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On Bailey's Transform and Expansion of Hypergeometric Functions-I

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Abstract: In this paper we introduce a new technique to obtain expansions of basic hypergeometric functions with the help of Bailey's transform and certain known transformations of truncated hypergeometric series. These results do not look possible with the help of the traditional method. Certain interesting special cases, both, new and known, have also been deduced.
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1. Introduction, Notations and Definitions

For $|q| < 1$ and α, real or complex, let

$$[\alpha; q]_n \equiv [\alpha]_n = \begin{cases} (1-\alpha)(1-\alpha q)(1-\alpha q^2)...(1-\alpha q^{n-1}); & n > 0 \\ 1 & ; \quad n = 0 \end{cases} \tag{1.1}$$

Accordingly,

$$[\alpha; q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n)$$

Also,

$$[a_1, a_2, a_3, ..., a_r; q]_n = [a_1; q]_n[a_2; q]_n[a_3; q]_n...[a_r; q]_n. \tag{1.2}$$

Now, we define a basic hypergeometric function

$${}_r\Phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s; q^\lambda \end{array}\right] = \sum_{n=0}^{\infty}\frac{[a_1, a_2, ..., a_r; q]_n z^n q^{\lambda n(n-1)/2}}{[q, b_1, b_2, ..., b_s; q]_n} \tag{1.3}$$

convergent for $|z| < \infty$ when $\lambda > 0$ and for $|z| < 1$ when $\lambda = 0$.
A generalized double basic hypergeometric function is defined as,

$$\Phi\left[\begin{array}{l} a_1, a_2, ..., a_r : \alpha_1, \alpha_2, ..., \alpha_{u_1}; \beta_1, \beta_2, ..., \beta_{v_1}; q; x, y \\ b_1, b_2, ..., b_s : \delta_1, \delta_2, ..., \delta_{u_2}; \gamma_1, \gamma_2, ..., \gamma_{v_2} \end{array}\right]$$

$$= \sum_{m,n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_{m+n}[\alpha_1, \alpha_2, ..., \alpha_{u_1}; q]_m}{[b_1, b_2, ..., b_s; q]_{m+n}[\delta_1, \delta_2, ..., \delta_{u_2}; q]_m} \times$$

$$\times \frac{[\beta_1, \beta_2, ..., \beta_{v_1}; q]_n x^m y^n}{[\gamma_1, \gamma_2, ..., \gamma_{v_2}; q]_n [q; q]_m [q; q]_n} \tag{1.4}$$

which converges for $|x|, |y| < 1$.
We also define a bi-basic hypergeometric series of one variable as,

$$\Phi \left[\begin{array}{l} a_1, a_2, ..., a_r; \alpha_1, \alpha_2, ..., \alpha_u; q, p; z \\ b_1, b_2, ..., b_s; \beta_1, \beta_2, ..., \beta_v \end{array} \right]$$

$$= \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_n [\alpha_1, \alpha_2, ..., \alpha_u; p]_n z^n}{[q, b_1, b_2, ..., b_s; q]_n [\beta_1, \beta_2, ..., \beta_v; p]_n} \tag{1.5}$$

which converges for max. $\{|q|, |p|, |z|\} < 1$.

We shall have the occasion to use the following well known Bailey's transform
If

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n} = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{r+2n}$$

Then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.6}$$

subject to the convergence of infinite series and that the sequences α_r, δ_r, u_r and v_r are rational functions of r alone.

2. Results to be used

In this section we shall develop certain results which will be used in the subsequent sections.
If in Bailey's transform we set $u_r = v_r = 1$ and $\delta_r = z^r$, then (1.6) leads to
Lemma 1

$$\sum_{n=0}^{\infty} \alpha_n z^n = (1 - z) \sum_{n=0}^{\infty} \beta_n z^n.$$

Again, if we set $u_r = v_r = 1$, we get

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r = \sum_{r=0}^{\infty} \delta_r - \sum_{r=0}^{n-1} \delta_r \tag{2.1}$$

which with,

$$\delta_r = \frac{[\alpha, \beta; p]_r p^r}{[p, \alpha\beta p; p]_r},$$

leads to,

$$\gamma_n = {}_2\Phi_1 \left[\begin{array}{l} \alpha, \beta; p \\ \alpha\beta p \end{array} \right] - {}_2\Phi_1 \left[\begin{array}{l} \alpha, \beta; p \\ \alpha\beta p \end{array} \right]_{n-1} \tag{2.2}$$

where,

$${}_r\Phi_s \left[\begin{array}{l} \alpha_1, \alpha_2, ..., \alpha_r; q; z \\ \beta_1, \beta_2, ..., \beta_s \end{array} \right]_N = \sum_{n=0}^{N} \frac{[\alpha_1, \alpha_2, ..., \alpha_r; q]_n z^n}{[q, \beta_1, \beta_2, ..., \beta_s; q]_n} \tag{2.3}$$

is a truncated hypergeometric series.
Now, using the following known sum of a truncated series (cf. Agarwal [1; app. II.8]),

$${}_2\Phi_1 \left[\begin{array}{l} \alpha, \beta; q; q \\ \alpha\beta q \end{array} \right]_n = \frac{[\alpha q, \beta q; q]_n}{[q, \alpha\beta q; q]_n}$$

we get,

$$\gamma_n = \frac{(1-\alpha\beta)}{(1-\alpha)(1-\beta)} \left[\frac{[\alpha, \beta; p]_\infty}{[p, \alpha\beta; p]_\infty} - (1-p^n) \frac{[\alpha, \beta; p]_n}{[\alpha\beta, p; p]_n} \right]$$

Thus, (1.6) leads to,
Lemma 2

$$\frac{(1-\alpha\beta)}{(1-\alpha)(1-\beta)} \sum_{n=0}^{\infty} \alpha_n \left[\frac{[\alpha, \beta; p]_\infty}{[p, \alpha\beta; p]_\infty} - (1-p^n) \frac{[\alpha, \beta; p]_n}{[\alpha\beta, p; p]_n} \right] = \sum_{n=0}^{\infty} \beta_n \frac{[\alpha, \beta; p]_n p^n}{[p, \alpha\beta p; p]_n} \tag{2.4}$$

Next, setting $u_r = v_r = 1$ and $\delta_r = \dfrac{[\alpha, \beta; p]_r p^r}{[\gamma, \alpha\beta p^2/\gamma; p]_r}$ in Bailey's transform we get

$$\gamma_n = \frac{[\alpha, \beta; p]_n p^n}{[\gamma, \alpha\beta p^2/\gamma; p]_n} \, {}_3\Phi_2 \left[\begin{array}{l} \alpha p^n, \beta p^n, p; p; p \\ \gamma p^n, \alpha\beta p^{n+2}/\gamma \end{array} \right]$$

Now, using the following summation due to Agarwal [2; p.79]

$${}_3\Phi_2 \left[\begin{array}{l} a, b, q; q; q \\ e, abq^2/e \end{array} \right]_N = \frac{(q-e)(e-abq)}{(aq-e)(e-bq)} \left[1 - \frac{[a, b; q]_{N+1}}{[e/q, abq/e; q]_{N+1}} \right]$$

and letting $N \to \infty$ we get, after simplification,

$$\gamma_n = \frac{(1-p/\gamma)(1-\alpha\beta p/\gamma)}{(1-\alpha p/\gamma)(1-\beta p/\gamma)}\left[\frac{[\alpha,\beta;p]_n}{[\gamma/p,\alpha\beta p/\gamma;p]_n} - \frac{[\alpha,\beta;p]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty}\right].$$

Thus, (1.6) leads to,

Lemma 3

$$\frac{(1-p/\gamma)(1-\alpha\beta p/\gamma)}{(1-\alpha p/\gamma)(1-\beta p/\gamma)}\times$$

$$\sum_{n=0}^{\infty}\alpha_n\left[\frac{[\alpha,\beta;p]_n}{[\gamma/p,\alpha\beta p/\gamma;p]_n} - \frac{[\alpha,\beta;p]_\infty}{[\gamma/p,\alpha\beta p/\gamma;p]_\infty}\right] = \sum_{n=0}^{\infty}\beta_n\frac{[\alpha,\beta;p]_n p^n}{[\gamma,\alpha\beta p^2/\gamma;p]_n}$$

The main aim of this paper is to establish certain expansions involving basic hypergeometric functions in terms of similar functions. These results do not appear to be established with the help of traditional methods. In order to establish our main results we shall require the following known results in our analysis;
Sear's transformation (cf. Gasper and Rahman [4;2.10.4, p.41])

$${}_4\Phi_3\left[\begin{matrix} q^{-n}, a, b, c; q; q \\ d, e, abcq^{1-n}/de \end{matrix}\right]$$

$$= \frac{[e/a, de/bc; q]_n}{[e, de/abc; q]_n}\,{}_4\Phi_3\left[\begin{matrix} q^{-n}, a, d/b, d/c; q; q \\ d, de/bc, aq^{1-n}/e \end{matrix}\right] \tag{2.5}$$

and Watson's transformation (cf. Gasper and Rahman [3;2.5.1, p. 35])

$${}_8\Phi_7\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, d, e, q^{-n}; q; a^2q^{n+2}/bcde \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d, aq/e, aq^{n+1} \end{matrix}\right]$$

$$= \frac{[aq, aq/de; q]_n}{[aq/d, aq/e; q]_n}\,{}_4\Phi_3\left[\begin{matrix} q^{-n}, d, e, aq/bc; q; q \\ aq/b, aq/c, deq^{-n}/a \end{matrix}\right] \tag{2.6}$$

3. Main Transformations

In this section we establish our transformations,
Letting $e \to q^{-n}$ in (2.5), we get

$${}_3\Phi_2\left[\begin{matrix} a, b, c; q; q \\ d, abcq/d \end{matrix}\right]_n$$

$$= \frac{[aq, bcq/d; q]_n}{[q, abcq/d; q]_n}\,{}_4\Phi_3\left[\begin{matrix} q^{-n}, a, d/b, d/c; q; q \\ d, aq, dq^{-n}/bc \end{matrix}\right] \tag{3.1}$$

Now, setting

$$\alpha_r = \frac{[a, b, c; q]_r q^r}{[q, d, abcq/d; q]_r}$$

we have

$$\beta_n = \frac{[aq, bcq/d; q]_n}{[q, abcq/d; q]_n} \, {}_4\Phi_3 \left[\begin{array}{l} q^{-n}, a, d/b, d/c; q; q \\ d, aq, dq^{-n}/bc \end{array} \right] \tag{3.2}$$

Now, Lemma 1 leads to,

$${}_3\Phi_2 \left[\begin{array}{l} a, b, c; q, zq \\ d, abcq/d \end{array} \right]$$

$$= (1-z)\Phi \left[\begin{array}{l} aq : a, d/b, d/c; bcq/d; q; bczq/d, z \\ abcq/d : d, aq; -----; \end{array} \right] \tag{3.3}$$

Next, putting the above values of α_n and β_n in Lemma 2, we get after some simplification

$$\frac{(1-\alpha)(1-\beta)}{(1-\alpha\beta)} \sum_{n=0}^{\infty} \frac{[\alpha, \beta; p]_n [aq, bcq/d; q]_n p^n}{[p, \alpha\beta p; p]_n [q, abcq/d; q]_n} \times$$

$${}_6\Phi_5 \left[\begin{array}{l} aq^{n+1}, a, d/b, d/c : \alpha p^n, \beta p^n; q, p; bcpq \\ d, aq, abcq^{n+1}/d : p^{n+1}, \alpha\beta p^{n+1} \end{array} \right]$$

$$= \frac{[\alpha, \beta; p]_\infty}{[p, \alpha\beta; p]_\infty} \, {}_3\Phi_2 \left[\begin{array}{l} a, b, c; q; q \\ d, abcq/d \end{array} \right] - \, {}_5\Phi_4 \left[\begin{array}{l} a, b, c; \alpha, \beta; q, p; q \\ d, abcq/d; p, \alpha\beta; \end{array} \right]$$

$$+ \, {}_5\Phi_4 \left[\begin{array}{l} a, b, c; \alpha, \beta; q, p; pq \\ d, abcq/d; p, \alpha\beta \end{array} \right] \tag{3.4}$$

Again, if we substitute the above values of α_n and β_n in Lemma 3, we get

$$\frac{(1-\alpha p/\gamma)(1-\beta p/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)} \sum_{n=0}^{\infty} \frac{[aq, bcq/d; q]_n [\alpha, \beta; p]_n p^n}{[q, abcq/d; q]_n [\gamma, \alpha\beta p^2/\gamma; p]_n}$$

$${}_6\Phi_5 \left[\begin{array}{l} a, d/b, d/c, aq^{n+1} : \alpha p^n, \beta p^n; q, p, bcpq \\ d, aq, abcq^{n+1}/d : \gamma p^{n+1}, \alpha\beta p^{n+2}/\gamma \end{array} \right]$$

$$= \, {}_5\Phi_4 \left[\begin{array}{l} a, b, c; \alpha, \beta; q, p; q \\ d, abcq/d; \gamma/p, \alpha\beta p/\gamma \end{array} \right]$$

$$- \frac{[\alpha, \beta; p]_\infty}{[\gamma/p, \alpha\beta p/\gamma; p]_\infty} \, {}_3\Phi_2 \left[\begin{array}{l} a, b, c; q; q \\ d, abcq/d \end{array} \right] \tag{3.5}$$

Further, taking $e = aq^{n+1}$ in (2.6), we get

$$
{}_6\Phi_5\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q; aq/bcd \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d \end{array}\right]_n
$$

$$
= \frac{[aq, dq; q]_n}{[q, aq/d; q]_n d^n}\ {}_4\Phi_3\left[\begin{array}{l} q^{-n}, aq^{n+1}, d, aq/bc; q; q \\ dq, aq/b, aq/c \end{array}\right] \tag{3.6}
$$

Now, taking $u_r = v_r = 1$ and

$$
\alpha_r = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q]_r (aq/bcd)^r}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d; q]_r}
$$

in Bailey's transform we get,

$$
\beta_n = \frac{[aq, dq; q]_n}{[q, aq/d; q]_n d^n}\ {}_4\Phi_3\left[\begin{array}{l} q^{-n}, aq^{n+1}, d, aq/bc; q; q \\ dq, aq/b, aq/c \end{array}\right] \tag{3.7}
$$

With the above values of α_n and β_n, Lemma 1 leads to the following interesting transformation

$$
(1-z)\sum_{n=0}^{\infty} \frac{[aq, dq; q]_n z^n}{[q, aq/d; q]_n d^n}\ {}_4\Phi_3\left[\begin{array}{l} q^{-n}, aq^{n+1}, d, aq/bc; q; q \\ dq, aq/b, aq/c \end{array}\right]
$$

$$
= {}_6\Phi_5\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q; azq/bcd \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d \end{array}\right] \tag{3.8}
$$

As $d \to \infty$, we get after some simplification

$$
(1-z)\sum_{n=0}^{\infty} \frac{[aq; q]_n (-z)^n q^{n(n+1)/2}}{[q; q]_n}\ {}_3\Phi_2\left[\begin{array}{l} q^{-n}, aq^{n+1}, aq/bc; q; 1 \\ aq/b, aq/c \end{array}\right]
$$

$$
= {}_5\Phi_4\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c; q; -azq/bc \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c; q \end{array}\right] \tag{3.9}
$$

The ${}_5\Phi_4$ on the right side is a well-poised while the ${}_3\Phi_2$ under the summation sign on the left is a Saalschützian.

With above value of α_n and the subsequent value of β_n given by (3.7) Lemma 3 leads to the following interesting transformation as $d \to \infty$,

$$
\frac{(1-\alpha p/\gamma)(1-p\beta/\gamma)}{(1-p/\gamma)(1-\alpha\beta p/\gamma)} \sum_{n=0}^{\infty} \frac{[aq; q]_n [\alpha, \beta; p]_n (-pq)^n q^{n(n+1)/2}}{[\gamma, \alpha\beta p^2/\gamma; p]_n [q; q]_n} \times
$$

$$
{}_5\Phi_4\left[\begin{array}{l} aq^{n+1}, aq^{n+2} : aq/bc : \alpha p^n, \beta p^n; q^2, q, p; p \\ ------- : aq/b, aq/c : \gamma p^n, \alpha\beta p^{n+2}/\gamma \end{array}\right]
$$

$$
= \frac{[\alpha, \beta; p]_\infty [aq, aq/bc; q]_\infty}{[\gamma/p, \alpha\beta p/\gamma; p]_\infty [aq/b, aq/c; q]_\infty}
$$

$$
+ {}_7\Phi_6\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c; \alpha, \beta; q, p; -aq/bc \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c; \gamma/p, \alpha\beta p/\gamma; q \end{array}\right] \tag{3.10}
$$

If we put $\gamma = \alpha p$ in (3.10), we get the following very important summation of a well-poised abnormal series due to Sears [5; 9.1]

$$
{}_5\Phi_4\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c; q; -aq/bc \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c; q \end{array}\right] = \frac{[aq, aq/bc; q]_\infty}{[aq/b, aq/c; q]_\infty} \tag{3.11}
$$

If $b, c \to \infty$ in the above, we get

$$
\sum_{n=0}^{\infty} \frac{[a;q]_n (1-q^{2n}) q^{n(3n-1)/2} (-a)^n}{(1-a)[q;q]_n} = [aq;q]_\infty \tag{3.12}
$$

As $a \to 1$ in (3.12), we get the following well known identity due to Euler,

$$
1 + \sum_{n=1}^{\infty} (1+q^n)(-)^n q^{n(3n-1)/2} = [q;q]_\infty \tag{3.13}
$$

4. Special Cases

In this section we shall deduce certain interesting special cases of our results established in the last section

If we equate the coefficients of z^n on both sides of (3.3), we get the following transformation,

$$
\frac{[a, b, c; q]_n q^n}{[d, aq, bcq/d; q]_n} = {}_4\Phi_3\left[\begin{array}{l} q^{-n}, a, d/b, d/c; q; q \\ d, aq, dq^{-n}/bc \end{array}\right]
$$

$$
-\frac{(1-q^n)(1-abcq^n/d)}{(1-aq^n)(1-bcq^n/d)}\, {}_4\Phi_3\left[\begin{array}{l} q^{1-n}, a, d/b, d/c; q; q \\ d, aq, dq^{1-n}/bc \end{array}\right] \tag{4.1}
$$

For c=d in (3.3), we get the following known result due to Denis et. al. [3; (5.7), p. 63]

$$
{}_2\Phi_1\left[\begin{array}{l} a, b; q; zq \\ abq \end{array}\right] = (1-z)\, {}_2\Phi_1\left[\begin{array}{l} aq, bq; q; z \\ abq \end{array}\right] \tag{4.2}
$$

Again, for a=0 in (3.3) we get yet another well known transformation, due to Euler,

$$
{}_2\Phi_1\left[\begin{matrix} b, c; q; zq \\ d \end{matrix}\right] = \frac{[bcqz/d;q]_\infty}{[zq;q]_\infty}\, {}_2\Phi_1\left[\begin{matrix} d/b, d/c; q; bczq/d \\ d \end{matrix}\right] \tag{4.3}
$$

Next, putting $c = 0, b = q$ and $d = aq$ in (3.3), after some simplification, we get

$$
\sum_{r=0}^{\infty} \frac{(zq)^r}{1-aq^r} = (1-z)(1-a) \sum_{n,r=0}^{\infty} \frac{[aq;q]_{n+r} z^{n+r} (-q)^r q^{r(r+1)/2}}{[q;q]_r [q;q]_n (1-aq^r)^2} \tag{4.4}
$$

Further, if we set $d \to c$ in (3.4), we get the following interesting transformation involving bi-basic hypergeometric series with two un-connected bases p and q, after some simplification

$$
\begin{aligned}
&\frac{[\alpha, \beta; p]_\infty [aq, bq; q]_\infty}{[p, \alpha\beta; p]_\infty [q, abq; q]_\infty} \\
&= \frac{(1-a)(1-b)(1-\alpha)(1-\beta)q}{(1-abq)(1-q)(1-\alpha\beta)}\, {}_4\Phi_3\left[\begin{matrix} \alpha p, \beta p : aq, bq; p, q; q \\ \alpha\beta p : q^2, abq^2 \end{matrix}\right] \\
&\quad + \frac{(1-\alpha)(1-\beta)}{(1-\alpha\beta)}\, {}_4\Phi_3\left[\begin{matrix} aq, bq : \alpha, \beta; q, p; p \\ abq : p, \alpha\beta p \end{matrix}\right]
\end{aligned} \tag{4.5}
$$

Now, equating the coefficients of z^n on both sides of (3.8), we get

$$
\begin{aligned}
&{}_4\Phi_3\left[\begin{matrix} q^{-n}, aq^{n+1}, d, aq/bc; q; q \\ dq, aq/b, aq/c \end{matrix}\right] \\
&- \frac{(1-q^n)(d-aq^n)}{(1-aq^n)(1-dq^n)}\, {}_4\Phi_3\left[\begin{matrix} q^{1-n}, aq^n, d, aq/bc; q; q \\ dq, aq/b, aq/c \end{matrix}\right] \\
&= \frac{(1-d)(1-aq^{2n})[b, c; q]_n (aq/bc)^n}{(1-dq^n)(1-aq^n)[aq/b, aq/c; q]_n}
\end{aligned} \tag{4.6}
$$

A number of other interesting special cases can also be deduced similarly.

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