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$\alpha gs\gamma-$CONTINUOUS FUNCTIONS AND $\alpha gs-(\gamma_1, \gamma_2)-$CONTINUOUS FUNCTIONS

N. K. Narmadha and N. Balamani

Department of Mathematics,
Avinashilingam Institute for Home Science and
Higher Education for Women, Coimbatore - 641043, Tamil Nadu, INDIA

E-mail : narmadhapadma2@gmail.com, nbalamani77@gmail.com

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Abstract: In this paper, we introduce and define $\alpha gs\gamma-$continuous functions and $\alpha gs-(\gamma_1, \gamma_2)-$continuous functions. Also, we obtain interrelation for the defined functions and analyse their properties.

Keywords and Phrases: Operation on $\alpha gs-$open sets, $\alpha gs\gamma-$open sets, $\alpha gs\gamma-$continuous, $\alpha gs-(\gamma_1, \gamma_2)-$continuous.

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1. Introduction

A continuous function exhibits no abrupt shifts or discontinuities and produces a steady variation in the output when the input varies smoothly. This fundamental concept underpins various areas in topology and its applications. Topology provides foundational notions for areas of mathematics that require a robust understanding of continuity. The concept of continuity and open sets plays a pivotal role in topology and its applications. Extending these ideas through operation γ enriches the theoretical framework and offers insights into more generalized structures, which can be applied to diverse fields like fuzzy topology, decision theory, and computational models.

The study of operation approach in topological spaces began with Kasahara [11], followed by Ogata [18], who introduced and analysed $\gamma-$open sets in such spaces. Sanjay Tahiliani [20] further defined $\beta-\gamma-$open sets by applying the γ operation to $\beta-$open sets. Carpintero et al. [8] explored the γ operation on $b-$open sets, while Ibrahim [9] extended this notion to $\alpha_\gamma-$open sets, applying γ to $\alpha-$open sets. Asaad made significant contributions, by investigating the γ operation on P_s-open sets [5], the properties of $g-\gamma-$open sets [6]. Asaad and Ahmad [1] introduced the concept of semi-generalized open sets via γ operation. Nazihah Ahmad and Asaad [17] studied the properties of semi generalized open sets through γ operation. Jayashree and Sivakamasundari [10] introduced the κ operation on $gs-$open sets, while Asaad and Ameen [2] studied the γ operation on $g\alpha-$open sets. In supra topological spaces, Asaad et al. [4] defined supra$-\gamma-$open sets. Asaad et al. [3] introduced bioperators on soft topological spaces. Mershia Rabuni and Balamani [12] advanced the study with γ operation on $\alpha g-$open sets. Narmadha and Balamani [14, 15, 16] further expanded the scope by defining $\alpha gs\gamma-$open sets, studying their properties, and analysing the separation axioms. Mizyed [13] explored new types of continuity, including $\beta-\gamma-$continuous functions, $\beta-\gamma-c-$continuous functions and $\beta c-\gamma-$continuous functions.

The primary objective of this study is to introduce and characterize $\alpha gs\gamma-$continuous functions and $\alpha gs-(\gamma_1,\gamma_2)-$continuous functions. Furthermore, the study aims to establish new properties, provide examples to illustrate these concepts, and identify future avenues for extending this line of research. This study focuses on expanding the understanding of open and continuous functions under operation γ, particularly in the context of $\alpha gs\gamma-$open sets. It provides a unified approach to analysing continuity in topological spaces.

2. Preliminaries

Fundamental definitions and results relevant to the study are given in this section.

Definition 2.1. [19] *A subset A of (X,τ) is called α generalized semi closed ($\alpha gs-$closed) if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X,τ). The complement of an $\alpha gs-$closed set is called $\alpha gs-$open. The set of all $\alpha gs-$open sets in (X,τ) is denoted by $\tau_{\alpha gs}$.*

Definition 2.2. [14] *Let (X,τ) be a topological space. An operation $\gamma:\tau_{\alpha gs}\to P(X)$ is a mapping from $\tau_{\alpha gs}$ to $P(X)$ such that $V\subseteq\gamma(V)$ for all $V\in\tau_{\alpha gs}$, the value of V under the operation γ is denoted by $\gamma(V)$.*

Definition 2.3. [14] *A non-empty subset A of a space (X,τ) with an operation γ on $\tau_{\alpha gs}$ is called an $\alpha gs\gamma-$open set of (X,τ) if for all $x\in A$, there exists an $\alpha gs-$open set U containing x such that $\gamma(U)\subseteq A$. The set of all $\alpha gs\gamma-$open sets is denoted by $\tau_{\alpha gs\gamma}$. The complement of an $\alpha gs\gamma-$open set is called $\alpha gs\gamma-$closed.*

Definition 2.4. [13] *Let* (X,τ) *and* (Y,σ) *be two topological spaces with an operation* γ *on* $\beta O(X)$*. Then* $f:(X,\tau)\to(Y,\sigma)$ *is called* $\beta-\gamma-$*continuous if for each* $x\in X$ *and for each open set* V *of* Y *containing* $f(x)$*, there exists a* $\beta-\gamma-$*open set* U *of* X *containing* x *such that* $f(U)\subseteq V$*.*

Definition 2.5. [20] *A mapping* $f:(X,\tau)\to(Y,\sigma)$ *is said to be* $\beta-(\gamma,b)-$*continuous if for each* $x\in X$ *and each* $\beta-$*open set* V *containing* $f(x)$*, there exists a* $\beta-$*open set* U *such that* $x\in U$ *and* $f(U^\gamma)\subseteq V^b$*.*

Definition 2.6. [16] *A topological space* (X,τ) *with an operation* γ *on* $\tau_{\alpha gs}$ *is said to be:*

(*i*) *An* $\alpha gs\gamma-T_0$ *space if for any two distinct points* x*,* y *in* X*, there exists an* $\alpha gs-$*open set* U *such that* $x\in U$ *and* $y\notin\gamma(U)$ *or there exists an* $\alpha gs-$*open set* U *such that* $y\in U$ *and* $x\notin\gamma(U)$*.*

(*ii*) *An* $\alpha gs\gamma-T_1$ *space if for any two distinct points* x*,* y *in* X*, there exist an* $\alpha gs-$*open set* U *containing* x *and an* $\alpha gs-$*open set* V *containing* y *such that* $y\notin\gamma(U)$ *and* $x\notin\gamma(V)$*.*

(*iii*) *An* $\alpha gs\gamma-T_2$ *space if for any two distinct points* x*,* y *in* X*, there exist an* $\alpha gs-$*open set* U *containing* x *and an* $\alpha gs-$*open set* V *containing* y *such that* $\gamma(U)$ *and* $\gamma(V)$ *are disjoint.*

Definition 2.7. [7] *Let* (X,τ) *and* (Y,τ') *be two topological spaces and* γ *an operation on* τ*. Then a function* $f:(X,\tau)\to(Y,\tau')$ *is said to be* $\gamma-$*continuous at* x *if for each open set* V *containing* $f(x)$*, there exists a* $\gamma-$*open set* U *containing* x *such that* $f(U)\subset V$*.*

Proposition 2.8. [15] *Let* (X,τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$*. Then for any subset* A *of* X *is* $\alpha gs\gamma-$*open if and only if* $A=\alpha gs_\gamma Int(A)$*.*

Proposition 2.9. [14] *Let* $\gamma:\tau_{\alpha gs}\to P(X)$ *be an operation on* $\tau_{\alpha gs}$ *and* A *be subset of* X*. Then* A *is* $\alpha gs\gamma-$*closed if and only if* $A=\alpha gscl_\gamma(A)$*.*

3. $\alpha gs\gamma-$ Continuous Functions and $\alpha gs-(\gamma_1,\gamma_2)-$ Continuous Functions

In the section we define and discuss new functions namely $\alpha gs-(\gamma_1,\gamma_2)-$continuous function and $\alpha gs\gamma-$ continuous function.

Here, (X,τ_1) and (Y,τ_2) be topological spaces. Let $\gamma_1:\tau_{\alpha gs}(X)\to P(X)$ and $\gamma_2:\tau_{\alpha gs}(Y)\to P(Y)$ be operations.

Definition 3.1. *A function* $f:(X,\tau_1)\to(Y,\tau_2)$ *is an* $\alpha gs-(\gamma_1,\gamma_2)-$*continuous if for all* $x\in X$ *and for all* $\alpha gs-$*open set* V *containing* $f(x)$*, there exists an*

$\alpha gs-$open set U containing x such that $f(\gamma_1(U)) \subseteq \gamma_2(V)$.

Theorem 3.2. *If a function $f : (X, \tau_1) \to (Y, \tau_2)$ is $\alpha gs - (\gamma_1, \gamma_2)-$continuous then*

(i) *$f(\alpha gsCl_{\gamma_1}(A)) \subseteq \alpha gsCl_{\gamma_2}(f(A))$ holds for all subset A of X.*

(ii) *$f^{-1}(W)$ is an $\alpha gs\gamma_1-$closed set in (X, τ_1) for all $\alpha gs\gamma_2-$closed set W of (Y, τ_2).*

Proof.

(i) Consider an $\alpha gs-$open set V containing $f(x)$. From the assumption, there exists an $\alpha gs-$open set U containing x such that $f(\gamma_1(U)) \subseteq \gamma_2(V)$. Let $f(x) \in f(\alpha gsCl_{\gamma_1}(A))$. Then $x \in \alpha gsCl_{\gamma_1}(A)$. By the definition of $\alpha gsCl_{\gamma_1}(A)$, $\gamma_1(U) \cap A \neq \varphi$. From this $\varphi \neq f(\gamma_1(U) \cap A) \subseteq f(\gamma_1(U)) \cap f(A) \subseteq \gamma_2(V) \cap f(A)$. Hence, $f(x) \in \alpha gsCl_{\gamma_2}(f(A))$.

(ii) Let W be an $\alpha gs\gamma_2-$closed set in (Y, τ_2). By (i), $f(\alpha gsCl_{\gamma_1}(f^{-1}(W))) \subseteq \alpha gsCl_{\gamma_2}(f(f^{-1}(W))) \subseteq \alpha gsCl_{\gamma_2}(W) = W$. Hence $\alpha gsCl_{\gamma_1}(f^{-1}(W)) \subseteq f^{-1}(W)$. Since $f^{-1}(W) \subseteq \alpha gsCl_{\gamma_1}(f^{-1}(W))$. Thus

$$f^{-1}(W) = \alpha gsCl_{\gamma_1}(f^{-1}(W))$$

By Proposition 2.9, $f^{-1}(W)$ is an $\alpha gs\gamma_1-$closed set in (X, τ_1).

Theorem 3.3. *If a function $f : (X, \tau_1) \to (Y, \tau_2)$ is injective $\alpha gs - (\gamma_1, \gamma_2)-$continuous and the space (Y, τ_2) is an $\alpha gs\gamma_2 - T_2$ space, then the space (X, τ_1) is an $\alpha gs\gamma_1 - T_2$ space.*

Proof. Consider two distinct points x_1, x_2 in X. Since f is an injective map. Then there exists a pair of distinct points $f(x_1)$, $f(x_2)$ in Y. From the $\alpha gs\gamma_2 - T_2$ space there exist $\alpha gs-$open set U_1 containing $f(x_1)$ and $\alpha gs-$open set U_2 containing $f(x_2)$ such that $\gamma_2(U_1) \cap \gamma_2(U_2) = \varphi$. From the definition of an $\alpha gs - (\gamma_1, \gamma_2)-$continuous function there exist $\alpha gs-$open set V_1 containing x_1 and $\alpha gs-$open set V_2 containing x_2 such that $f(\gamma_1(V_1)) \subseteq \gamma_2(U_1)$ and $f(\gamma_1(V_2)) \subseteq \gamma_2(U_2)$. Hence, $\gamma_1(V_1) \cap \gamma_1(V_2) = \varphi$. Therefore (X, τ_1) is an $\alpha gs\gamma_1 - T_2$ space.

Theorem 3.4. *If a function $f : (X, \tau_1) \to (Y, \tau_2)$ is injective $\alpha gs - (\gamma_1, \gamma_2)-$continuous and the space (Y, τ_2) is an $\alpha gs\gamma_2 - T_1$ space, then the space (X, τ_1) is an $\alpha gs\gamma_1 - T_1$ space.*

Proof. Let x_1, x_2 be two distinct points in X. Since f is an injective map, there exist two distinct points $f(x_1)$, $f(x_2)$ in Y. By an $\alpha gs\gamma_2 - T_1$ space there exist an

$\alpha gs-$open set U_1 containing $f(x_1)$ and an $\alpha gs-$open set U_2 containing $f(x_2)$ such that $f(x_2)\notin\gamma_2(U_1)$ and $f(x_1)\notin\gamma_2(U_2)$. Since f is an $\alpha gs-(\gamma_1,\gamma_2)-$continuous function, there exists an $\alpha gs-$open set V_1 containing x_1 such that $f(\gamma_1(V_1))\subseteq\gamma_2(U_1)$ and there exists an $\alpha gs-$open set V_2 containing x_2 such that $f(\gamma_1(V_2))\subseteq\gamma_2(U_2)$. Hence (X,τ_1) is an $\alpha gs\gamma_1-T_1$ space.

Theorem 3.5. *If a function* $f:(X,\tau_1)\to(Y,\tau_2)$ *is injective* $\alpha gs-(\gamma_1,\gamma_2)-$*continuous and the space* (Y,τ_2) *is an* $\alpha gs\gamma_2-T_0$ *space, then the space* (X,τ_1) *is an* $\alpha gs\gamma_1-T_0$ *space.*

Proof. Consider two distinct points x_1, x_2 in X. Since f is an injective map, there exist two distinct points $f(x_1)$, $f(x_2)$ in Y. From the definition of $\alpha gs\gamma_2-T_0$, there exists an $\alpha gs-$open set U such that $f(x_1)\in U$ and $f(x_2)\notin\gamma_2(U)$ or $f(x_2)\in U$ and $f(x_1)\notin\gamma_2(U)$. Since f is an $\alpha gs-(\gamma_1,\gamma_2)-$continuous function, there exists an $\alpha gs-$open set V containing x_1 such that $f(\gamma_1(V))\subseteq\gamma_2(U)$. Hence (X,τ_1) is an $\alpha gs\gamma_1-T_0$ space.

Proposition 3.6. *Let* (X,τ_1), (Y,τ_2) *and* (Z,τ_3) *be topological spaces and* γ_1, γ_2 *and* γ_3 *be operations on* $\tau_{\alpha gs}(X)$, $\tau_{\alpha gs}(Y)$ *and* $\tau_{\alpha gs}(Z)$ *respectively. If* $f:(X,\tau_1)\to(Y,\tau_2)$ *is* $\alpha gs-(\gamma_1,\gamma_2)-$*continuous and* $g:(Y,\tau_2)\to(Z,\tau_3)$ *is* $\alpha gs-(\gamma_2,\gamma_3)-$*continuous then* $gof:(X,\tau_1)\to(Z,\tau_3)$ *is* $\alpha gs-(\gamma_1,\gamma_3)-$*continuous.*

Proof. Let $x\in X$ and V be $\alpha gs-$open set of Z containing $g(f(x))$. Since g is $\alpha gs-(\gamma_2,\gamma_3)-$continuous there exists an $\alpha gs-$open set U of Y containing $f(x)$ such that $g(\gamma_2(U))\subseteq\gamma_3(V)$. Similarly f is $\alpha gs-(\gamma_1,\gamma_2)-$continuous there exists an $\alpha gs-$open set R of X containing x such that $f(\gamma_1(R))\subseteq\gamma_2(U)$. Thus $g(f(\gamma_1(R)))\subseteq g(\gamma_2(U))\subseteq\gamma_3(V)$. Hence gof is $\alpha gs-(\gamma_1,\gamma_3)-$continuous.

Definition 3.7. *Let* (X,τ_1) *and* (Y,τ_2) *be topological spaces and* γ *be an operation on* $\tau_{\alpha gs}(X)$. *Then* $f:(X,\tau_1)\to(Y,\tau_2)$ *is called* $\alpha gs\gamma-$*continuous if for all* $x\in X$ *and for all open set* V *of* Y *containing* $f(x)$, *there exists an* $\alpha gs\gamma-$*open set* U *of* X *containing* x *such that* $f(U)\subseteq V$.

Remark 3.8. *An* $\alpha gs-(\gamma_1,\gamma_2)-$*continuous function and* $\alpha gs\gamma-$*continuous function are independent as seen below.*

Example 3.9. Let $X=Y=\{a,b\}$. Then $\tau_1=\tau_2=\{\varphi,\{a\},X\}$ and $\tau_{\alpha gs}(X)=\tau_{\alpha gs}(Y)=\{\varphi,\{a\},X\}$. An operation γ_1 and an operation γ_2 are defined as follows.

$$\gamma_1(A)=A\ \forall\ A\in\tau_{\alpha gs}(X) \text{ and } \gamma_2(B)=Y\ \forall\ B\in\tau_{\alpha gs}(Y)$$

A function $f:(X,\tau_1)\to(Y,\tau_2)$ is defined as follows.

$$f(x)=\begin{cases} a; & x=b \\ b; & x=a \end{cases} \quad \forall\ x\in X$$

Then $\tau_{\alpha gs\gamma_1} = \{\varphi, \{a\}, X\}$. Hence f is an $\alpha gs-(\gamma_1,\gamma_2)-$continuous function but not an $\alpha gs\gamma-$continuous function.

Example 3.10. Let $X = Y = \{a, b, c\}$. Then $\tau_1 = \tau_2 = \{\varphi, X\}$ and $\tau_{\alpha gs}(X) = \tau_{\alpha gs}(Y) = P(X)$. An operation γ_1 and an operation γ_2 are defined as follows.

$$\gamma_1(A) = \begin{cases} A & \text{if } A = \{a,b\} \text{ or } \{a,c\} \text{ or } \{b,c\} \\ X & \text{otherwise} \end{cases} \qquad \text{and } \gamma_2(B) = B$$

$\forall\ A \in \tau_{\alpha gs}(X)$ and $\forall\ B \in \tau_{\alpha gs}(Y)$ respectively. A function f is defined as follows.

$$f(x) = x\ \forall\ x \in X$$

Then $\tau_{\alpha gs\gamma_1} = \{\varphi, \{a,b\}, \{b,c\}, \{a,c\}, X\}$. Hence f is an $\alpha gs\gamma-$continuous function but not an $\alpha gs-(\gamma_1,\gamma_2)-$continuous function.

Theorem 3.11. *Let (X,τ_1) and (Y,τ_2) be topological spaces and γ be an operation on $\tau_{\alpha gs}(X)$. Then the function $f : (X,\tau_1) \to (Y,\tau_2)$ is $\alpha gs\gamma-$continuous iff the inverse image of every open set in Y is an $\alpha gs\gamma-$open set in X.*

Proof. Consider $x \in X$. Let V be an open set containing $f(x)$ in Y. Then $f^{-1}(V)$ is an $\alpha gs\gamma-$open set containing x in X and $f(f^{-1}(V)) \subseteq V$. Hence f is an $\alpha gs\gamma-$continuous function.

Conversely assume that f is an $\alpha gs\gamma-$continuous function. Let V be an open set containing $f(x)$ in Y. Then there exists an $\alpha gs\gamma-$open set U of X containing x such that $f(U) \subseteq V$. Hence $f^{-1}(V)$ is an $\alpha gs\gamma-$open set in X.

Proposition 3.12. *Every $\gamma-$continuous function is an $\alpha gs\gamma-$continuous function.*

Proof. Obvious.

Remark 3.13. *An $\alpha gs\gamma-$continuous function need not be $\gamma-$continuous as observed from the following example.*

Example 3.14. Let $X = \{a, b, c, d\} = Y$. Then $\tau_1 = \tau_2 = \{\varphi, \{a, b\}, X\}$ and $\tau_{\alpha gs}(X) = \tau_{\alpha gs}(Y) = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. An operation γ on $\tau_{\alpha gs}(X)$ is defined as follows.

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \text{ or } \{b\} \\ X & \text{otherwise} \end{cases} \qquad \forall\ A \in \tau_{\alpha gs}(X)$$

Thus $\tau_\gamma = \{\varphi, X\}$ and $\tau_{\alpha gs\gamma} = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. A function $f : (X,\tau_1) \to (Y,\tau_2)$ is defined as follows.

$$f(x) = x\ \forall\ x \in X$$

Hence f is an $\alpha gs\gamma-$continuous function but not $\gamma-$continuous.

Proposition 3.15. *Let* $f:(X,\tau_1)\to(Y,\tau_2)$ *be a function and* γ *be an operation on* $\tau_{\alpha gs}(X)$*. If for each open set* S *of* Y, $Int(f^{-1}(S))=\alpha gs_\gamma Int(f^{-1}(S))$ *then the following statements are equivalent.*

(i) f *is continuous.*

(ii) f *is* $\alpha gs\gamma-$*continuous.*

Proof. (i)⇒(ii) Assume that f is continuous. Let $x\in X$ and S be an open set containing $f(x)$ in Y. By the assumption, $f^{-1}(S)$ is an open set in X. Thus $f^{-1}(S)=Int(f^{-1}(S))=\alpha gs_\gamma Int(f^{-1}(S))$. By Proposition 2.8, $f^{-1}(S)$ is an $\alpha gs\gamma-$open set. Hence f is $\alpha gs\gamma-$continuous.

(ii)⇒(i) Let S be an open set in Y. By (ii) and Theorem 3.11, $f^{-1}(S)$ is an $\alpha gs\gamma-$open set in X. By Proposition 2.8, $f^{-1}(S)=\alpha gs_\gamma Int(f^{-1}(S))=Int(f^{-1}(S))$. Then $f^{-1}(S)$ is an open set in X. Hence f is continuous.

Remark 3.16. *Composition of two* $\alpha gs\gamma-$*continuous functions need not be* $\alpha gs\gamma-$*continuous as observed from the following example.*

Example 3.17. Let $X=Y=Z=\{a,b,c\}$ and $\tau_1=\tau_2=\tau_3=\{\varphi,\{a\},X\}$. Then $\tau_{\alpha gs}(X)=\tau_{\alpha gs}(Y)=\tau_{\alpha gs}(Z)=\{\varphi,\{a\},\{a,b\},\{a,c\},X\}$. An operation γ on $\tau_{\alpha gs}(X)$ is defined as follows.

$$\gamma(A)=\begin{cases} A & \text{if } b\notin A\\ cl(A) & \text{if } b\in A\end{cases}\qquad \forall A\in\tau_{\alpha gs}(X)$$

Thus $\tau_{\alpha gs\gamma}=\{\varphi,\{a\},\{a,c\},X\}$ in X. The functions $f:(X,\tau_1)\to(Y,\tau_2)$ and $g:(Y,\tau_2)\to(Z,\tau_3)$ are defined as follows.

$$f(x)=\begin{cases} a; & x=a\\ c; & x=b\\ b; & x=c\end{cases}\quad \forall\, x\in X \text{ and } g(x)=\begin{cases} a; & x=a \text{ and } x=c\\ b; & x=b\end{cases}\quad \forall\, x\in Y$$

Here f and g are $\alpha gs\gamma-$ continuous but $gof:(X,\tau_1)\to(Z,\tau_3)$ is not $\alpha gs\gamma-$continuous.

Proposition 3.18. *If a function* $f:(X,\tau_1)\to(Y,\tau_2)$ *is* $\alpha gs\gamma-$*continuous then for each point* $x\in X$ *and for each open set* V *containing* $f(x)$*, there exists an* $\alpha gs-$*open set* U *in* X *such that* $x\in U$ *and* $f(U)\subseteq V$*.*

Proof. Trivial.

Remark 3.19. *Converse of Proposition 3.18 need not be true as shown below.*

Example 3.20. Let $X = Y = \{a, b, c\}$ and $\tau_1 = \tau_2 = \{\varphi, \{a\}, X\}$. Then $\tau_{\alpha gs}(X) = \tau_{\alpha gs}(Y) = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. An operation γ on $\tau_{\alpha gs}(X)$ is defined as follows.

$$\gamma(A) = \begin{cases} A & \text{if } b \notin A \\ cl(A) & \text{if } b \in A \end{cases} \quad \forall\ A \in \tau_{\alpha gs}(X)$$

Thus $\tau_{\alpha gs\gamma} = \{\varphi, \{a\}, \{a, c\}, X\}$ in X. The function $f : (X, \tau_1) \to (Y, \tau_2)$ is defined as follows.

$$f(x) = \begin{cases} a; & x = a \text{ and } x = b \\ b; & x = c \end{cases} \quad \forall\ x \in X$$

Here for each point $x \in X$ and for each open set V containing $f(x)$, there exists an $\alpha gs-$open set U in X such that $x \in U$ and $f(U) \subseteq V$ but f is not an $\alpha gs\gamma-$continuous function.

4. Conclusion

This paper explores the concepts of $\alpha gs - (\gamma_1, \gamma_2)-$continuous functions and $\alpha gs\gamma-$continuous functions, aiming to enrich the theoretical framework of topology by examining their fundamental properties and interrelations. These functions serve to generalize and unify existing notions of continuity, providing a broader perspective on the behaviour of mappings under specific operations.

The findings of this study pave the way for future research to extend the analysis of bi-operations in various topological spaces. Potential areas of expansion include the application of these concepts to compactness, connectedness, and separation axioms, as well as their implications in more complex structures. This work also provides a foundation for future investigations into the broader applicability of $\alpha gs\gamma-$continuous functions and their variants in both theoretical and applied contexts.

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