$$

Accordingly,

$$[\alpha;q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n)$$

Also,

$$[a_1,a_2,a_3,...,a_r;q]_n = [a_1;q]_n[a_2;q]_n[a_3;q]_n...[a_r;q]_n. \tag{1.2}$$

Now, we define a basic hypergeometric function

$${}_r\Phi_s\left[\begin{array}{l} a_1,a_2,...,a_r;q;z \\ b_1,b_2,...,b_s;q^\lambda \end{array}\right] = \sum_{n=0}^{\infty}\frac{[a_1,a_2,...,a_r;q]_n z^n q^{\lambda n(n-1)/2}}{[q,b_1,b_2,...,b_s;q]_n} \tag{1.3}$$

convergent for $|z| < \infty$ when $\lambda > 0$ and for $|z| < 1$ when $\lambda = 0$.
A generalized double basic hypergeometric function is defined as,

$$\Phi\left[\begin{array}{l} a_1, a_2, ..., a_r : \alpha_1, \alpha_2, ..., \alpha_{u_1}; \beta_1, \beta_2, ..., \beta_{v_1}; q; x, y \\ b_1, b_2, ..., b_s : \delta_1, \delta_2, ..., \delta_{u_2}; \gamma_1, \gamma_2, ..., \gamma_{v_2} \end{array}\right]$$

$$= \sum_{m,n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_{m+n}[\alpha_1, \alpha_2, ..., \alpha_{u_1}; q]_m}{[b_1, b_2, ..., b_s; q]_{m+n}[\delta_1, \delta_2, ..., \delta_{u_2}; q]_m} \times$$

$$\times \frac{[\beta_1, \beta_2, ..., \beta_{v_1}; q]_n x^m y^n}{[\gamma_1, \gamma_2, ..., \gamma_{v_2}; q]_n [q; q]_m [q; q]_n} \tag{1.4}$$

which converges for $|x|, |y| < 1$.
A bi-basic hypergeometric series of one variable is defined as,

$$\Phi\left[\begin{array}{l} a_1, a_2, ..., a_r; \alpha_1, \alpha_2, ..., \alpha_u; q, p; z \\ b_1, b_2, ..., b_s; \beta_1, \beta_2, ..., \beta_v \end{array}\right]$$

$$= \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_n [\alpha_1, \alpha_2, ..., \alpha_u; p]_n z^n}{[q, b_1, b_2, ..., b_s; q]_n [\beta_1, \beta_2, ..., \beta_v; p]_n} \tag{1.5}$$

which converges for max. $\{|q|, |p|, |z|\} < 1$.

We shall have the occasion to use the following well known Bailey's transform
If

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \tag{1.6}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n} = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{r+2n} \tag{1.7}$$

Subject to convergence of the infinite series defining γ_n, then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.8}$$

subject to the convergence of the series on both side of (1.8) and that the parameters α_r, δ_r, u_r and v_r are rational functions of r alone.

In a recent communication Singh and Singh [3] established a number of interesting expansions of basic hypergeometric functions in terms of similar functions with the help of the truncated series and Bailey's transform. Some very interesting particular cases were also deduced. In this paper we make use of certain transformations of truncated series and Bailey's transform to establish certain more general expansions of basic hypergeometric in terms of similar functions.

We shall use the following known results due to Denis [2],

$$ {}_3\Phi_2\left[\begin{array}{l} q^{-n}, a, c; q; bdq^n/ac \\ b, d \end{array}\right] = \frac{[b/a;q]_n}{[b;q]_n}\, {}_3\Phi_2\left[\begin{array}{l} a, d/c, q^{-n}; q; q \\ d, aq^{1-n}/b \end{array}\right] \tag{1.9} $$

$$ {}_3\Phi_2\left[\begin{array}{l} q^{-n}, a, d/c; q; q \\ aq^{1-n}/b, d \end{array}\right] = \frac{[b, bd/ac;q]_n[a, bd/a;q]_\infty}{[b/a, bd/a;q]_n[d,b;q]_\infty}\, {}_3\Phi_2\left[\begin{array}{l} b/a, d/a, bdq^n/ac; q; a \\ bd/ac, bdq^n/a \end{array}\right] \tag{1.10} $$

$$ {}_3\Phi_2\left[\begin{array}{l} q^{-n}, a, d/c; q; q \\ aq^{1-n}/b, d \end{array}\right] = \frac{[b, bd/ac;q]_n}{[b/a, bd/a;q]_n}\, {}_3\Phi_2\left[\begin{array}{l} b/a, b/c, q^{-n}; q; dq^n \\ b, bd/ac \end{array}\right] \tag{1.11} $$

and

$$ {}_3\Phi_2\left[\begin{array}{l} q^{-n}, a, d/c; q; q \\ aq^{1-n}/b, d \end{array}\right] = \frac{[bd/a, d/c;q]_\infty[b, bd/ac;q]_n}{[d, bd/ac;q]_\infty[b/a, bd/a;q]_n}\, {}_3\Phi_2\left[\begin{array}{l} c, b/a, bq^n; q; d/c \\ b, bdq^n/a \end{array}\right] \tag{1.12} $$

Now, letting $d \to q^{-n}$ in (1.9) yields

$$ {}_2\Phi_1\left[\begin{array}{l} a, c; q, b/ac \\ b \end{array}\right]_n = \frac{[b/a;q]_n}{[b;q]_n}\, {}_2\Phi_1\left[\begin{array}{l} a, q^{-n}; q; q \\ aq^{1-n}/b \end{array}\right] \tag{1.13} $$

Again, when $b \to q^{-n}$ in (1.9), we get

$$ {}_2\Phi_1\left[\begin{array}{l} a, c; q, d/ac \\ d \end{array}\right]_n = \frac{[aq;q]_n}{a^n[q;q]_n}\, {}_3\Phi_2\left[\begin{array}{l} a, d/c, q^{-n}; q; q \\ d, aq \end{array}\right] \tag{1.14} $$

Further, setting $b = aq$ in (1.10), we get

$$ {}_2\Phi_1\left[\begin{array}{l} a, d/c; q; q \\ d \end{array}\right]_n = \frac{[aq, dq/c;q]_n[a, dq;q]_\infty}{[q, dq;q]_n[d, aq;q]_\infty}\, {}_3\Phi_2\left[\begin{array}{l} q, d/a, dq^{1+n}/c; q; a \\ dq/c, dq^{n+1} \end{array}\right] \tag{1.15} $$

Also, for $b = aq$ (1.11) yields

$$ {}_2\Phi_1\left[\begin{matrix} a, d/c; q; q \\ d \end{matrix}\right]_n = \frac{[aq, dq/c; q]_n}{[q, d; q]_n}\ {}_3\Phi_2\left[\begin{matrix} q, aq/c, q^{-n}; q; dq^n \\ aq, dq/c \end{matrix}\right] \tag{1.16} $$

Next, taking $b = aq$ in (1.12), we get

$$ {}_2\Phi_1\left[\begin{matrix} a, d/c; q; q \\ d \end{matrix}\right]_n = \frac{[d/c, dq; q]_\infty [aq, dq/c; q]_n}{[d, dq/c; q]_\infty [q, dq; q]_n}\ {}_3\Phi_2\left[\begin{matrix} q, c, aq^{n+1}; q; d/c \\ aq, dq^{n+1} \end{matrix}\right] \tag{1.17} $$

2. Lemmas

In this section we shall establish the following lemmas which will be used in establishing our main results,

Lemma 1

If

$$ \beta_n = \sum_{r=0}^{n} \alpha_r \tag{2.1} $$

then

$$ \sum_{n=0}^{\infty} \alpha_n z^n = (1 - z) \sum_{n=0}^{\infty} \beta_n z^n \tag{2.2} $$

and

Lemma 2

If (2.1) is true

then

$$ \sum_{n=0}^{\infty} \alpha_n \left\{ \frac{[\alpha, \beta; p]_n}{[\gamma/p, \alpha\beta p/\gamma; p]_n} - \frac{[\alpha, \beta; p]_\infty}{[\gamma/p, \alpha\beta p/\gamma; p]_\infty} \right\} = \frac{(1 - \alpha p/\gamma)(1 - \beta p/\gamma)}{(1 - p/\gamma)(1 - \alpha\beta p/\gamma)} \sum_{n=0}^{\infty} \beta_n \frac{[\alpha, \beta; p]_n p^n}{[\gamma, \alpha\beta p^2/\gamma; p]_n} \tag{2.3} $$

Proof

Setting $u_r = v_r = 1$ and $\delta_r = z^r$ in Bailey's transform (1.6)-(1.8), we get Lemma 1.

To prove lemma 2, set $u_r = v_r = 1$ and

$$ \delta_r = \frac{[\alpha, \beta; p]_r p^r}{[\gamma, \alpha\beta p^2; p]_r} $$

in Bailey's transform (1.6)-(1.8). This leads to

$$\beta_n = \sum_{n=0}^{n} \alpha_r \qquad \gamma_n = \sum_{n=0}^{\infty} \frac{[\alpha, \beta; p]_{r+n} p^{r+n}}{[\gamma, \alpha\beta p/\gamma; p]_{r+n}}$$

Now, simplifying the value of γ_n with the help of the following known result due to Agarwal [1, p. 79],

$$ {}_3\Phi_2 \left[\begin{array}{l} \alpha, \beta, p; p; p \\ \gamma, \alpha\beta p^2/\gamma \end{array} \right]_N = \frac{(p-\gamma)(\gamma-\alpha\beta p)}{(\alpha p-\gamma)(\gamma-\beta p)} \left\{ 1 - \frac{[\alpha, \beta; p]_{N+1}}{[\gamma/p, \alpha\beta p/\gamma; p]_{N+1}} \right\},$$

which, for $N \to \infty$ yields

$$ {}_3\Phi_2 \left[\begin{array}{l} \alpha, \beta, p; p; p \\ \gamma, \alpha\beta p^2/\gamma \end{array} \right] = \frac{(p-\gamma)(\gamma-\alpha\beta p)}{(\alpha p-\gamma)(\gamma-\beta p)} \left\{ 1 - \frac{[\alpha, \beta; p]_{\infty}}{[\gamma/p, \alpha\beta p/\gamma; p]_{\infty}} \right\},$$

Now, application of the above result in the simplification of the value for γ_n leads to the proof of lemma 2.

3. Main Results

In this section we shall discuss certain interesting expansions of basic hypergeometric functions

(i) If we set

$$\alpha_r = \frac{[a, c; q]_r (b/ac)^r}{[q, b; q]_r},$$

we get

$$\beta_n = \frac{[b/a; q]_n}{[b; q]_n} \, {}_2\Phi_1 \left[\begin{array}{l} a, d/c; q; q \\ aq^{1-n}/b \end{array} \right]_n$$

with the above values in Lemma 1, we get,

$$\sum_{n=0}^{\infty} \frac{[b/a; q]_n z^n}{[b; q]_n} \, {}_2\Phi_1 \left[\begin{array}{l} a, d/c; q; q \\ aq^{1-n}/b \end{array} \right]_n = (1-z)^{-1} \, {}_2\Phi_1 \left[\begin{array}{l} a, c; q; bz/ac \\ b \end{array} \right] \qquad (3.1)$$

Comparison of the coefficients of z^n on both sides of the above result will lead to interesting relation between truncated series and a terminating one.

If we make use of the above values of α_n and β_n in Lemma 2, we get the following interesting expansions,

$$\frac{(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)} \sum_{n=0}^{\infty} \frac{[cq, b/a; q]_n [\alpha, \beta; p]_n p^n}{[b, cq; q]_n [\gamma, \alpha\beta p^2/\gamma; p]_n} \times$$

$$
{}_4\Phi_3\left[\begin{array}{l} cq^{n+1}, aq : \alpha p^n, \beta p^n q \quad q, p; bp/ac \\ -- \, bq^n : \gamma p^n, \alpha\beta p^{n+2}/\gamma \end{array}\right]
$$

$$
= {}_4\Phi_3\left[\begin{array}{l} a, c; \alpha, \beta; q, p; b/ac \\ b; \gamma/p, \alpha\beta p/\gamma \end{array}\right] - \frac{[b/a, b/c; q]_\infty [\alpha, \beta; p]_\infty}{[b, b/ac; q]_\infty [\gamma/p, \alpha\beta p/\gamma; p]_\infty} \qquad (3.2)
$$

This is a transformation of a bi-basic series in terms of a double bi-basic series, both on unconnected base p and q.

(ii) Next, putting

$$
\alpha_r = \frac{[a, c; q]_r (d/ac)^r}{[q, d; q]_r}
$$

in (2.1) and using (1.14) we get

$$
\beta_n = \frac{[aq; q]_n}{a^n [q; q]_n}\, {}_3\Phi_2\left[\begin{array}{l} a, d/c, q^{-n}; q; q \\ aq, d \end{array}\right] \qquad (3.3)
$$

Now, (2.2) with the above values of α_r and β_r yields

$$
(1-z)\sum_{n=0}^{\infty} \frac{[aq; q]_n z^n}{a^n [q; q]_n}\, {}_3\Phi_2\left[\begin{array}{l} a, d/c, q^{-n}; q; q \\ aq, d \end{array}\right] = {}_2\Phi_1\left[\begin{array}{l} a, c; q; zd/ac \\ d \end{array}\right] \qquad (3.4)
$$

Next, with $a \to \infty$ in (3.4), we get, after some simplifications,

$$
{}_1\Phi_1\left[\begin{array}{l} c; q; -zd/c \\ d; q \end{array}\right] = [z; q]_\infty\, {}_1\Phi_1\left[\begin{array}{l} d/c; q; z \\ d \end{array}\right] \qquad (3.5)
$$

Again, if we substitute the above value of α_n and β_n from (3.3) in (2.3), we get after some simplification

$$
\frac{(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)} \sum_{n=0}^{\infty} \frac{[aq; q]_n [\alpha, \beta; p]_n (p/a)^n}{[q; q]_n [\gamma, \alpha\beta p^2/\gamma; p]_n}\, {}_3\Phi_2\left[\begin{array}{l} a, d/c, q^{-n}; q; q \\ aq, d \end{array}\right]
$$

$$
= {}_4\Phi_3\left[\begin{array}{l} a, c; \alpha, \beta; q, p; d/ac \\ d; \gamma/p, \alpha\beta p/\gamma \end{array}\right] - \frac{[\alpha, \beta; p]_\infty [d/a, d/c; q]_\infty}{[\gamma/p, \alpha\beta p/\gamma; p]_\infty [d, d/ac; q]_\infty} \qquad (3.6)
$$

This is an expansion of a bi-basic series in terms of another bi-basic series.

(iii) Further, setting

$$
\alpha_r = \frac{[a, d/c; q]_r q^r}{[q, d; q]_r}
$$

in (2.1) and making use of (1.15), we get

$$
\beta_n = \frac{[aq, dq/c; q]_n [a, dq; q]_\infty}{[q, dq; q]_n [aq, d; q]_\infty}\, {}_3\Phi_2\left[\begin{array}{l} q, d/a, dq^{n+1}/c; q; a \\ dq/c, dq^{n+1} \end{array}\right] \qquad (3.7)
$$

Now, substituting the above values of α_n and β_n in (2.2), we get the following interesting transformation

$$\frac{(1-a)(1-z)}{(1-d)}\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n}{[q,dq;q]_n}\ {}_5\Phi_4\left[\begin{matrix} dq^{n+1}/c, dq^{n+2}/c : q, d/a, aq^{n+1}; q^2, q; az \\ dq^{n+1}, deq^{n+2} : q^{n+1}, dq/c \end{matrix}\right]$$

$$= {}_2\Phi_1\left[\begin{matrix} a, d/c; zq \\ d \end{matrix}\right] \tag{3.8}$$

Also, with the substitution of the above values α_n and β_n in Lemma 2 we get,

$$\frac{(1-a)(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-d)(1-p/\gamma)(1-\alpha\beta p/\gamma)}\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n[\alpha,\beta;p]_n p^n}{[q,dq;q]_n[\gamma,\alpha\beta p^2/\gamma;p]_n}\times$$

$${}_7\Phi_6\left[\begin{matrix} dq^{n+1}/c, dq^{n+2}/c : q, d/a, aq^{n+1} : \alpha p^n, \beta p^n; q^2, q, p; ap \\ dq^{n+1}/c, dq^{n+2} : dq/c, q^{1+n}, dq/c : \gamma p^n, \alpha\beta p^{n+2}/\gamma \end{matrix}\right]$$

$$= {}_4\Phi_3\left[\begin{matrix} a, d/c; \alpha, \beta; q, p; q \\ d; \gamma/p, \alpha\beta p/\gamma \end{matrix}\right] - \frac{[\alpha,\beta;p]_\infty[dq/c,a;q]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty[q,d;q]_\infty}\ {}_2\Phi_1\left[\begin{matrix} q, d/a; q; q \\ dq/c \end{matrix}\right] \tag{3.9}$$

Putting a=d in (3.9), we get

$${}_3\Phi_2\left[\begin{matrix} d/c : \alpha, \beta; q, p; q \\ --- : \gamma/p, \alpha\beta p/\gamma \end{matrix}\right] - \frac{[\alpha,\beta;p]_\infty[dq/c;q]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty[q;q]_\infty}$$

$$= \frac{(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)}\ {}_3\Phi_2\left[\begin{matrix} dq/c : \alpha, \beta; q, p; p \\ d/q : \gamma, \alpha\beta p^2/\gamma \end{matrix}\right] \tag{3.10}$$

Now, if we set d=c in (3.10), we get the following known summation due to Agarwal [1; page 79],

$${}_3\Phi_2\left[\begin{matrix} \alpha, \beta, p; p; p \\ \gamma, \alpha\beta p^2/\gamma \end{matrix}\right] = \frac{(1-p/\gamma)(1-\alpha\beta p/\gamma)}{(1-\alpha p/\gamma)(1-\beta p\gamma)}\left\{1 - \frac{[\alpha,\beta;p]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty}\right\} \tag{3.11}$$

(iv) Again, setting

$$\alpha_r = \frac{[a,d/c;q]_r q^r}{[q,d;q]_r}$$

in (2.1) and applying (1.16), we get

$$\beta_n = \frac{[aq,dq/c;q]_n}{[q,d;q]_n}\ {}_3\Phi_2\left[\begin{matrix} q, aq/c, q^{-n}; q; dq^n \\ aq, dq/c \end{matrix}\right] \tag{3.12}$$

The above values of α_n and β_n in Lemma 1 lead to,

$$(1-z)\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n z^n}{[q,d;q]_n}\ {}_3\Phi_2\left[\begin{matrix} q,aq/c,q^{-n};q;dq^n \\ aq,dq/c \end{matrix}\right] = {}_2\Phi_1\left[\begin{matrix} a,d/c;q;zq \\ d \end{matrix}\right] \tag{3.13}$$

which with c=q reduces to

$${}_2\Phi_1\left[\begin{matrix} a,d/q;q;zq \\ d \end{matrix}\right] = \frac{[azq;q]_\infty}{[zq;q]_\infty}\ {}_2\Phi_2\left[\begin{matrix} q,a;q;-zd \\ d,azq;q \end{matrix}\right] \tag{3.14}$$

If we substitute the above values of α_n and β_n in Lemma 2, we get after some simplifiaction

$$\frac{(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)}\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n[\alpha,\beta;p]_n p^n}{[q,d;q]_n[\gamma,\alpha\beta p^2/\gamma;p]_n}\ {}_3\Phi_2\left[\begin{matrix} q,aq/c,q^{-n};q;dq^n \\ aq,dq/c \end{matrix}\right]$$

$$= {}_4\Phi_3\left[\begin{matrix} a,d/c;\alpha,\beta;q,p;q \\ d;\gamma/p,\alpha\beta p/\gamma \end{matrix}\right] - \frac{[\alpha,\beta;p]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty}\ {}_3\Phi_2\left[\begin{matrix} a,d/c;q;q \\ d \end{matrix}\right] \tag{3.15}$$

(v) Further, setting

$$\alpha_r = \frac{[a,d/c;q]_r q^r}{[q,d;q]_r}$$

in (2.1) and using (1.17), we get

$$\beta_n = \frac{(1-d/c)[aq,dq/c;q]_n}{(1-d)[q,dq;q]_n}\ {}_3\Phi_2\left[\begin{matrix} q,c,aq^{n+1};q;d/c \\ aq,dq^{n+1} \end{matrix}\right]. \tag{3.16}$$

with the above values of α_n and β_n, Lemma 1 yields

$${}_2\Phi_1\left[\begin{matrix} a,d/c;q;zq \\ d \end{matrix}\right] = \frac{(1-d/c)(1-z)}{(1-d)}\times$$

$$\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n z^n}{[q,dq;q]_n}\ {}_3\Phi_2\left[\begin{matrix} q,c,aq^{n+1};q;d/c \\ aq,dq^{n+1} \end{matrix}\right] \tag{3.17}$$

Now, with the above values of α_n and β_n Lemma 2 leads to

$$\frac{(1-d/c)(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-d)(1-p/\gamma)(1-\alpha\beta p/\gamma)}$$

$$\sum_{n=0}^{\infty}\frac{[aq,dq/c;q]_n[\alpha,\beta;p]_n p^n}{[q,dq;q]_n[\gamma,\alpha\beta p^2/\gamma;p]_n}\ {}_3\Phi_2\left[\begin{matrix} q,c,aq^{n+1};q;d/c \\ aq,dq^{n+1} \end{matrix}\right]$$

$$= {}_4\Phi_3 \left[\begin{array}{l} a, d/c : \alpha, \beta; q, p; q \\ d : \gamma, \alpha\beta p/\gamma \end{array} \right]$$

$$-\frac{[\alpha, \beta; p]_\infty}{[\gamma/p, \alpha\beta p/\gamma; p]_\infty} \, {}_2\Phi_1 \left[\begin{array}{l} a, d/c; q; q \\ d \end{array} \right] \tag{3.18}$$

Scores of similar results can be established using the method followed in this paper which otherwise do not look possible with the help of traditional methods.

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