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SUM CONNECTIVITY MATRIX AND ENERGY OF A 3-UNIFORM T_2HYPERGRAPH

Sharmila D., Sujitha S.* and Angel Jebitha M. K.*

Department of Mathematics,
Arunachala College of Engineering for Women,
Nagercoil, Kanyakumari - 629203, Tamil Nadu, INDIA

E-mail : sharmilareegan10@gmail.com

*PG and Research Department of Mathematics,
Holy Cross College (Autonomous),
Nagercoil, Kanyakumari - 629004, Tamil Nadu, INDIA

E-mail : sujitha.s@holycrossngl.edu.in, angeljebitha@holycrossngl.edu.in

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Abstract: Let H be a 3-uniform T_2 hypergraph with order $n \geq 5$. The sum connectivity matrix of H, denoted by SC(H) is defined as the square martix of order n, whose $(i,j)^{th}$ entry is $\frac{1}{\sqrt{d_i+d_j}}$ if x_i and x_j are adjacent and zero for other cases. The sum connectivity energy SCE(H) of H is the sum of the absolute values of the eigenvalues of SC(H). It is shown that, for a 3-uniform T_2 hypergraph $\lfloor SCE(H) \rfloor \leq \left\lfloor \frac{n}{2} \right\rfloor + 2$.

Keywords and Phrases: T_2 hypergraph; 3-uniform T_2 hypergraph; sum connectivity matrix; sum connectivity energy.

2020 Mathematics Subject Classification: 05C65.

1. Introduction

The basic definitions and terminologies of a hypergraph are not given here and we refer it [2] and [8]. The concept of hypergraph was introduced by Berge in 1967. In 2017, Seena V and Raji Pilakkat were introduced Hausdorff hypergraph, T_0 hypergraph and T_1 hypergraph. Based on [5] and [6] S. Sujitha and D. Sharmila introduced T_2 hypergraph and studied adjacency matrix, Randic matrix, Zagreb matrix and its corresponding energy [7]. The sum connectivity index is a relatively recent concept compared to other well-established indices like the Wiener index or the Zagreb indices. It emerged from the broader field of topological indices, which has been extensively used in chemical graph theory since the 1940s and 1950s. In 2009, Bo Zhou and Nenad Trinajstic introduced the sum connectivity index, for simplicity we call it SCI [1].

The energy of the hypergraph and adjacency energy of the hypergraph introduced in [3, 4]. In this paper we study the sum connectivity energy and bound of the 3-uniform T_2 hypergraph. Throughout this article, H is a simple connected 3-uniform T_2 hypergraph with order n, and size m, where the order and size are the minimum numbers of edges needed to define a 3-uniform T_2 hypergraph. The following definitions and theorems are used in the sequel.

Definition 1.1. [7] *A hypergraph $H = (X, D)$ is said to be a T_2 hypergraph if for any three distinct vertices u, v and w in X there exist a hyperedge containing u and v but not w and another hyperedge containing w but not u and v.*

Result 1.2. [7]

(i) The minimum number of edges need to define a T_2 hypergraph is $\left[\frac{2n+5}{4}\right]$, where n is the number of vertices.

(ii) For a T_2 hypergaph H, the minimum degree $\delta(H) = 2$.

(iii) For a T_2 hypergraph H, rank $r(H)=\left[\frac{2n+1}{4}\right]$, where $n \geq 5$.

Definition 1.3. *A T_2 hypergraph $H = (X, D)$ is said to be a 3-uniform T_2 hypergraph if every hyperedge contains exactly three vertices.*

Definition 1.4. [1] *The sum connectivity matrix of a T_2 hypergraph H is defined by* $SC(H)=\begin{cases} \frac{1}{\sqrt{d_i+d_j}} & if x_i x_j \in D \\ 0 & otherwise \end{cases}$

Definition 1.5. [1] *The sum connectivity energy of a T_2 hypergraph H is defined by sum of the absolute values of the sum connectivity matrix of H.*

2. Sum connectivity matrix and energy of a 3-uniform T_2 Hypergraph

In this section, we explore the sum connectivity matrix and energy of a 3-uniform T_2 hypergraph. We then illustrate these concepts with specific examples, providing a clear and concrete understanding of the structural characteristic of a T_2 hypergraph.

Illustration 2.1. *Consider a 3-uniform T_2 hypergraph H given in Figure 1 with 10 vertices and 11 edges.*

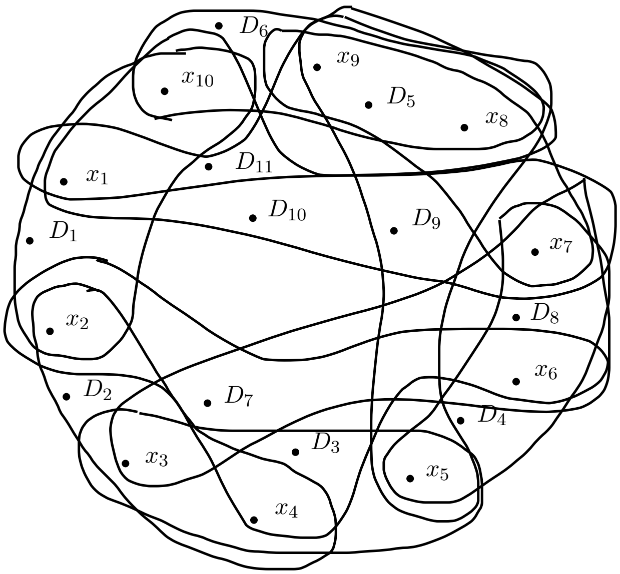

Figure 1: 3-uniform T_2 hypergraph

The sum connectivity matrix of H is given by

$$SC(H)= \begin{pmatrix} 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{7}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & 0 \\ 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{7}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{7}} & 0 & \frac{1}{\sqrt{7}} & 0 & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & 0 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{7}} \\ \frac{1}{\sqrt{7}} & 0 & 0 & 0 & \frac{1}{\sqrt{7}} & 0 & \frac{1}{\sqrt{8}} & 0 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{7}} \\ \frac{1}{\sqrt{7}} & 0 & 0 & 0 & \frac{1}{\sqrt{7}} & 0 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & 0 & \frac{1}{\sqrt{7}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & \frac{1}{\sqrt{7}} & 0 \end{pmatrix}$$

The sum connectivity eigen values of SC(H) are

$\lambda = 2.0595, 1.0888, .4528, -.119, -.35, -.41, -.41, -.5972, -.773, -.9419$

Therefore, In figure 1, the sum connectivity energy SCE(H)= $\sum_{i=1}^{n} |\lambda_i| = 7.2022$

Result 2.2. Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then

$\left\lfloor \frac{n}{2} \right\rfloor + 2 = \lfloor SCE(H) \rfloor = 7$
The Table 1, provide the sum connectivity energy of a 3-uniform T_2 hypergraph with respect to its order.

No of Vertices	SCE(H)	$\left\lfloor \frac{n}{2} \right\rfloor + 2$
5	3.31	4
6	4.31	5
7	5.02	5
8	6.04	6
9	6.36	6
10	7.20	7
...	...	...
n	...	$\left\lfloor \frac{n}{2} \right\rfloor + 2$

Table 1: Sum connectivity energy of a 3-uniform T_2 hypergraph

Result 2.3. For a 3-uniform T_2 hypergraph $\left\lceil \sum_{i=1}^{n} \lambda_i^2 \right\rceil = \Delta + \delta + 1$ with $n \geq 6$.

Observation 2.4. *For a 3-uniform T_2 hypergraph the independence number $\alpha(H) = 2$.*

Observation 2.5. *For a 3-uniform T_2 hypergraph the independence number $\alpha(H) \leq n - 3 - S + min(3, S)$.*

Theorem 2.6. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then $SCE(H) < \sqrt{2(n - \alpha(H))(\Delta + \delta + 1)}$.*

Proof. Let $\lambda_1, \lambda_2, \lambda_3$ be three positive sum connectivity eigenvalues of H and let $\xi_1, \xi_2, ..., \xi_s$ be the s negative sum connectivity eigenvalues of H such that n-3-S=0.
We have $\alpha \leq n - 3 - S + min(3, S)$
Hence $3 \leq n - \alpha(H)$ and $S \leq n - \alpha(H)$,here $\alpha(H) = 2$
Since $\sum_{i=1}^{3} \lambda_i + \sum_{i=1}^{S} \xi_i = 0$
From Cauchy-Schwarz inequality,

SCE(H)=$2\sum_{i=1}^{3} \lambda_i \leq 2\sqrt{3 \sum_{i=1}^{3} \lambda_i^2}$

Also SCE(H)=$2\sum_{i=1}^{S} \xi_i \leq 2\sqrt{S \sum_{i=1}^{S} \xi i^2}$

$\frac{SCE(H)^2}{2} = \frac{SCE(H)^2}{4} + \frac{SCE(H)^2}{4} \leq 3 \sum_{i=1}^{3} \lambda_i^2 + S \sum_{i=1}^{S} \xi_i^2$

$\leq (n-\alpha(H))\sum_{i=1}^{3}\lambda_i^2+(n-\alpha(H))\sum_{i=1}^{S}\xi_i^2$
$\leq (n-\alpha(H))\sum_{i=1}^{2}\lambda_i^2+(n-\alpha(H))\sum_{i=1}^{S}\xi_i^2$
$=(n-\alpha(H))(\sum_{i=1}^{2}\lambda_i^2+\sum_{i=1}^{S}\xi_i^2)$
$=(n-\alpha(H))(\lambda_1^2+\lambda_2^2+\lambda_3^2+\xi_1^2+\xi_2^2+,...,+\xi_n^2)$
$<(n-\alpha(H))\left[\sum_{i=1}^{n=S+3}\lambda_i^2\right]$
Hence $SCE(H) < \sqrt{2(n-\alpha(H))(\Delta+\delta+1)}$.

Illustration 2.7. *Consider a 3-uniform T_2 hypergraph with order 10.*
In H, SCE(H)= 7.2022, $\alpha(H)=2, \Delta+\delta+1=8$, $\sqrt{2(n-\alpha(H))(\Delta+\delta+1)}=11.31$
SCE(H)=7.2022 $<\sqrt{2(n-\alpha(H))(\Delta+\delta+1)}=11.31$
Hence, the Theorem 2.6 is verified.

Result 2.8. Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then $\lambda_1+\lambda_2 \leq \sqrt{2\delta+\frac{3}{5}}$. The sharp bound holds for n=8 in H.

Theorem 2.9. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then*
SCE(H) $<\sqrt{2\delta+\frac{3}{5}}+\sqrt{(n-2)(\Delta+\delta+1)}$.

Proof. From Cauchy schwarz inequality,
$\sum_{i=3}^{n}\lambda_i \leq \sqrt{(\sum_{i=3}^{n}\lambda_i^2)(\sum_{i=3}^{n}1)}$
$SCE(H)-(\lambda_1+\lambda_2)\leq \sqrt{[(\sum_{i=1}^{n}\lambda_i^2)-(\lambda_1^2+\lambda_2^2)](n-2)}$
$<\sqrt{n-2}\sqrt{\left[\sum_{i=1}^{n}\lambda_i^2\right]-(\lambda_1^2+\lambda_2^2)}$
$SCE(H)<\sqrt{n-2}\sqrt{(\Delta+\delta+1)-(\lambda_1^2+\lambda_2^2)}+(\lambda_1+\lambda_2)$
$SCE(H)<\sqrt{2\delta+\frac{3}{5}}+\sqrt{n-2}\sqrt{(\Delta+\delta+1)-(\lambda_1^2+\lambda_2^2)}$
Let h(s,t)= $\sqrt{2\delta+\frac{3}{5}}+\sqrt{n-2}\sqrt{(\Delta+\delta+1)-(s^2+t^2)}$
Differentiate partially with respect to s and t,
$h_s=\frac{-s\sqrt{n-2}}{\sqrt{(2\delta+\frac{3}{5})-s^2-t^2}}$
$h_t=\frac{-t\sqrt{n-2}}{\sqrt{(2\delta+\frac{3}{5})-s^2-t^2}}$
Stationary points are given by $h_s=0$ and $h_t=0$
$h_s=0 \Rightarrow \frac{-s\sqrt{n-2}}{\sqrt{(2\delta+\frac{3}{5})-s^2-t^2}}=0 \Rightarrow s=0$

$h_t = 0 \Rightarrow \frac{-t\sqrt{n-2}}{\sqrt{(2\delta+\frac{3}{5})-s^2-t^2}} = 0 \Rightarrow t = 0$
$h_{ss} = -\frac{(\Delta+\delta+1-t^2)\sqrt{n-2}}{[\Delta+\delta+1-s^2-t^2]^{\frac{3}{2}}}$
$h_{tt} = -\frac{(\Delta+\delta+1-s^2)\sqrt{n-2}}{[\Delta+\delta+1-s^2-t^2]^{\frac{3}{2}}}$
$h_{st} = -\frac{st\sqrt{n-2}}{[\Delta+\delta+1-s^2-t^2]^{\frac{3}{2}}}$
At (0,0),$h_{ss} = -\sqrt{\frac{n-2}{\Delta+\delta+1}} < 0$
$h_{tt} = -\sqrt{\frac{n-2}{\Delta+\delta+1}} < 0$
$h_{st} = 0$
T $= h_{ss}h_{tt} - (h_{st})^2 > 0$
The maximum value at (0,0) is h(0,0)=$\sqrt{2\delta + \frac{3}{5}} + \sqrt{(n-2)(\Delta+\delta+1)}$
Hence SCE(H) $< \sqrt{2\delta + \frac{3}{5}} + \sqrt{(n-2)(\Delta+\delta+1)}$.

Illustration 2.10. *Consider a 3-uniform T_2 hypergraph with order 10.*
In H, SCE(H)= 7.2022,$\sqrt{2\delta + \frac{3}{5}} = 2.57, \Delta + \delta + 1 = 8,$
$\sqrt{2\delta + \frac{3}{5}} + \sqrt{(n-2)(\Delta+\delta+1)} = 10.57$
SCE(H)=7.2022<10.57
Hence, Theorem 2.9. is verified.

Theorem 2.11. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then* $\sqrt{\Delta+\delta+1} < SCE(H) < \sqrt{n(\Delta+\delta+1)}$.

Proof. From Cauchy - Schwarz inequality,
$(\sum_{i=1}^{n} \lambda_i)^2 \leq n \sum_{i=1}^{n} \lambda_i^2 \leq n \left[\sum_{i=1}^{n} \lambda_i^2\right]$
$(SCE(H))^2 < n(\Delta+\delta+1)$
Therefore SEC(H)$< \sqrt{n(\Delta+\delta+1)} \ldots (1)$
Also, $(SCE(H))^2 = (\sum_{i=1}^{n} \lambda_i)^2 > \left[\sum_{i=1}^{n} \lambda_i^2\right] = \Delta+\delta+1$
SCE(H)$> \sqrt{\Delta+\delta+1}$
Hence ,$\sqrt{\Delta+\delta+1} < SCE(H) < \sqrt{n(\Delta+\delta+1)}$ By (1).

Illustration 2.12. *Consider a 3-uniform T_2 hypergraph with order 10.*
In H, SCE(H)= 7.2022, $\Delta+\delta+1 = 8$,
$\sqrt{\Delta+\delta+1} = 2.828 < SCE(H) = 7.2022 < \sqrt{n(\Delta+\delta+1)} = 8.94$
Hence, Theorem 2.11. is verified.

Theorem 2.13. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then*

$SCE(H) < \frac{\Delta+\delta+1}{n} + \sqrt{(n-1)(\Delta+\delta+1) - (\frac{\Delta+\delta+1}{n})^2}$.

Proof. By Cauchy - Schwarz inequality,

$(\sum_{i=2}^{n} \lambda_i)^2 \leq (n-1) \sum_{i=2}^{n} \lambda_i^2$

$(\sum_{i=1}^{n} \lambda_i - \lambda_1)^2 \leq (n-1)(\sum_{i=1}^{n} \lambda_i^2 - \lambda_1^2)$

$(SCE(H) - \lambda_1)^2 < (n-1)(\left\lceil \sum_{i=1}^{n} \lambda_i^2 \right\rceil - \lambda_1^2)$

$(SCE(H) - \lambda_1)^2 < (n-1)[(\Delta+\delta+1) - \lambda_1^2]$

$SCE(H) < \lambda_1 + \sqrt{(n-1)[(\Delta+\delta+1) - \lambda_1^2]}$

$Let S(t) = t + \sqrt{(n-1)[(\Delta+\delta+1) - t^2]}$

For decreasing function, S'(t)$\leq 0 \Rightarrow 1 - \frac{t(n-1)}{\sqrt{(n-1)(\Delta+\delta+1-t^2)}} \leq 0$

$\Rightarrow t \geq \sqrt{\frac{\Delta+\delta+1}{n}}$

Since $\Delta+\delta+1 \geq n$, we have $\sqrt{\frac{\Delta+\delta+1}{n}} \leq \frac{\Delta+\delta+1}{n} \leq \lambda_1$

Therefore $S(\lambda_1) \leq S(\frac{\Delta+\delta+1}{n})$

Hence, SCE(H)$\leq S(\lambda_1) < S(\frac{\Delta+\delta+1}{n})$

i.e.,SCE(H)$< S(\lambda_1) < S(\frac{\Delta+\delta+1}{n})$

i.e., $SCE(H) < \frac{\Delta+\delta+1}{n} + \sqrt{(n-1)(\Delta+\delta+1) - (\frac{\Delta+\delta+1}{n})^2}$.

Illustration 2.14. *Consider a 3-uniform T_2 hypergraph with order 10.*
In H, SCE(H)= 7.2022, $\Delta+\delta+1 = 8$,

$SCE(H) = 7.2022 < \frac{\Delta+\delta+1}{n} + \sqrt{(n-1)(\Delta+\delta+1) - (\frac{\Delta+\delta+1}{n})^2} = 8.94$

Hence, Theorem 2.13. is verified.

Result 2.16. Let H be a 3-uniform T_2 hypergraph with $n \geq 5$.
Then $\lambda_1 \geq \sqrt{\frac{n^2+9n+6}{46}}$. The sharp bound holds for n=10 in H.

Result 2.17. Let H be a 3-uniform T_2 hypergraph with $n \geq 5$.
Then $\lceil \lambda_1 \rceil = \left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil$.

Theorem 2.17. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then*
$SCE(H) < \left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil + \frac{(n-1)(\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil)^2}{(detSC(H))^{\frac{1}{n}}}$.

Proof. We have $\lceil \lambda_1 \rceil = \left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil > [detSC(H)]^{\frac{1}{n}}$

$\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil \sum_{i=2}^{n} \lambda_i > [detSC(H)]^{\frac{1}{n}} \sum_{i=2}^{n} \lambda_i$

Since $\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil > |\lambda_i| \, \forall i = 2, 3, ...n$

$(n-1)(\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil)^2 > (detSC(H))^{\frac{1}{n}}(SCE(H) - \lambda_1)$

$> (detSC(H))^{\frac{1}{n}}(SCE(H) - \lceil \lambda_1 \rceil)$

$\frac{(n-1)(\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil)^2}{(detSC(H))^{\frac{1}{n}}} > (SCE(H) - \lceil \lambda_1 \rceil)$

$\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil + \frac{(n-1)(\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil)^2}{(detSC(H))^{\frac{1}{n}}} > SCE(H).$

Illustration 2.18. *Consider a 3-uniform T_2 hypergraph with order 10.*

In H, SCE(H)= 7.20, $\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil = 3, (detSC(H))^{\frac{1}{n}} = 0.5611$

$\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil + \frac{(n-1)(\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil)^2}{(detSC(H))^{\frac{1}{n}}} = 3 + \frac{9\times(3)^2}{0.5611} = 147.36 > 7.20 = SCE(H)$

Hence, Theorem 2.17. is verified.

Theorem 2.19. *Let H be a 3-uniform T_2 hypergraph with $n \geq 5$. Then*

$n[detSC(H)]^{\frac{1}{n}} < SCE(H) < n\frac{\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil^2}{[detSC(H)]^{\frac{1}{n}}}.$

Proof. From an arithmetic and geometric mean inequality,

$\frac{\sum_{i=1}^{n} \lambda_i}{n} > (detSC(H))^{\frac{1}{n}}$

$\sum_{i=1}^{n} \lambda_i > n(detSC(H))^{\frac{1}{n}} \cdots (1)$

We have $\lceil \lambda_1 \rceil = \left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil > [detSC(H)]^{\frac{1}{n}}$

$\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil \sum_{i=1}^{n} \lambda_i > [detSC(H)]^{\frac{1}{n}} \sum_{i=1}^{n} \lambda_i$

Since $\lceil \lambda_1 \rceil > |\lambda_i| \, \forall i = 2, 3, ...n$

$n \left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil^2 > SCE(H)[detSC(H)]^{\frac{1}{n}}$

$n\frac{\left\lceil \sqrt{\frac{n^2+9n+6}{46}} \right\rceil^2}{[detSC(H)]^{\frac{1}{n}}} > SCE(H) \cdots (2)$

From (1) and (2) we obtain the inequality.

Theorem 2.20. *Let H be a 3- uniform T_2 hypergraph with $n \geq 6$. Then $SCE(H) \leq \frac{S}{n} + \sqrt{\frac{S}{n}} + \sqrt{(n-2)[\Delta + \delta + 1 - \frac{S}{n} - (\frac{S}{n}^2)]}$. Equality holds if $n = 6$ in H.*

Proof. By Cauchy - Schwarz inequality,

$(\sum_{i=2}^{n-1} \lambda_i)^2 \leq (n-2) \sum_{i=2}^{n-1} \lambda_i^2$

$(\sum_{i=1}^{n} \lambda_i - |\lambda_1| - |\lambda_n|)^2 \leq (n-2)[\sum_{i=1}^{n} \lambda_i^2 - \lambda_1^2 - \lambda_n^2]$

$\leq (n-2)[\left\lceil \sum_{i=1}^{n} \lambda_i^2 \right\rceil - \lambda_1^2 - \lambda_n^2]$

$SCE(H) \leq |\lambda_1| + |\lambda_n| + \sqrt{(n-2)[\Delta + \delta + 1 - \lambda_1^2 - \lambda_n^2]}$

Define $S(a,b) = a + b + \sqrt{(n-2)[\Delta + \delta + 1 - a^2 - b^2]}$...(1)

Where $a = \lambda_1$ and $b = \lambda_n$

Differentiate (1) with respect to a and b

$S_a = 1 - \frac{a(n-2)}{(n-2)(\Delta+\delta+1-a^2-b^2)}$

$S_b = 1 - \frac{b(n-2)}{(n-2)(\Delta+\delta+1-a^2-b^2)}$

$S_{aa} = -\frac{\sqrt{n-2(\Delta+\delta+1-b^2)}}{(\Delta+\delta+1-a^2-b^2)^{\frac{3}{2}}}$

$S_{bb} = -\frac{\sqrt{n-2(\Delta+\delta+1-a^2)}}{(\Delta+\delta+1-a^2-b^2)^{\frac{3}{2}}}$

$S_{ab} = -\frac{\sqrt{n-2}(ab)}{(\Delta+\delta+1-a^2-b^2)^{\frac{3}{2}}}$

Stationary points are given by $S_a = 0$ and $S_b = 0$

$S_a = 0 \Rightarrow 1 - \frac{a(n-2)}{(n-2)(\Delta+\delta+1-a^2-b^2)}$

$S_b = 0 \Rightarrow 1 - \frac{b(n-2)}{(n-2)(\Delta+\delta+1-a^2-b^2)}$

Then $a^2 + b^2(n-1) = \Delta + \delta + 1 \cdots (2)$

$b^2 + a^2(n-1) = \Delta + \delta + 1 \cdots (3)$

From (2) and (3)

$a = b = \sqrt{\frac{\Delta+\delta+1}{n}}$

At (a,b), $S_{aa} = S_{bb} = -\frac{\sqrt{n}(n-1)}{\sqrt{\Delta+\delta+1}(n-2)}$

$S_{ab} = -\frac{\sqrt{n}}{\sqrt{\Delta+\delta+1}(n-2)}$

T$=(S_{aa})(S_{bb}) - (S_{ab})^2 = \frac{n^2}{(\Delta+\delta+1)(n-2)} > 0$

$S(a,b) = S(\sqrt{\frac{\Delta+\delta+1}{n}}, \sqrt{\frac{\Delta+\delta+1}{n}}) = \sqrt{n(\Delta+\delta+1)}$

Also $S(a,b)$ decreases in the interval, $\sqrt{\frac{\Delta+\delta+1}{n}} < \frac{S}{n} < a = |\lambda_1|$

where $S = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{d_i+d_j}}$ $0 < b = |\lambda_n| < \sqrt{\frac{S}{n}} < \sqrt{\frac{\Delta+\delta+1}{n}}$.

Thus $S(|\lambda_1|, |\lambda_n|) \leq S(\frac{S}{n}, \sqrt{\frac{S}{n}}) \leq S(\sqrt{\frac{\Delta+\delta+1}{n}}, \sqrt{\frac{\Delta+\delta+1}{n}})$.

Hence, $SCE(H) \leq \frac{S}{n} + \sqrt{\frac{S}{n}} + \sqrt{(n-2)[\Delta+\delta+1-\frac{S}{n}-(\frac{S}{n}^2)]}$.

Illustration 2.21. *Consider a 3-uniform T_2 hypergraph with n=10.*

In H, $SCE(H) = 7.2022$, $S = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\sqrt{d_i+d_j}} = 20.2345, \Delta+\delta+1 = 8$

$SCE(H) = 7.2022 < 2.023 + 1.422 + \sqrt{8(8-2.0234-2.0234626)^2}$

$7.20234 < 7.3277$.

Hence, Theorem 2.20. is verified.

3. Conclusion

In this article, we studied sum connectivity matrix and its energy for a 3-uniform T_2 hypergraph. Also we found the bound of the sum connectivity energy of a 3-uniform T_2 hypergraph using various graph parameters. Among these bound we speculate,

$$SCE(H) < \frac{S}{n} + \sqrt{\frac{S}{n}} + \sqrt{(n-2)[\Delta+\delta+1-\frac{S}{n}-(\frac{S}{n}^2)]} < \sqrt{n(\Delta+\delta+1)}$$
$$= \frac{\Delta+\delta+1}{n} + \sqrt{(n-1)(\Delta+\delta+1)-(\frac{\Delta+\delta+1}{n})^2} < \sqrt{2\delta+\frac{3}{5}} + \sqrt{(n-2)(\Delta+\delta+1)}$$
$$< \sqrt{2(n-\alpha(H))(\Delta+\delta+1)} < \left\lceil\sqrt{\frac{n^2+9n+6}{46}}\right\rceil + \frac{(n-1)(\left\lceil\sqrt{\frac{n^2+9n+6}{46}}\right\rceil)^2}{(detSC(H))^{\frac{1}{n}}}.$$

Hence, $\frac{S}{n} + \sqrt{\frac{S}{n}} + \sqrt{(n-2)[\Delta+\delta+1-\frac{S}{n}-(\frac{S}{n}^2)]}$ yields the approximate sum connectivity energy of a 3-uniform T_2 hypergraph.

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