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NANO WEAKLY $g\#$ IRRESOLUTE FUNCTIONS AND NANO WEAKLY $g\#$ HOMEOMORPHISMS IN NANO TOPOLOGICAL SPACES

P. Kalaivani and R. Nithyakala

Department of Mathematics,
Vidyasagar College of Arts and Science,
Udumalpet, Tirupur District, Tamil Nadu, INDIA

E-mail : kalaiudt6@gmail.com, nithyaeswar11@gmail.com

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Abstract: The purpose of this paper is to extend the study of $NWg\#$-closed and $NWg\#$-open sets using properties of sets. Also introduce the notion of Nano Weakly $g\#-(NWg\#)$ irresolute functions and Nano Weakly $g\#-(NWg\#)$ homeomorphisms in Nano topological spaces and studied some of their basic characterizations. Also define the new sets such as $NWg\#$ kernel and $NWg\#$ surface by using $NWg\#$- open sets and $NWg\#$ - closed sets. The basic characterizations with nano interior and nano closure are discussed. Also Nano infimum and Nano supremum of sets are introduced using greatest lower bound and least upper bound properties.

Keywords and Phrases: $NWg\#ker$, $NWg\#$ surf, $NWg\#$- continuous functions, $NWg\#$ - irresolute functions, $NWg\#$- homeomorphisms.

2020 Mathematics Subject Classification: 54C05.

1. Introduction

The topological spaces have many applications for different types of sets like fuzzy sets, nano sets, permutation sets and soft sets. In 2013 Lellis Thivagar. M and Carmel Richard [7] established Nano topology and introduced Nano continuity, Nano irresolute, Nano open mappings and Nano homeomorphism. The term irresoluteness was introduced by Crossley S.G and Hildebrand S.K [3] in 1972.

Sulochanadevi. P and Bhuvaneswari. K [9] introduced nano regular generalized irresolute maps in nano topological spaces. In 2016 Bhuvaneswari. K and Ezhilarasi. A [2] revealed the idea of nano semi generalized homeomorphisms in nano topological spaces.

This research introduce some terminologies of the sets called $NWg\#$ kernel and $NWg\#$ surface of the sets using $NWg\#$ - open sets and $NWg\#$ - closed sets in Nano topological spaces. Nano infimum and Nano supremum of sets are introduced based on the set inclusion principles and are defined using the concepts of greatest lower bound and least upper bound properties. The Nano Hasse diagram for the $NWg\#$- closed sets and $NWg\#$- open sets have been constructed by utilizing the set inclusion principles and examining the distinctive features of the new sets. The $NWg\#$ - irresolute functions and $NWg\#$- homeomorphism in nano topological spaces are introduced and investigate their relationship with other existing Nano homeomorphisms.

Symbols	Abbreviations
$NWg\#$ - closed (open)	Nano Weakly $g\#$ - closed set (open set)
$NWg\#ker(A)$	Nano Weakly $g\#$ kernel of a set A
$NWg\#surf(A)$	Nano Weakly $g\#$ surface of a set A
$NWg\#inf(A)$	Nano Weakly $g\#$ infimum of a set A
$NWg\#sup(A)$	Nano Weakly $g\#$ supremum of a set A

2. Preliminaries

Definition 2.1. [7] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space. Let $X \subseteq U$.*
1. The Lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is

$$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\},$$

where $R(X)$ denotes the equivalence class determined by X.
2. The Upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}.$$

3. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X) - L_R(X)$.

Example 2.2. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{c\}, \{b, d\}\}$ let $X = \{a, d\} \subseteq U$. Now

U	$R(x) = U/R$	$L_R(X) = R(x) \subseteq X$	$U_R(X) = R(x) \cap X \neq \phi$
$\{a\}$	$\{a\}$	$\surd$	$\surd$
$\{b\}$	$\{b, d\}$	$\times$	$\surd$
$\{c\}$	$\{c\}$	$\times$	$\times$
$\{d\}$	$\{b, d\}$	$\times$	$\surd$

From the above table,
1. The Lower approximation of X with respect to R is $L_R(X) = \{a\}$.
2. The Upper approximation of X with respect to R is $U_R(X) = \{a, b, d\}$.
3. The boundary region of X with respect to R is $B_R(X) = \{b, d\}$.

Definition 2.3. [7] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$.*
Then $\tau_R(X)$ satisfies the following axioms:
1. U and $\Phi \in \tau_R(X)$,
2. The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
$\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ is the Nano Topological Space.

Example 2.4. From example 2.2 The Nano topology on U with respect to X is $\tau_R(X) = \{U, \Phi, \{a\}, \{a, b, d\}, \{b, d\}\}$.

Definition 2.5. [7] *$(U, \tau_R(X))$ is a Nano Topological Space with respect to X and if $A \subseteq U$, then*

- *The Nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $Nint(A)$, which is the largest nano open subset of A.*
- *The Nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$, which is the smallest nano closed set containing A.*

Definition 2.6. [7] *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano Topological Spaces. A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano continuous if the inverse image* *of every Nano open set in $(V, \tau_{R'}(Y))$ is Nano open in $(U, \tau_R(X))$.*

Definition 2.7. [5] *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano Topological Spaces. A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano Weakly g#- continuous (NWg# - continuous) if the inverse image of every nano open set in $(V, \tau_{R'}(Y))$ is NWg#-open in $(U, \tau_R(X))$.*

Definition 2.8. [7] *Let $(U, \tau_R(X))$ be a Nano Topological Space and $A \subseteq U$. The set Nano kernel of A is defined by $Nker(A) = \bigcap\{U : A \subseteq U, U \in \tau_R(X)\}$. It is denoted by $Nker(A)$.*

Definition 2.9. [6] *A relation R on a set A is called a partially ordered set or poset if the relation R is reflexive, antisymmetric and transitive.*

Definition 2.10. [6] *A simplified form of the digraph of a partial ordering relation on a finite set that contains the information of the relation is called a Hasse diagram.*

Definition 2.11. [6] *Let A be subset of a partially ordered set $(P, \leq)$ and if y is an element of P such that $a \leq y$ for all elements $a \in A$, then y is called an upper bound of A.*

Definition 2.12. [6] *Let A be subset of a partially ordered set $(P, \leq)$ and if x is an element of P such that $x \leq a$ for all elements $a \in A$, then x is called the lower bound of A.*

Definition 2.13. [6] *The element l is called the least upper bound (LUB) or Supremum of the subset A of a poset $(P, \leq)$ if l is an upper bound that is less than every other upper bound of A.*

Definition 2.14. [6] *The element g is called the greatest lower bound (GLB) or Infimum of the subset A of a poset $(P, \leq)$ if g is a lower bound that is greater than every other Lower bound of A.*

Definition 2.15. [5] *A Nano Topological Space $(U, \tau_R(X))$ is NWg# - compact if every NWg# open cover of $(U, \tau_R(X))$ has a finite sub cover.*

Definition 2.16. [5] *A Nano Topological Space $(U, \tau_R(X))$ is NWg# - connected if it cannot be expressed as the disjoint union of two nonempty NWg#- open sets.*

3. Nano Weakly g# Kernel and Nano Weakly g# Surface of a set

In this section we define and study the properties of Nano Weakly g# kernel and Nano Weakly g# surface of a set in Nano Topological Spaces.

Definition 3.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The Nano hasse diagram of the nano topological space is a digraph that contains the relation between its nano subsets.*

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then the Nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ and $\tau_R^c(X) = \{U, \phi, \{b, c, d\}, \{a, b\}, \{b\}\}$.

$NWg\#$ - open sets are $\{U, \phi, \{a\}, \{d\}, \{c\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}\}$.

$NWg\#$ - closed sets are $\{U, \phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.

Nano Hasse Diagram of $NWg\#$ - closed sets and $NWg\#$ - open sets

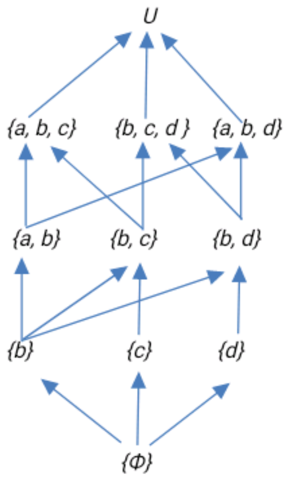

Figure 3.1

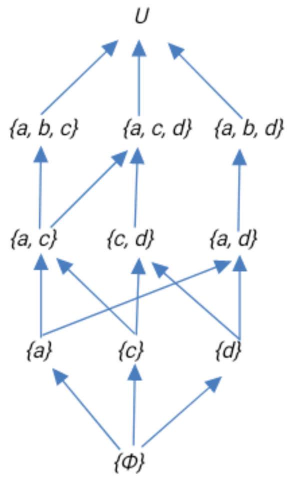

Figure 3.2

Definition 3.3. *Let $(U, \tau_R(X))$ be a nano topological space. The set $NWg\#$ kernel of A is defined as the intersection of all $NWg\#$ - open sets of U containing A and it is denoted by $NWg\#ker(A)$. $NWg\#ker(A) = \bigcap\{B \mid A \subseteq B : B \text{ is } NWg\# - open\}$.*

Definition 3.4. *Let $(U, \tau_R(X))$ be a Nano topological space. The set $NWg\#$ surface of A is defined as the union of all $NWg\#$- closed sets of U contained in A and it is denoted by $NWg\#surf(A)$.*

$$NWg\#surf(A) = \cup\{B \mid B \subseteq A; B \text{ is } NWg\# \text{ closed}\}.$$

From Figure 3.1, $NWg\#surf(b) = \{b\} \cup \{\phi\} = \{b\}$.
From Figure 3.2, $NWg\#ker(c) = \{c\}$.
$NWg\#surf(a) = \{\phi\}$.
$NWg\#surf(c) = \{c\}$.
$NWg\#surf(a, b, c) = \{a, b, c\}$.
$NWg\#surf(a, b) = \{a, b\}$.
$NWg\#ker(a, d) = \{a, d\}$.
$NWg\#ker(b, c, d) = U$.
$NWg\#ker(b) = U$.
$NWg\#ker(a, b, d) = \{a, b, d\}$.

Remark 3.5. *The $NWg\#ker(A)$ is the smallest $NWg\#$ open set that contains A and the set $NWg\#surf(A)$ is the largest $NWg\#$- closed subset of A.*

Remark 3.6. *The subsets which are not in the hasse diagram have $NWg\#surf$ as $\{\phi\}$ and $NWg\#ker$ as U.*

Theorem 3.7. *Let A and B be the subsets of $(U, \tau_R(X))$ then*
(i) $NWg\#ker(U) = U$ and $NWg\#ker(\phi) = \phi$
(ii) $A \subseteq NWg\#ker(A)$
(iii) If $A \subseteq B$, B is $NWg\#$ open set then $NWg\#ker(A) \subseteq B$
(iv) If $A \subseteq B$ then $NWg\#ker(A) \subseteq NWg\#ker(B)$
Proof. (i) Since U is the only $NWg\#$ - open set containing U, $NWg\#ker(U) = U$. Also ϕ is the only $NWg\#$- open set containing ϕ. Hence $NWg\#ker(\phi) = \phi$.
(ii) From definition 3.3 the proof is obvious.
(iii) Assume that $A \subseteq B$, B is $NWg\#$ - open. $NWg\#ker(A) = \cap\{B : B$ is $NWg\#-$ open and $A \subseteq B\}$. Also $NWg\#ker(A) \subseteq \{NWg\# - openset \supset A.\}$ In particular, the set B. Hence $NWg\#ker(A) \subseteq B$.
(iv)Let $A \subseteq B$ then by definition 3.3. $NWg\#ker(B) = \cap\{H : H\ isNWg\#$ - open and $B \subseteq H\}$. Now H is $NWg\#$ - open and $B \subseteq H$, by (iii) $NWg\#ker(B) \subseteq H$. Since $A \subseteq B,\ A \subseteq B \subseteq H$, which is $NWg\#$- open set $\Rightarrow NWg\#ker(A) \subseteq H$. Also $NWg\#ker(A) \subseteq \cap\{H : H\ is\ NWg\#$- open and $B \subseteq H\} = NWg\#ker(B)$. Hence $NWg\#ker(A) \subseteq NWg\#ker(B)$.

Theorem 3.8. *If a subset A of $(U, \tau_R(X))$ is $NWg\#$- open then $NWg\#ker(A) = A$ but not conversely.*
Proof. Let A be a $NWg\#$-open subset of U. By Theorem 3.7, $A \subseteq NWg\#ker(A)$. Also A is $NWg\#$- open set contained in A which implies $NWg\#ker(A) \subseteq A$. Hence $NWg\#ker(A) = A$. The converse of the above theorem is not true as seen from the following example.

Example 3.9. In example 3.2 Let $A = \{a, b\}$ then $NWg\#ker(\{a, b\}) = \{a, b\}$ but $\{a, b\}$ is not $NWg\#$- open set in U.

Remark 3.10. *G is any subset of $(U, \tau_R(X))$ then*
(i)$NWg\#int(G) \subseteq NWg\#\ker(G)$
(ii)$NWg\#surf(G) \subseteq NWg\#cl(G)$
From Example 3.2 $NWg\#int(\{a, b\}) = \{a\} \subseteq NWg\#\ker(\{a, b\}) = \{a, b\}$
$NWg\#surf(\{a, c\}) = \{c\} \subseteq NWg\#cl(\{a, c\}) = \{a, b, c\}$.

Theorem 3.11. *Let A and B be the subsets of $(U, \tau_R(X))$. Then*
(i) $NWg\#surf(U) = U$ and $NWg\#surf(\phi) = \phi$
(ii) $NWg\#surf(A) \subseteq A$.

(iii) If B is any $NWg\#$ closed set contained in A then $B \subseteq NWg\#surf(A)$
(iv) If $A \subseteq B$ then $NWg\#surf(A) \subseteq NWg\#surf(B)$
Proof. (i) U is the only $NWg\#$ - closed set contained in U. By definition 3.4 $NWg\#\text{surf}(U) = U$. Similarly, ϕ is the only $NWg\#$- closed set contained in ϕ. Hence $NWg\#\text{surf}(\phi) = \phi$.
(ii) The proof is obvious.
(iii) Let A be subset of $(U, \tau_R(X))$. $B \subseteq A$, B is $NWg\#$- closed set. Since $NWg\#surf(A)$ is the union of all $NWg\#$- closed sets contained in A, $NWg\# surf(A)$ is contains every $NWg\#$ - closed set contained in A. In particular, the set B. Hence $B \subseteq NWg\# \ surf(A)$.
(iv) Let A and B be the subsets of $(U, \tau_R(X))$ such that $A \subseteq B$. By definition 3.4 $NWg\#surf(A) = \cup\{H : H \subseteq A : H \text{ is } NWg\#closed\}$. Since A is a subset of B, $NWg\#surf(A) \subseteq \cup\{H : H \text{ is } NWg\#closedandH \subseteq B\} = NWg\#surf(B)$.
Hence $NWg\#surf(A) \subseteq NWg\#surf(B)$.

Theorem 3.12. *If a subset A of $(U, \tau_R(X))$ is $NWg\#$- closed then $NWg\# surf(A) = A$ but not conversely.*
Proof. Let A be a $NWg\#$- closed subset of $(U, \tau_R(X))$. Then $NWg\#surf(A) \subseteq A$. Also A is $NWg\#$- closed set contained in A. By Theorem 3.10, $A \subseteq NWg\# surf(A)$. Hence $NWg\#surf(A) = A$.
The converse of the above theorem is not true is seen from the following example.

Example 3.13. In example 3.2 $surf(\{c, d\}) = \{c, d\}$ but it is not $NWg\#$ - closed in $(U, \tau_R(X))$.

4. Nano Infimum and Nano Supremum of subsets

This section defines the Nano infimum and Nano supremum of two subsets in nano topological spaces.

Definition 4.1. *The Nano supremum of a set A or the least upper bound of A (LUB) is the smallest $NWg\#$-open set that contains A. The Nano infimum of a set A or the greatest lower bound (GLB) is the largest $NWg\#$-closed subset of A.*

Definition 4.2. *If A and B are any two subsets of the nano topological space $(U, \tau_R(X))$. Then the Nano infimum of A and B is defined as the greatest lower bound of A and B. The Nano infimum is the largest $NWg\#$ -closed subset of A and B.*
$Ninf(A, B) = GLB(A, B) = \{largest\ NWg\# - closed\ subset\ of\ A\ and\ B)\}$.

Definition 4.3. *If A and B are any two subsets of the nano topological space $(U, \tau_R(X))$. The Nano supremum of A and B is defined as the least upper bound*

of A and B. The Nano supremum is the smallest $NWg\#$ -open set that contains both A and B.
$Nsup(A, B) = LUB(A, B) = \{Smallest NWg\#-open\ set\ that\ contains\ both\ A\ and\ B\}$.

Example 4.4. Let $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ *and* $X = \{a, b\}$ *then the Nano topology*
$\tau_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$ *and* $\tau_{R^c}(X) = \{U, \phi, \{b, c, d\}, \{a, c\}, \{c\}\}$.
$NWg\#-$*closed sets are* $\{U, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.
$NWg\#-$*open sets are* $\{\phi, U, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}, \{a\}\}$.

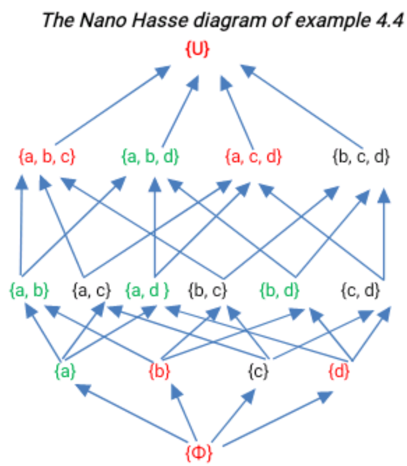

Figure 4.1

$\{a\}, \{a, d\}), \{a, b\}, \{b, d\}$ and $\{a, b, d\}$ are $NWg\#$- open sets,
$\{c\}, \{a, c\}, \{b, c\}, \{c, d\}$ and $\{b, c, d\}$ are $NWg\#$-closed sets and
$\{\Phi\}, \{b\}, \{d\}, \{a, b, c\}, \{a, c, d\}$ and U are both $NWg\#$-open and $NWg\#$-closed sets. From Figure 4.1,

$Ninf(\{b, c\}, \{b, c, d\}) = GLB(\{b, c\}, \{b, c, d\}) = \{b, c\}$ $Nsup(\{a\}, \{a, d\}) = LUB(\{a\}, \{a, d\}) = \{a, d\}$

$Ninf(\{a, b\}, \{a, b, d\}) = \{b\}$ $Nsup(\{c\}, \{a, b, c\}) = \{a, b, c\}$

$Ninf(\{a, b\}\{b, c\},) = \{\Phi\}$ $Nsup(\{b, c\}, \{b, d\}) = U$

$Ninf(\{a, c, d\}, \{b, c, d\}) = \{c\}$ $Nsup(\{c\}, \{a, b, c\}) = \{a, b, c\}$

From the above Hasse diagram the GLB of an open set and a closed set is (Φ). The LUB of an open set and a closed set is U.

5. On Nano Weakly $g\#$ - Irresolute Functions in Nano Topological Spaces

In this section a new form of irresolute functions called Nano Weakly $g\#$ - irresolute functions in nano topological spaces is introduced and some of their properties are analyzed.

Definition 5.1. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y)))$ is Nano Weakly $g^{\#}$ -irresolute $NWg^{\#}$ -irresolute) if the inverse image of every $NWg^{\#}$ -open set in $(V, \tau_{R'}(Y))$ is$NWg\#$-open set in $(U, \tau_R(X))$.*

Example 5.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, c\}$ then the nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then the nano topology $\tau_{R'}(Y) = \{V, \phi, \{y\}, \{x, z\}, \{x, y, z\}\}$.
Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = y$, $f(b) = x$, $f(c) = z$, and $f(d) = w$. Then $f^{-1}(y) = \{a\}$, $f^{-1}(x, z) = \{b, c\}$, and $f^{-1}(x, y, z) = \{a, b, c\}$. Thus the inverse image of every $NWg\#$ -open set in $(V, \tau_{R'}(Y))$ is $NWg\#$-open in $(U, \tau_R(X))$. Hence f is $NWg\#$-irresolute map.

Theorem 5.3. *If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ are two $NWg^{\#}$- irresolute functions then their composition is also $NWg\#$ - irresolute.*

Proof. Let $(U, \tau_R(X))$, $(V, \tau_{R'}(Y))$, and $(W, \tau_{R''}(Z))$ be any three nano topological spaces and let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be two $NWg\#$-irresolute functions. The composition of two functions $(g \circ f) : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$. Assume that H is $NWg\#$ -open set in $(W, \tau_{R''}(Z))$. Since g is $NWg\#$-irresolute, $g^{-1}(H)$ is $NWg\#$-open in $(V, \tau_{R'}(Y))$. Also f is $NWg\#$ -irresolute, $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$ is $NWg\#$-open in $(U, \tau_R(X))$. Then for each $NWg\#$-open set H in $(W, \tau_{R''}(Z))$, $(g \circ f)^{-1}(H)$ is $NWg\#$-open in $(U, \tau_R(X))$. Hence $(g \circ f)$ is $NWg\#$-irresolute in nano topological spaces.

Theorem 5.4. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ and $g : (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ be such that $g \circ f : (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is $NWg\#$-closed function.*
(i) If f is nano continuous and injective then g is $NWg\#$-closed.
(ii) If g is $NWg\#$-irresolute and injective then f is $NWg\#$-closed.

Proof. (i) Assume that $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is nano continuous and injective. Let $g : (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ be such that $g \circ f : (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is $NWg\#$-closed function. Let A be a nano closed subset of $(V, \tau'_R(Y))$. Since f is nano continuous, $f^{-1}(A)$ is nano closed in $(U, \tau_R(X))$. Since $g \circ f$ is $NWg\#$ -closed

and f is injective, $(g \circ f)(f^{-1}(A)) = g(A)$ is $NWg\#$-closed in $(W, \tau''_R(Z))$. Hence the map g is $NWg\#$-closed.
(ii) Assume that $g : (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is $NWg\#$-irresolute and injective. Let H be a nano closed set of $(U, \tau_R(X))$. Then $U - H$ is nano open set of $(U, \tau_R(X))$. Since $g \circ f$ is $NWg\#$-closed, $(g \circ f)(H)$ is $NWg\#$-closed in $(W, \tau''_R(Z))$. $(W - (g \circ f)(H))$ is $NWg\#$-open in $(W, \tau''_R(Z))$. Since g is $NWg\#$-irresolute, $g^{-1}(W - (g \circ f)(H)) = V - g^{-1}(g(f(H))) = V - f(H)$ is $NWg\#$-open in $(V, \tau'_R(Y))$. Therefore $f(H)$ is $NWg\#$-closed in $(V, \tau'_R(Y))$. Thus for every nano closed set H of $(U, \tau_R(X)), f(H)\ is\ NWg\# - closedin\ (V, \tau'_R(Y))$. Hence f is $NWg\#$ -closed function in nano topological spaces.

Theorem 5.5. *If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $NWg\#$- irresolute then it is $NWg\#$- continuous but not conversely.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be $NWg\#$-irresolute and let G be a nano open set in $(V, \tau_{R'}(Y))$. Since every nano open set is $NWg\#$ -open, G is $NWg\#$ -open in $(V, \tau'_R(Y))$. Also f is $NWg\#$- irresolute $f^{-1}(G)$ is $NWg\#$ - open in $(U, \tau_R(X))$. Thus the inverse image of every nano open set in $(V, \tau'_R(Y))$ is $NWg\#$ - open in $(U, \tau_R(X))$. Hence f is $NWg\#$ - continuous.

The contrary of the preceding theorem is not true as evidenced by the following example.

Example 5.6. In example 5.2 define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = x,\ f(b) = y,\ f(c) = z,\ f(d) = w$. The map f is $NWg\#$ -continuous. Now $f^{-1}(y, z, w) = \{b, c, d\}$ which is not $NWg\#$ -open in $(U, \tau_R(X))$. Hence f is not $NWg\#$ -irresolute.

Theorem 5.7. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a bijective function such that the image of every nano αg- open set in $(U, \tau_R(X))$ is nano αg -open set in $(V, \tau'_R(Y))$ and $NWg\#$- continuous, then f is $NWg\#$-irresolute.*

Proof. Assume that$f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is bijective function such that the image of every nano ag-open set in $(U, \tau_R(X))$ is nano ag- open set in$(V, \tau'_R(Y))$ and $NWg\#$-continuous. Let G be a $NWg\#$- closed set in $(V, \tau'_R(Y))$. Let $f^{-1}(G) \subseteq K$, where K is nano αg - open set in $(U, \tau_R(X))$ which implies $G \subseteq f(K)$. By assumption $f(K)$ is nano αg- open set in $(V, \tau'_R(Y))$. Since G is $NWg\#$- closed in $(V, \tau'_R(Y))$, $Ncl(Nint(G)) \subseteq f(K) \Rightarrow f^{-1}(Ncl(Nint(G))) \subseteq K$. Since f is $NWg\#$- continuous and $Ncl(Nint(G))$ is nano closed in $(V, \tau'_R(Y))$,
$f^{-1}(Ncl(Nint(G)))$ is $NWg\#$- closed in $(U, \tau_R(X))$. Since $f^{-1}(Ncl(Nint(G))) \subseteq K$ and $f^{-1}(Ncl(Nint(G)))$ is $NWg\#$- closed, $Ncl(Nint(f^{-1}(Ncl(Nint(G))))) \subseteq K\ implies$
$Ncl(Nint(f^{-1}(G))) \subseteq K \Rightarrow f^{-1}(G)$ is $NWg\#$- closed in $(U, \tau_R(X))$. Hence f is

$NWg\#$- irresolute.

Theorem 5.8. *If $f : (U, \tau_R(X)) \longrightarrow (V, \tau'_R(Y))$ is $NWg\#$ - irresolute and $g : (V, \tau'_R(Y)) \longrightarrow (W, \tau''_R(Z))$ is $NWg\#$ - continuous then $g \circ f : (U, \tau_R(X)) \longrightarrow (W, \tau''_R(Z))$ is $NWg\#$ - continuous.*

Proof. Let $f : (U, \tau_R(X)) \longrightarrow (V, \tau'_R(Y))$ be $NWg\#$ - irresolute and $g : (V, \tau'_R(Y)) \longrightarrow (W, \tau''_R(Z))$ is $NWg\#$ - continuous. Let A be a nano open set in $(W, \tau''_R(Z))$. g is $NWg\#$ - continuous, for each open set A in $(W, \tau''_R(Z)), g^{-1}(A)$ is $NWg\#$ - open in $(V, \tau'_R(Y))$. Since f is $NWg\#$ - irresolute, $f^{-1}(g^{-1}(A))$ is $NWg\#$ - open in $(U, \tau_R(X))$. Thus $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $NWg\#$ - open in $(U, \tau_R(X))$. Therefore, $g \circ f$ is $NWg\#$ - continuous.

Theorem 5.9. *If $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is a surjective, $NWg\#$- irresolute function. If $(U, \tau_R(X))$ is $NWg\#$- compact then $(V, \tau'_R(Y))$is $NWg\#$- compact.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be surjective and $NWg\#$- irresolute. By theorem 5.5. f is $NWg\#$- continuous. The function f is surjective, $NWg\#$-continuous function if $(U, \tau_R(X))$ is $NWg\#$ compact then $(V, \tau'_R(Y))$ is nano compact. Every nano compact set is $NWg\#$- compact. Hence $(V, \tau'_R(Y))$ is $NWg\#$-compact in nano topological spaces.

Theorem 5.10. *If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is surjective, $NWg\#$ - irresolute and $(U, \tau_R(X))$ is $NWg\#$ -connected then $(V, \tau_{R'}(Y))$ is $NWg\#$ - connected.*

Proof. *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be surjective, $NWg\#$ - irresolute and $(U, \tau_R(X))$ is $NWg\#$- connected. Suppose $(V, \tau_{R'}(Y))$ is not $NWg\#$- connected then $V = A \cup B$ and $A \cap B = \{\phi\}$where* A and B are $NWg\#$ open sets in $(V, \tau_{R'}(Y))$. Since f is $NWg\#$ - irresolute and surjection, $U = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty $NWg\#$- open subsets of $(U, \tau_R(X))$. This contradicts the fact that $(U, \tau_R(X))$ is $NWg\#$ - connected. Therefore $(V, \tau_{R'}(Y))$ is $NWg\#$ - connected.

6. Nano Weakly $g\#-$ Homeomorphism in Nano Topological Spaces

In this section we define and study the concept of Nano Weakly $g\#$-homeomorphism in nano topological spaces and obtain some of its properties.

Definition 6.1. *The function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be Nano Weakly g# - homeomorphism ($NWg\#$- homeomorphism) if*
(i) f is bijective
(ii) f is $NWg\#$ -continuous
(iii) f is $NWg\#$ - open map

Example 6.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, c\}$ then the Nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V =$

$\{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$. Then the nano topology $\tau_{R'}(Y) = \{V, \phi, \{y\}, \{x, z\}, \{x, y, z\}\}$. Define the bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = y, f(b) = x, f(c) = z,$ and $f(d) = w$. Now $f(a) = y, f(\{b, c\}) = \{x, z\}, f(\{a, b, c\}) = \{x, y, z\}$. Thus f is $NWg\#$- open map. Then $f^{-1}(y) = a, f^{-1}(\{x, z\}) = \{b, c\}$ and $f^{-1}(\{x, y, z\}) = \{a, b, c\}$. Thus f is $NWg\#$- continuous. Hence f is $NWg\#$ - homeomorphism in Nano Topological Spaces.

Theorem 6.3. *Every Nano homeomorphism is a $NWg\#$- homeomorphism but not conversely.*

Proof. Let$f : (U, \tau_R(X)) \to (V, \tau_R(Y))$ be a nano homeomorphism. The mapping f is bijective, nano continuous and nano open map. Every nano continuous function is $NWg\#$- continuous function and every nano open map is $NWg\#$- open map. Therefore, f is $NWg\#$- continuous, $NWg\#$ - open map and bijective. Hence f is $NWg\#$ - homeomorphism.

The contrary of the preceding theorem is not true as evidenced by the following example.

Example 6.4. In example 6.2. Define the bijective function $f : (U, \tau_R(X)) \longrightarrow (V, \tau_R'(Y))$ by $f(a) = \{x\}, f(b) = \{z\}, f(c) = \{y\},$ and $f(d) = \{w\}$. Then $f(a) = \{x\}, f(\{b, c\}) = \{y, z\}$,
$f(\{a, b, c\}) = \{x, y, z\}$ are $NWg\#$- open set in $(V, \tau_R'(Y))$. Then f is $NWg\#$ homeomorphism. The function f is not nano open map since $f(a) = \{x\}$ is not nano open in $(V, \tau_R'(Y))$. The map f is not nano continuous since $f^{-1}(\{y\}) = \{c\}$ is not nano open in $(U, \tau_R(X))$. Hence f is not nano homeomorphism.

Theorem 6.5. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then the following statements are true.*
(i)Every $N\alpha$ - homeomorphism is $NWg\#$ - homeomorphism but not conversely.
(ii)Every Nr - homeomorphism is $NWg\#$ - homeomorphism but not conversely.
(iii)Every $Ng\alpha$ - homeomorphism is $NWg\#$ - homeomorphism but not conversely.
(iv)Every $N\alpha g$- homeomorphism is $NWg\#$ - homeomorphism but not conversely.

Proof. The proof is similar to Theorem 6.3.

The contrary of the preceding theorem is not true as evidenced by the following examples.

Example 6.6. In example 6.4 the function f is $NWg\#$ - homeomorphism. The function f is not nano α - homeomorphism since $f(a) = \{x\}$ is not nano α - open in $(V, \tau_{R'}(Y))$ for the nano open set $\{a\}$ in $(U, \tau_R(X))$. Also f is not nano α - continuous since $f^{-1}(y) = \{c\}$ is not nano α - open set in $(U, \tau_R(X))$ for the nano open set $\{y\}$ in $(V, \tau_{R'}(Y))$.

Example 6.7. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, c\}$ then the nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R'(Y) = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then $\tau_{R'}(Y) = \{V, \Phi, \{y\}, \{x, z\}, \{x, y, z\}\}$. Define the bijective function
$f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = y, f(b) = x, f(c) = z$ and $f(d) = w$. Then f is $NWg\#$ - homeomorphism. Then f is not nano regular open map since $f(\{a, b, c\}) = \{x, y, z\}$ is not nano regular - open set in $(V, \tau_{R'}(Y))$. Hence $f$is not Nano regular - (Nr) homeomorphism.

Example 6.8. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{b, d\}$ then the Nano open sets of U with respect to X is $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then the Nano open sets of V with respect to Y is $\tau_{R'}(Y) = \{V, \phi, \{y\}, \{x\}, \{y, x\}\}$. Define bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = x, f(c) = y \; and \; f(d) = z$. Hence f is $NWg\#$ - homeomorphism but f is not nano αg - continuous since the open set $\{y\}$ in $(V, \tau_{R'}(Y)), f^{-1}(y) = \{c\}$ is not $N\alpha g$ - open set in $(U, \tau_R(X))$. Hence f is not $N\alpha g$- homeomorphism.

Theorem 6.9. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{a, b\}$ then the nano open sets of U with respect to X is $\tau_R(X) = \{U, \phi, \{b\}, \{a, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$ then the nano open sets of V with respect to Y is $\tau_R(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define bijective function $f : (U, \tau_R(X)) \to (V, \tau_R(Y))$ by $f(a) = x, f(b) = y, f(c) = w$ and $f(d) = z$. Hence f is $NWg\#$ - homeomorphism. But f is not $N\alpha g$ - continuous since $f^{-1}(y, w) = \{b, c\}$ is not $N\alpha g$- open set in $(U, \tau_\alpha(X))$. Hence f is not $N\alpha g$ - homeomorphism.*

Theorem 6.10. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces and $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then*
(i)Every $NWg\#$- homeomorphism is $N\beta$-homeomorphism but not conversely.
(ii) Every $NWg\#$ - homeomorphism is $Ngsp$ - homeomorphism but not conversely.
Proof of (i). Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano $NWg\#$ - homeomorphism. The map f is bijective, $NWg\#$ - continuous and $NWg\#$ - open map. Every $NWg\#$ - continuous function is $N\beta$ - continuous function and every $NWg\#$ - open map is $N\beta$-open map. Hence f is $N\beta$- continuous, $N\beta$ - open map and bijective. Therefore f is $N\beta$ - homeomorphism.
The proof of (ii) is similar to proof of (i).
The contrary of the preceding theorem is not true as evidenced by the

following example.

Example 6.11. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the Nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ and the nano topology $\tau_{R'}(Y) = \{V, \emptyset, \{y\}, \{x, z\}, \{x, y, z\}\}$. Define the bijective map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = \{y\}, f(b) = \{z\}, f(c) = \{x\}, f(d) = \{w\}$. Then f is $N\beta$ - homeomorphism and $Ngsp$ - homeomorphism. But it is not $NWg\#$ - homeomorphism since $f^{-1}(\{x, z\}) = \{b, c\}$ is not $NWg\#$ - open in $(U, \tau_R(X))$.
The relation between various types of Nano homeomorphisms are given by

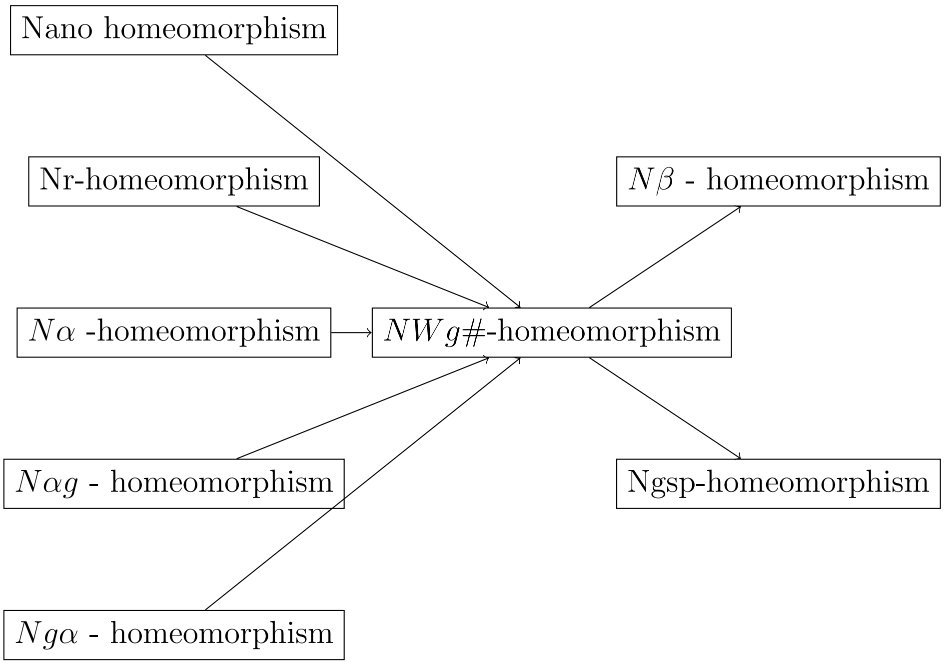

Figure 5.1

The reverse implications of Figure 5.1 are not true in the above diagram.

Theorem 6.12. *Let the bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be $NWg\#$ - homeomorphism if and only if f is $NWg\#$- continuous and $NWg\#$ - closed map.*
Proof. Assume that the bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $NWg\#$ - homeomorphism. Then f is $NWg\#$ - continuous and $NWg\#$ - open map. Let G be a nano closed set in $(U, \tau_R(X))$. Now $U - G$ is nano open set in $(U, \tau_\alpha(X))$. By assumption f is $NWg\#$ - open map then $f(U - G)$ is $NWg\#$ - open set in $(V, \tau_{R'}(Y))$. Now $f(U) - f(G) = V - f(G)$ is $NWg\#$ - open set in $(V, \tau_{R'}(Y))$. Therefore $f(G)$ is $NWg\#$ -closed set in $(V, \tau_{R'}(Y))$ for every nano closed set G in

$(U, \tau_R(X))$. Hence the function f is $NWg\#$ - closed map.
Conversely assume that $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $NWg\#$ - continuous and $NWg\#$ - closed. Let A be a nano open set in $(U, \tau_R(X))$. Now $U - A$ is nano closed in $(U, \tau_R(X))$. By assumption f is $NWg\#$ - closed map, $f(U - A)$ is $NWg\#$ - closed in $(V, \tau_{R'}(Y))$. Now $f(U - A) = f(U) - f(A) = V - f(A)$ is $NWg\#$ - closed in $(V, \tau_{R'}(Y))$. Therefore $f(A)$ is $NWg\#$ - open in $(V, \tau_{R'}(Y))$ for every nano open set A in $(U, \tau_R(X))$. Thus f is $NWg\#$ - open map. Hence f is $NWg\#$ - homeomorphism.

Theorem 6.13. *A bijective function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $NWg\#$ - continuous the following statements are equivalent.*
(i)f is $NWg\#$ - open map.
(ii) f is $NWg\#$ - homeomorphism.
(iii)f is $NWg\#$ - closed map.
Proof. $(i) \implies (ii)$ The bijective function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $NWg\#$ - continuous and $NWg\#$- open map. By definition 6.1, f is $NWg\#$- homeomorphism.
$(ii) \implies (iii)$ Assume that f is $NWg\#$ - homeomorphism f is bijective, $NWg\#$- continuous and $NWg\#$- open map. Let A be a nano closed set in $(U, \tau_R(X))$. Now $U - A$ is nano open set in $(U, \tau_R(X))$. By assumption f is $NWg\#$ - open map, $f(U - A)$ is $NWg\#$ - open in $(V, \tau_{R'}(Y))$ and $f(U - A) = f(U) - f(A) = V - f(A)$ is $NWg\#$ -open in $(V, \tau_{R'}(Y))$. Thus $f(A)$ is $NWg\#$ - closed in $(V, \tau_{R'}(Y))$ for every nano closed set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ - closed map.
$(iii) \implies (i)$ Assume that f is $NWg\#$- closed map. Let B be a nano open set in $(U, \tau_R(X))$. Then $U - B$ is nano closed set in $(U, \tau_R(X))$. By assumption, $f(U - B)$ is $NWg\#$- closed set in $(V, \tau_{R'}(Y))$. Now $f(U - B) = f(U) - f(B) = V - f(B)$ is $NWg\#$ - closed in $(V, \tau_{R'}(Y))$. Hence $f(B)$ is $NWg\#$ - open in $(V, \tau_{R'}(Y))$ for every nano open set B in $(U, \tau_R(X))$. Thus f is $NWg\#$- open map.

Remark 6.14. *The composition of two $NWg\#$ - homeomorphism is $NWg\#$ - homeomorphism as seen in the following example.*

Example 6.15. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, c\}$ then the Nano topology $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then the nano topology $\tau_{R'}(Y) = \{V, \phi, \{y\}, \{x, z\}, \{x, y, z\}\}$.
Define the bijective map $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ by $f(a) = y, f(b) = x, f(c) = z$ and $f(d) = w$. Now $f(a) = y, f(\{b, c\}) = \{x, z\}, f(\{a, b, c\}) = \{x, y, z\}$. Then f is $NWg\#$ - homeomorphism.
Let $W = \{p, q, r, s\}$ with $W/R'' = \{\{p\}, \{r\}, \{q, s\}\}$ and $Z = \{p, q\}$ then the nano

topology with respect to W is $\tau_{R''}(W) = \{W, \phi, \{p\}, \{q,s\}, \{p,q,s\}\}$. Define the bijective map $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ by $g(x) = q, g(y) = p, g(z) = s$, and $g(w) = r$. Now $g(y) = \{p\}, g(\{x,z\}) = \{q,s\}, g(\{x,y,z\}) = \{p,q,s\}$. Then g is $NWg\#$ - homeomorphism.
The composition of two homeomorphisms g and f is given by $(g \circ f) : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ by $(g \circ f)(a) = p, (g \circ f)(b) = q, (g \circ f)(c) = s, (g \circ f)(d) = r$. Then $(g \circ f) : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is $NWg\#$ - homeomorphism.

7. Conclusion

In this article we have introduced the concept of two new classes of subsets called $NWg\#$ kernel and $NWg\#$ surface of a nano topological space. Moreover, $NWg\#$ kernel of a subset is the superior set in which it is the smallest $NWg\#$-open set that contains it and $NWg\#$ surface of a subset is the inferior set which has the largest $NWg\#$ - closed set contained in it. The nano hasse diagram for the $NWg\#$ - closed and $NWg\#$ -open sets are used to discover nano infimum and nano supremum of two subsets by finding the GLB and LUB of the sets. The later part Nano weakly $g\#$ - irresolute functions and Nano weakly $g\#$- homeomorphisms in nano topological spaces are introduced. Some of its properties have been discussed with suitable examples. The proposed definitions can be applied and extended to future research with some applications.

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References

[1] Basavaraj M. Ittanagi and Mohan V., On $S\alpha rw$- continuous and $S\alpha rw$- irresolute Maps in Topological Spaces, Journal of Ultra Scientist of Physical Sciences, Vol. 30(1) (2018), 66-79.

[2] Bhuvaneswai K. and Ezhilarasi A., Nano Semi Generalized Homeomorphisms in Nano Topological Spaces, International Journal of Mathematical Achieve, Vol. 7 No. 7 (2016).

[3] Crossley S. G. and Hildebrand S. K., Semi Topological Properties." Fund. Math., 74 (1972), 233-254.

[4] Jafari S. and Rajesh N., Weak and strong form of $\tilde{g}$ irresolute functions, University of Nis Faculty of Science and Mathematics, Vol. 21, No. 2 (2007), 199-209.

[5] Kalaivani P. and Nithyakala R., Nano Weakly $g\#$ Closed maps and Nano Weakly $g\#$ Open maps in Nano Topological Spaces, South East Asian Journal of Mathematics and Mathematical Sciences, Vol. 20, No. 2 (2024), 425-438.

[6] Kenneth H. Rosen, Discrete Mathematics and Its Applications, McGraw Hill Education, 7th Edition (2017).

[7] Lellis Thivagar M. and Carmel Richard, On Nano Continuity, Mathematical Theory and Modeling, Vol. 3, No. 7 (2013), 32-37.

[8] Nethaji O., Selvi G., Jackson S. and Thangamayan V., Nano $JD*$- Homeomorphisms, South East Asian Journal of Mathematics and Mathematical Sciences, Vol. 19, No. 2 (2023), 393-402.

[9] Sulochanadevi P. and Bhuvaneswari K., Nano Regular generalized Irresolute maps in Nano Topological Spaces, IJSRP, Vol. 10, Issue 12 (2020), 505-512.