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A NEW GRAPH TOPOLOGY ON DECOMPOSITION OF GRAPH

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Abstract: In this paper, we explore the extension of topological concepts to graph theory by defining a graph topology as a collection of sub-graphs within a graph G that satisfy properties analogous to the axioms of point-set topology. Specifically, we focus on the edge-induced sub-graph topology, where open sets are sub-graphs formed by subsets of the edge set E of G. Building upon this framework, we introduce the concept of an N -graph topological space, generated by these edge-induced sub-graphs. This novel approach facilitates a deeper exploration of the interplay between graph-theoretical structures and topological spaces, potentially leading to new insights and applications in both fields.

Keywords and Phrases: Graph Topology, Edge-Induced sub graph Topology, N -Graph Topological Space, N-Graph Topology, N-open sub graph, N-closed sub graph, N-graph Interior, N-graph closure.

2020 Mathematics Subject Classification: 05C62, 05C75, 54A05.

1. Introduction

The primary objective of this study is to visually demonstrate key concepts from topology. By observing graphical representations of topological spaces and properties on a two-dimensional surface, several topological results become more intuitive and engaging. Initial discussions of these concepts can be traced back to work in [[1], [2], [3], [4], [6], [9], [12], [11]]. In addition, the notion of a topology in a graph, defined through its subgraphs, was introduced and analyzed in [10].

Pawlak's work in [13] laid the groundwork for rough set theory by exploring the concepts of approximations and boundary regions of a set. Building upon these foundational studies, this paper introduces a new topology called N-Graph topology. This topology is based on an approximation operator for sub graphs within an undirected, non-empty graph G. The study also investigates the properties of N-closed graphs and the N -closure of a sub graph within the context of an N-Graph topological space. For a detailed discussion on the definitions and terminology used in graph theory, refer to [[5], [7], [8], [14]].

2. Upper and Lower approximation of Subgraph

In this section, we explore the concepts of lower and upper approximations for a sub graph H within a non-empty simple graph G. These approximations are important in understanding the relationships between a sub graph and the larger graph, especially when we consider the graph as a topological space. The approximations are based on the collection $\mathfrak{R}$(G), which consists of distinct edge-induced sub graphs of G. These sub graphs are derived from the edge sets of G, and we study how their union and intersection can help describe the structure of the sub graph H. In graph theory and topology, these operators are useful tools for developing topological spaces over graphs, where the approximation operators describe the relationships between various sub graphs and their surroundings. The N-Graph topology introduced in this paper benefits from these approximations by providing a framework to analyze and visualize topological properties of graphs.

Upper approximation N^*(H)

The upper approximation of a sub graph H, denoted as N^*(H), captures the sub graphs that "overlap" with H in some way, but may not be fully contained within it. Formally, we define the upper approximation as: $N^*(\mathrm{H}) = \{\cup H_i : H_i \in \mathfrak{R}(\mathrm{G})$and $H_i \cap \mathrm{H} \neq \phi\}$.

This definition suggests that N^*(H)includes all the edge-induced sub graphs H_i from $\mathfrak{R}$(G) that share at least one edge with H. Intuitively, this represents the idea of a "loose" or "larger" approximation of H, where the graph can extend beyond the boundaries of H, but still retains some overlap.

Lower approximation N_*(H)

The lower approximation of a sub graph H, denoted as N_*(H), is more restrictive. It includes only those edge-induced sub-graphs that are completely contained within H. Formally, the lower approximation is defined as:$N_*(\mathrm{H})=\{\cup H_i : H_i \in \mathfrak{R}(\mathrm{G})$ and $H_i \subseteq \mathrm{H}\}$.

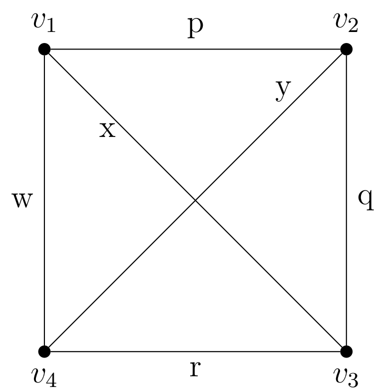
Thus, N_*(H) consists of all the sub graphs in $\mathfrak{R}$(G) that are subsets of H, meaning that they fit entirely within the boundaries of H. This approximation can be thought of as the "tightest" or "smallest" approximation of H, where the graph cannot extend beyond H.

Boundary Region B_N(H)

The boundary region of H in G is defined as the difference between the upper and lower approximations: B_N(H)= N^*(H)$-N_*$(H). The boundary region captures the elements that are in the upper approximation but not in the lower approximation. In other words, it consists of sub graphs from $\mathfrak{R}$(G) that "partially" overlap with H but are not fully contained within it. This can be seen as the "border" or "boundary" between H and the rest of the graph, highlighting areas where H is not completely represented.

Example 2.1. let's consider the non-empty graph G, with edge set E(G) = $\{p, q, r, w, x, y\}$, and the collection of distinct edge-induced sub graphs $\mathfrak{R}$(G) of G. We define the set of sub graphs $\mathfrak{R}$(G) as:

- H_1 with edges $E(H_1) = \{p\}$.
- H_2 with edges $E(H_2) = \{q\}$.
- H_3 with edges $E(H_3) = \{r, w\}$.
- H_4 with edges $E(H_4) = \{x, y\}$.



Consider a sub graph H of G with edge set E(G) = $\{p, q, x\}$. This is a relatively small subgraph of G consisting of three edges.

The lower approximation N_*(H) captures the sub graphs that are contained entirely within H. These are the sub graphs that can be viewed as precise representations of H without extending beyond it. In this case, the lower approximation of H consists of sub graphs whose edge sets are subsets of $E(H)$. From the collection $\mathfrak{R}$(G), the

only sub graph that is fully contained within H is H_1, which corresponds to the edge p, and H_2, which corresponds to the edge q. Therefore, the lower approximation of H is the sub graph N_*(H) with edge set: $E(N_*(\mathrm{H})) = \{p, q\}$. Thus, N_*(H) is a spanning sub graph of G that consists of just the edges p and q, without the edge x.
The upper approximation N^*(H) represents a more "extended" version of H, capturing all sub graphs that intersect with H. These sub graphs may include edges that are not fully contained within H, but that share some part of it. From the collection $\mathfrak{R}$(G), the sub graphs that intersect H are:

- H_1 with edge P, which intersects H.
- H_2 with edge q, which intersects H.
- H_4 with edges x and y, which intersects H through the edge x.

Therefore, the upper approximation of H includes all the edges that intersect with H, so the edge set of N^*(H) is: $E(N^*(\mathrm{H})) = \{p, q, x, y\}$. Thus, N^*(H) is a super graph of H, extending beyond the original edges of H to include the edge y from H_4.
The boundary region B_N(H) is defined as the difference between the upper and lower approximations, and it represents the edges that lie in th=e upper approximation but not in the lower approximation. Intuitively, it highlights the "boundary" or the parts of the graph that are adjacent to but not entirely contained within H. The boundary region of H is calculated as: $E(B_N(\mathrm{H})) = \{x, y\}$. Thus, the boundary region consists of the edges x and y, which are part of the graph but not fully contained within the sub graph H. These edges form the boundary between the sub graph H and the rest of the graph G. This example demonstrates how the approximation operators can be used to understand the structure of a sub graph in the context of the entire graph.

Theorem 2.2. *Let* G *be a non-empty simple graph, and let* $\mathfrak{R}$(G) *denote the collection of distinct edge-induced sub graphs of* G*. If* H *is a sub graph of* G*, the following properties hold for the lower and upper approximations* N_*(H) *and* N^*(H)*of* H*:*

1. $N_*(\mathrm{H}) \subseteq \mathrm{H} \subseteq N^*(\mathrm{H})$.
2. $N_*(\mathrm{G}) = \mathrm{G} = N^*(\mathrm{G})$.
3. $N^*(N_*(\mathrm{H})) \subseteq \mathrm{H} \subseteq N_*(N^*(\mathrm{H}))$.
4. If $\mathrm{H} = \emptyset$, then $N_*(\emptyset) = \emptyset = N^*(\emptyset)$

Proof. The proof of these properties follows directly from the definitions of the lower and upper approximation operators.

1. Inclusion property: By definition,N_*(H) consists of the union of sub graphs that are entirely contained within H, and $N^($H) consists of the union of sub graphs that intersect H. Therefore, it is clear that $N_*(\mathrm{H}) \subseteq \mathrm{H} \subseteq N^*(\mathrm{H})$.

2. Equality for the Whole Graph: The graph G is both a subset of itself and intersects with itself, so applying the lower and upper approximation operators to G yields $N_*(\mathrm{G}) = \mathrm{G} = N^*(\mathrm{G})$

3. Compositions of Approximations The inclusion:$N^*(N_*(\mathrm{H})) \subseteq \mathrm{H} \subseteq N_*(N^*(\mathrm{H}))$ follows from the properties of the approximation operators.The lower approximation N_*(H) limits the sub graphs to those contained within H, and applying the upper approximation to N_*(H) does not extend beyond H. Similarly, N^*(H) is the union of sub graphs that intersect H, and applying the lower approximation to N^*(H) will only yield sub graphs that are subsets of H.

4. Empty Graph Case: For the empty graph $\mathrm{H} = \emptyset$, both the lower and upper approximations of $\emptyset$ are also empty, so $N_*(\emptyset) = \emptyset = N^*(\emptyset)$.

Example 2.3. Consider the aforementioned graph in Example 2.1 with $\mathfrak{R}(\mathrm{G}) = \{H_1, H_2, H_3\}$ where $E((H_1) = \{p, r\}$, $E(H_2) = \{x, y\}$, $E(H_3) = \{q, w\}$ and H is a spanning sub graph of G with edges $E(\mathrm{H}) = \{p, q, x\}$.Consequently, the approximations of H are $N_*(\mathrm{H}) =$, $N^*(\mathrm{H}) = \mathrm{G}$ and $B_N(\mathrm{H}) = \mathrm{G}$.

Theorem 2.4. *Let* G *be a non-empty simple graph, and let* H *and* T *be sub graphs of* G*. Consider the approximation operators* $N_*(.)$ *and* $N^*(.)$*, applied to these sub graphs. The following properties hold:*

1. *Inclusion property of sub graphs: If* $\mathrm{H} \subseteq \mathrm{T}$, $N_*(\mathrm{H}) \subseteq N_*(\mathrm{T})$ *and* $N^*(\mathrm{H}) \subseteq N^*(\mathrm{T})$.

2. *Union of lower approximations:* $[N_*(H) \cup N_*(T)] \subseteq N_*(H \cup T)$

3. *Intersection of lower approximations:* $(N_*(\mathrm{H}) \cup N_*(\mathrm{T})) \subseteq N_*[\mathrm{H} \cup \mathrm{T})$.

4. *Union of upper approximations:* $N^*(\mathrm{H} \cup \mathrm{T}) = N^*(\mathrm{H}) \cup N^*(\mathrm{T})$.

5. *Intersection of upper approximations:* $N^*(\mathrm{H} \cap \mathrm{T}) \subseteq N^*(\mathrm{H}) \cap N^*(\mathrm{T})$.

Proof. The properties of approximations substantiate Theorem 2.4.

Example 2.5. To illustrate the aforementioned theorem, consider Example 2.1 with $\mathfrak{R}(\mathrm{G}) = \{H_1, H_2, H_3, H_4\}$ where $E(H_1) = \{p\}$, $E(H_2) = \{q\}$, $E(H_3) = \{r, w\}$, $E(H_4) = \{x, y\}$. Let H and T be spanning subgraphs of G, where $E(\mathrm{H}) = \{p, q, x\}$ and $E(\mathrm{T}) = \{p\}$. The lower approximation of H and T are spanning subgraphs with edge set $\{p, q\}$ and $\{p\}$ respectively. Similarly, the upper approximations of H and T are super spanning subgraphs with edge sets $\{x, y, p, q\}$ and $\{p\}$ respectively.

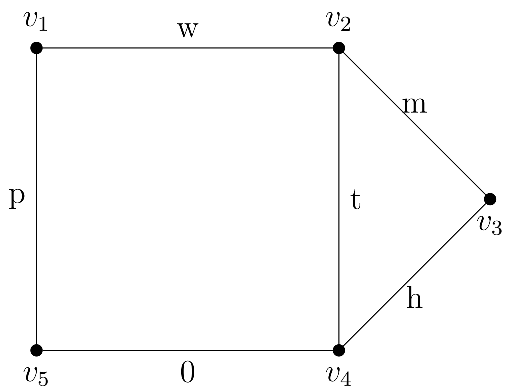
3. N-Graph Topology

Definition 3.1. *Consider a non-empty simple graph* $\mathrm{G} = (V, E)$ *and a collection of distinct edge induced subgraphs of* G*, generated by the subsets of E. For any subgraph* H *of* G*, define* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N_*(\mathrm{H}), N^*(\mathrm{H}), B_N(\mathrm{H})\}$ *where* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *satisfies the following axioms:*

- *The graph* G *(full graph) itself and the empty graph* $\emptyset$ *belong to* $\Gamma_{\mathfrak{R}}(\mathrm{H})$.
- *An arbitrary union of members of* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *is in* $\Gamma_{\mathfrak{R}}(\mathrm{H})$.
- *A finite intersection of members of* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *is in* $\Gamma_{\mathfrak{R}}(\mathrm{H})$.

The collection $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *is termed a N-graph topology on* G*. The pair* $(\mathrm{G}, \Gamma_{\mathfrak{R}}(\mathrm{H}))$ *is designated as the N-Graph topological space. The elements of* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *are referred to as N-open subgraphs in* G*, and the complement of a N-open subgraph is termed a N-closed subgraph of* $\Gamma_{\mathfrak{R}}(\mathrm{H})$*. A subgraph that is both a N-open subgraph and a N-closed subgraph is designated as a N-clopen subgraph.*

Example 3.2. Consider a non-empty simple graph G with edges set $\{w, m, h, o, p, t\}$ and a collection of distinct edge-induced subgraphs of G,$\mathfrak{R}(\mathrm{G}) = \{H_1, H_2, H_3, H_4\}$ where $E(H_1) = \{w\}$, $E(H_2) = \{m\}$, $E(H_3) = \{o, p\}$, $E(H_4) = \{h, t\}$.



Given for a subgraph H of G and $E(H) = \{w, m, t\}$,the N-Graph topology of G is the collection $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N_*(\mathrm{H}), N^*(\mathrm{H}), B_N(\mathrm{H})\}$ and $E(N_*(\mathrm{H})) = \{w, m\}$,

$E(N^*(\mathrm{H})) = \{w, m, h, t\}$, $E(B_N(\mathrm{H})) = \{h, t\}$.

Definition 3.3. *For a subgraph* H *of non-empty simple graph* G*. the Indiscrete N-graph topology is defined as* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\emptyset, \mathrm{G}\}$*, a collection of the trivial subgraphs of* G*.*

Theorem 3.4. *For a subgraph* H *of non-empty simple graph* G *and* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *is the N-Graph topology on* G*, the set* $B = \{\mathrm{G}, N_*(\mathrm{H}), B_N(\mathrm{H})\}$ *is the basis for* $\Gamma_{\mathfrak{R}}(\mathrm{H})$*.*
Proof. To demonstrate that B is a basis for $\Gamma_{\mathfrak{R}}(\mathrm{H})$, it must satisfy the resulting specifications:

1. For each edge $e \in \mathrm{G}$, there exists at least one basis element $\mathrm{B_i} \in \mathrm{B}$ containing an edge e.

2. Since $\mathrm{G} \cap N_*(\mathrm{H}) = N_*(\mathrm{H})$, $\mathrm{G} \cap B_N(\mathrm{H}) = B_N(\mathrm{H})$ and$N_*(\mathrm{H}) \cap B_N(\mathrm{H}) = \emptyset$, if an edge e be appropriate to the intersection of two basis elements B_1 and B_2 of B , there exists a basis element B_3 containing edge e such that $B_3 \subseteq B_1 \cap B_2$.Thus, B is a basis for $\Gamma_{\mathfrak{R}}(\mathrm{H})$.

Proposition 3.5. *For a subgraph* H *of non-empty simple graph* G*:*

1. *If* $N_*(\mathrm{H}) = \emptyset$ *and* $N^*(\mathrm{H}) = \mathrm{G}$*, then* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset\}$

2. *If* $N_*(\mathrm{H}) = N^*(\mathrm{H}) = \mathrm{H}$*, then* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \mathrm{H}, \emptyset\}$

3. *If* $N_*(\mathrm{H}) = \emptyset$,$N^*(\mathrm{H}) \neq \mathrm{G}$*, then* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N^*(\mathrm{H})\}$

4. *If* $N_*(\mathrm{H}) \neq \emptyset$,$N^*(\mathrm{H}) = \mathrm{G}$*, then* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N_*(\mathrm{H})\}$

5. *If* $N_*(\mathrm{H}) \neq N^*(\mathrm{H})$ *where* $N_*(\mathrm{H}) \neq \emptyset$ *and* $N^*(\mathrm{H}) \neq \mathrm{G}$*, then* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N_*(\mathrm{H}), N^(\mathrm{H}), B_N(\mathrm{H})\}$*.*

Proposition 3.6. *For a traversing subgraph* H *of non-empty simple graph* G *and* $\mathfrak{R}(\mathrm{G})$ *is the collection of distinct edge induced subgraphs isomorphic to* H *of* G*, the N-graph topology for subgraph* H *is* $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, \mathrm{H}\}$*.*
Proof. Consider the spanning subgraph H of G. By the definition of lower and upper approximations of graphs, it is evident that $N_*(\mathrm{H}) = \emptyset$ and $N^*(\mathrm{H}) = H$. Hence the N-graph topology of any subgraph H is $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, \mathrm{H}\}$.

4. N-interior and N-closure of subgraph in N-Graph topological space

The new concepts of interior and closure operator of subgraphs in N-Graph topological spaces were introduced in this section and discussed some properties.

Definition 4.1. *Let* $\mathrm{G} = (V, E)$ *be a graph and* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *is a N-graph topological* *space for any subgraph* H *of* G*. Let* S *be a subgraph of* G*. For an edge* $e \in E(\mathrm{S}), \mathrm{S}$ *is a neighbourhood graph of e, there exist* $H_i \in \Gamma_{\mathfrak{R}}(\mathrm{H})$*, provided* $e \in E(H_i)$ *and* $H_i \subset \mathrm{S}$*. Now e is called an edge interior of* S*. The collection of all edge interiors of* S *is denoted by* S′*. The N-interior subgraph of* S *is the subgraph generated by* S′*. That is* $N_H Int(\mathrm{S}) = . < \mathrm{S}' >$*.*

Definition 4.2. *Let* $\mathrm{G} = (V, E)$ *be a graph and let* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *be a N-graph topology on* G*. Let* K *be an edge induced subgraph of* G*. An edge* $e \in E(\mathrm{G}) - E(\mathrm{K})$ *is a edge limit of* K *if for all open subgraphs of* K′ *in* $\Gamma_{\mathfrak{R}}(\mathrm{H})$ *with* $e \in E(\mathrm{K}')$*,* $E(\mathrm{K}) \cap E(\mathrm{K}')$ *is a non-empty edge set. The collection of all edge limits of an edge induced subgraph K is denoted by* K^**. The N-closure of subgraph* K *is defined as edge-induced subgraph generated by* $E(\mathrm{K}) \cup \mathrm{K}^*$ *and it is denoted by* $N_H Cl(S)$*.Additionally,* $N_H Int(\mathrm{S})$ *is the largest N-open subgraph of* S *and* $N_H Cl(\mathrm{S})$ *is the smallest super graph N-closed subgraph of* S*.*

Definition 4.3. *A subgraph* S *of a N-graph topology on* G *is called N -dense subgraph if* $N_H Cl(\mathrm{S}) = \mathrm{G}$*.*

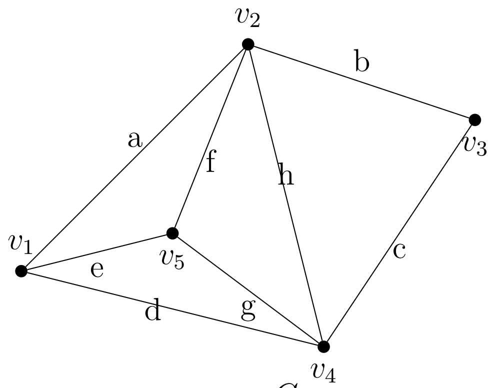
Proposition 4.4. *Let* G *be a non-empty simple graph and* H *be the spanning subgraph of* G*, then* H *is the only N- dense subgraph in* $\Gamma_{\mathfrak{R}}(\mathrm{H})$*.*
Proof. Since the complements of all elements of $\Gamma_{\mathfrak{R}}(\mathrm{H})$ are N -closed subgraph and G is the only N-closed subgraph which H is the subgraph of G. Hence $N_H Cl(\mathrm{S}) = \mathrm{G}$. Therefore, H is the only N- dense subgraph in $\Gamma_{\mathfrak{R}}(\mathrm{H})$.

Proposition 4.5. *Let* G *be a non-empty simple graph and* S, T *be the subgraphs of* G*, then the following statements hold:*

1. $N_{\mathrm{H}} Int(\mathrm{S}) \subseteq \mathrm{S} \subseteq N_{\mathrm{H}} Cl(\mathrm{S})$
2. $[N_{\mathrm{H}} Int(\mathrm{S}) \cup N_{\mathrm{H}} Int(\mathrm{T})] \subset N_{\mathrm{H}} Int[\mathrm{S} \cup \mathrm{T}]$
3. $N_{\mathrm{H}} Int(\mathrm{S} \cap \mathrm{T}) = N_{\mathrm{H}} Int(\mathrm{S}) \cap N_{\mathrm{H}} Int(\mathrm{T})$

Example 4.6. Consider a non-empty simple graph G with edges $\{a, b, c, d, e, f, g, h\}$ and $\mathfrak{R}(\mathrm{G}) = \{H_1, H_2, H_3\}$ where $E(H_1) = \{a, b, c\}$,$E(H_2) = \{d, e\}$,$E(H_3) = \{f, g, h\}$.

For spanning subgraph H of G and $E(\mathrm{H}) = \{a, b, c, d\}$, the N-graph topology is defined as $\Gamma_{\mathfrak{R}}(\mathrm{H}) = \{\mathrm{G}, \emptyset, N_*(\mathrm{H}), N^*(\mathrm{H}), B_N(\mathrm{H})\}$ where $E(N_*(\mathrm{H})) = \{a, b, c\}$, $E(N^*(\mathrm{H})) = \{a, b, c, d, e\}, E(B_N(\mathrm{H})) = \{d, e\}$. For subgraph $\mathrm{S} \subseteq \mathrm{G}$ and $E(\mathrm{S}) = \{a, b, c\}$,then $N_H Cl(\mathrm{S})$ and $N_H Int(\mathrm{S})$ as follows:



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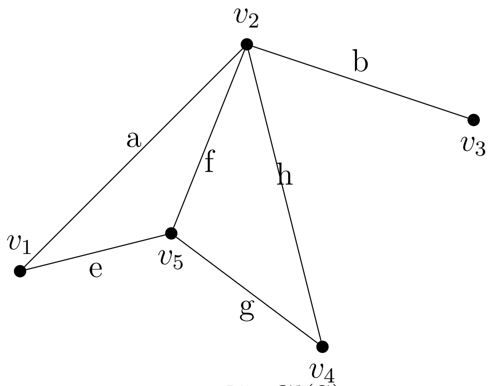
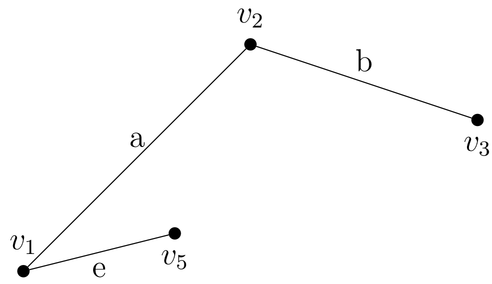


$\mathrm{N_H}Cl(\mathrm{S})$



$\mathrm{N_H}Int(\mathrm{S})$

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