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MORE ON HOMEOMORPHISMS IN FUZZY HYPERSOFT TOPOLOGICAL SPACES AND THEIR APPLICATION IN COVID-19 DIAGNOSIS USING COTANGENT SIMILARITY MEASURE

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Abstract: In this article, FH_yS homeomorphism, FH_yS semi homeomorphism, FH_yS δ homeomorphism, FH_yS pre homeomorphism, FH_yS δ pre homeomorphism, FH_yS δ semi homeomorphism, FH_yS $\delta\alpha$ homeomorphism, FH_yS e- homeomorphism, FH_yS e^* homeomorphism and various forms of FH_yS C homeomorphisms in FH_yS topological spaces are introduced and studied. Also, we have discussed the properties of various forms of FH_yS homeomorphisms. Moreover, a new cotangent similarity measure for FH_yS sets is introduced and applied in the Covid-19 diagnosis using an example.

Keywords and Phrases: FH_yS homeomorphism, FH_yS δ homeomorphism, FH_yS e-homeomorphism, FH_yS e-C homeomorphism, cotangent similarity measure.

2020 Mathematics Subject Classification: 03E72, 54A10, 54A40, 54C05, 54C10.

1. Introduction

Real-world decision-making problems in fields like engineering, computer science, medicine, artificial intelligence, management, economics and social sciences often involve inadequate and uncertain data. The conventional mathematical methods cannot deal with these sort of problems due to the imprecise data. The fuzzy set with membership value in [0,1] was introduced by Zadeh [37] in 1965 to deal with the real-world decision-making problems involving uncertainty. In fuzzy set, every element of the universe is a member of the set but with some value or degree of belongingness called as membership value of an element which lies between 0 and 1. The fuzzy topological space was developed by Chang [10]. In 1999, the soft set theory was introduced by Molodstov [19]. Soft set is a collection of parameters which describe the characteristics, properties or attributes of the objects. Soft set theory has many applications in various fields such as data analysis, optimization, decision making, forecasting etc. Consequently, the soft topological spaces were developed by Shabir and Naz [30].

By replacing function with the cartesian product of a multi-argument function with a different set of attributes, the concept of a soft set is extended to a hypersoft set and subsequently to plithogenic set by Smarandache [31]. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Abbas et al. [2] defined the basic operations on hypersoft sets and hypersoft point in all the universe of discourses. The topological structures of fuzzy hypersoft (briefly, FH_yS) set, intuitionistic hypersoft set and neutrosophic hypersoft set were developed by Ajay and Charisma [4]. FH_yS topology and intuitionistic hypersoft topology are generalized by the general framework neutrosophic hypersoft topology. FH_yS semi-open sets were defined and an application in multiattribute group decision making were developed by Ajay et al. [5].

Saha [27] defined δ-open sets in fuzzy topological spaces. The δ-open sets were introduced by Vadivel et al. [34] in neutrosophic topological spaces and Surendra et al. [32, 33] in neutrosophic hypersoft topological spaces. In 2019, Acikgoz and Esenbel [1] defined neutrosophic soft δ-topology. The notion of e-open sets were introduced by Ekici [16] in a general topology, Seenivasan et al. [29] in fuzzy topological space, Chandrasekar et al. [9] in intuitionistic fuzzy topological space, Vadivel et al. [35] in neutrosophic topological spaces, Revathi et al. [22] in neutrosophic soft topological spaces and Aranganayagi et al. [7] in neutrosophic hypersoft topological spaces. Aras and Bayramov [8] introduced neutrosophic soft continuity in neutrosophic soft topological spaces. The concepts of e-continuity, e-irresolute maps, e-open maps, e-closed maps and e-homeomorphisms were developed by Vadivel et al. [35, 36] in neutrosophic topological spaces and Revathi et al. [23, 24, 25, 26] in neutrosophic soft topological spaces. Ahsan et al. [3] studied a theoretical and analytical approach for fundamental framework of composite mappings on FH_yS classes. Aranganayagi et al. [6] studied more on open maps and closed maps in FH_yS topological spaces and developed an application in diagnosing Covid-19 using cotangent similarity measure.

Das et al. [11, 12, 13, 14, 15], Granados et al. [17] and Mukherjee et al. [20, 21] provided valuable insights into fuzzy set theory, hypersoft sets, topology, decision-making models and their applications. Saqlain et al. [28] studied single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft set. Jafar et al. [18] studied trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection.

In hypersoft environment, some kind of open sets and maps are introduced and their applications are studied so far. No investigation on homeomorphisms is initiated. There is a need to study homeomorphisms in the hypersoft environment because it is a fundamental concept in topology and has many applications in contemporary mathematics. As hypersoft set involves multi attributes, the homeomorphisms developed in hypersoft environment can be applied in the decision-making problems with more parameters. This leads us to develop homeomorphisms via stronger and weaker forms of open sets in fuzzy hypersoft topological spaces.

In this paper, we develop the concept of FH_yS homeomorphism, semi homeomorphism, δ homeomorphism, pre homeomorphism, δ pre homeomorphism, δ semi homeomorphism, $\delta\alpha$ homeomorphism, e-homeomorphism, e^* homeomorphism and various forms of C homeomorphisms in FH_yS topological spaces and some of their basic properties are analyzed with examples. Also, an application in Covid-19 diagnosis is explained with the algorithm and example using cotangent similarity measure for FH_yS sets.

2. Preliminaries

Definition 2.1. [37] *Let Θ be an initial universe. A function λ from Θ into the unit interval I is called a fuzzy set in Θ. For every $\chi \in \Theta$, $\lambda(\chi) \in I$ is called the* *grade of membership of* χ *in* λ. *Some authors say that* λ *is a fuzzy subset of* Θ *instead of saying that* λ *is a fuzzy set in* Θ. *The class of all fuzzy sets from* Θ *into the closed unit interval* I *will be denoted by* I^{Θ}.

Definition 2.2. [19] *Let* Θ *be an initial universe,* Υ *be a set of parameters and* $\mathcal{P}(\Theta)$ *be the power set of* Θ. *A pair* $(\tilde{\theta}, \zeta)$ *is called the a soft set over* Θ *where* $\tilde{\theta}$ *is a mapping* $\tilde{\theta} : \Upsilon \to \mathcal{P}(\Theta)$. *In other words, the soft set is a parametrized family of subsets of the set* Θ.

Definition 2.3. [31] *Let* Θ *be an initial universe and* $\mathcal{P}(\Theta)$ *be the power set of* Θ. *Consider* $\upsilon_1, \upsilon_2, \upsilon_3, ..., \upsilon_n$ *for* $n \geq 1$, *be* n *distinct attributes, whose corresponding attribute values are respectively the sets* $\Upsilon_1, \ \Upsilon_2, \ ..., \ \Upsilon_n$ *with* $\Upsilon_i \cap \Upsilon_j = \emptyset$, *for* $i \neq j$ *and* $i, j \in \{1, 2, ..., n\}$. *Then the pair* $(\tilde{\theta}, \ \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n)$ *where* $\tilde{\theta} : \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n \to \mathcal{P}(\Theta)$ *is called a hypersoft set over* Θ.

Definition 2.4. [2] *Let* Θ *be an initial universal set and* $\Upsilon_1, \Upsilon_2, ..., \Upsilon_n$ *be pairwise disjoint sets of parameters. Let* $\mathcal{P}(\Theta)$ *be the set of all fuzzy sets of* Θ. *Let* E_i *be the nonempty subset of the pair* Υ_i *for each* $i = 1, 2, ..., n$. *A* FH_yS *set (briefly,* FH_ySs*) over* Θ *is defined as the pair* $(\tilde{\theta}, E_1 \times E_2 \times ... \times E_n)$ *where* $\tilde{\theta} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(\Theta)$ *and* $\tilde{\theta}(E_1 \times E_2 \times ... \times E_n) = \{(\upsilon, \langle \chi, \mu_{\tilde{\theta}(\upsilon)}(\chi) \rangle : \chi \in \Theta) : \upsilon \in E_1 \times E_2 \times ... \times E_n \subseteq \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n\}$ *where* $\mu_{\tilde{\theta}(\upsilon)}(\chi)$ *is the membership value such that* $\mu_{\tilde{\theta}(\upsilon)}(\chi) \in [0, 1]$.

Definition 2.5. [2] *Let* $(\tilde{\theta_1}, \zeta_1)$ *and* $(\tilde{\theta_2}, \zeta_2)$ *be two* FH_ySs*'s over* Θ. *Then* $(\tilde{\theta_1}, \zeta_1)$ *is the* FH_yS *subset of* $(\tilde{\theta_2}, \zeta_2)$ *if* $\mu_{\tilde{H}(\upsilon)}(\chi) \leq \mu_{\tilde{G}(\upsilon)}(\chi)$.

It is denoted by $(\tilde{\theta_1}, \zeta_1) \subseteq (\tilde{\theta_2}, \zeta_2)$.

Definition 2.6. [2] *Let* $(\tilde{\theta_1}, \zeta_1)$ *and* $(\tilde{\theta_2}, \zeta_2)$ *be* FH_ySs*'s over* Θ. $(\tilde{\theta_1}, \zeta_1)$ *is equal to* $(\tilde{\theta_2}, \zeta_2)$ *if* $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = \mu_{\tilde{\theta_2}(\upsilon)}(\chi)$.

Definition 2.7. [2] *A* FH_ySs $(\tilde{\theta_1}, \zeta)$ *over* Θ *is said to be null* FH_yS *set if* $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = 0, \ \forall \ \upsilon \in \zeta$ *and* $\chi \in \Theta$. *It is denoted by* $\tilde{0}_{(\Theta,\Upsilon)}$.

A FH_ySs $(\tilde{\theta_2}, \zeta)$ *over* Θ *is said to be absolute* FH_yS *set if* $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = 1 \ \forall \upsilon \in \zeta$ *and* $\chi \in \Theta$. *It is denoted by* $\tilde{1}_{(\Theta,\Upsilon)}$.

Clearly, $\tilde{0}^c_{(\Theta,\Upsilon)} = \tilde{1}_{(\Theta,\Upsilon)}$ *and* $\tilde{1}^c_{(\Theta,\Upsilon)} = \tilde{0}_{(\Theta,\Upsilon)}$.

Definition 2.8. [2] *Let* $(\tilde{\theta_1}, \zeta)$ *be* FH_ySs *over* Θ. $(\tilde{\theta_1}, \zeta)^c$ *is the complement of* $(\tilde{\theta_1}, \zeta)$ *if* $\mu^c_{\tilde{\theta}(\upsilon)}(\chi) = \tilde{1}_{(\Theta,\Upsilon)} - \mu_{\tilde{\theta}(\upsilon)}(\chi)$ *where* $\forall \ \upsilon \in \zeta$ *and* $\forall \chi \in \Theta$. *It is clear that* $((\tilde{\theta_1}, \zeta)^c)^c = (\tilde{\theta_1}, \zeta)$.

Definition 2.9. [2] *Let* $(\tilde{\theta_1}, \zeta_1)$ *and* $(\tilde{\theta_2}, \zeta_2)$ *be* FH_ySs*'s over* Θ. *Extended union* $(\tilde{\theta_1}, \zeta_1) \cup (\tilde{\theta_2}, \zeta_2)$ *is defined as*

$$\mu((\tilde{\theta_1},\zeta_1)\cup(\tilde{\theta_2},\zeta_2)) = \begin{cases} \mu_{\tilde{\theta_1}(\upsilon)}(\chi) & if \ \ \upsilon\in\zeta_1-\zeta_2 \\ \mu_{\tilde{\theta_2}(\upsilon)}(\chi) & if \ \ \upsilon\in\zeta_2-\zeta_1 \\ max\{\mu_{\tilde{\theta_1}(\upsilon)}(\chi),\mu_{\tilde{\theta_2}(\upsilon)}(\chi)\} & if\ \upsilon\in\zeta_1\cap\zeta_2 \end{cases}$$

Definition 2.10. [2.4] *Let* $(\tilde{\theta_1},\zeta_1)$ *and* $(\tilde{\theta_2},\zeta_2)$ *be* FH_ySs*'s over* Θ*. Extended intersection* $(\tilde{\theta_1},\zeta_1)\cap(\tilde{\theta_2},\zeta_2)$ *is defined as*

$$\mu((\tilde{\theta_1},\zeta_1)\cap(\tilde{\theta_2},\zeta_2)) = \begin{cases} \mu_{\tilde{\theta_1}(\upsilon)}(\chi) & if\upsilon\in\zeta_1-\zeta_2 \\ \mu_{\tilde{\theta_2}(\upsilon)}(\chi) & if\upsilon\in\zeta_2-\zeta_1 \\ min\{\mu_{\tilde{\theta_1}(\upsilon)}(\chi),\mu_{\tilde{\theta_2}(\upsilon)}(\chi)\} & if\upsilon\in\zeta_1\cap\zeta_2 \end{cases}$$

Definition 2.11. [4] *Let* (Θ,Υ) *be the family of all* FH_ySs*'s over the universe set* Θ *and* $\tau\subseteq FH_ySs(\Theta,\Upsilon)$*. Then* τ *is said to be a* FH_yS *topology (briefly,* FH_ySt*) on* Θ *if*

(i) $\tilde{0}_{(\Theta,\Upsilon)}$ *and* $\tilde{1}_{(\Theta,\Upsilon)}$ *belongs to* τ

(ii) the union of any number of FH_ySs*'s in* τ *belongs to* τ

(iii) the intersection of finite number of FH_ySs*'s in* τ *belongs to* τ*.*

Then (Θ,Υ,τ) *is called a* FH_yS *topological space (briefly,* FH_ySts*) over* Θ*. Each member of* τ *is said to be* FH_yS *open set (briefly,* FH_ySos*). A* FH_ySs $(\tilde{\theta_1},\zeta)$ *is called a* FH_yS *closed set (briefly,* FH_yScs*) if its complement* $(\tilde{\theta_1},\zeta)^c$ *is* FH_ySos*.*

Definition 2.12. [4] *Let* (Θ,Υ,τ) *be a* FH_ySts *over* Θ *and* $(\tilde{\theta_1},\zeta)$ *be a* FH_ySs *in* Θ*. Then,*

(i) the FH_yS *interior (briefly,* FH_ySint*) of* $(\tilde{\theta_1},\zeta)$ *is defined as* $FH_ySint(\tilde{\theta_1},\zeta) = \cup\{(\tilde{\theta_2},\zeta):(\tilde{\theta_2},\zeta)\subseteq(\tilde{\theta_1},\zeta)$ *where* $(\tilde{\theta_2},\zeta)$ *is* $FH_ySos\}$*.*

(ii) the FH_yS *closure (briefly,* FH_yScl*) of* $(\tilde{\theta_1},\zeta)$ *is defined as* $FH_yScl(\tilde{\theta_1},\zeta) = \cap\{(\tilde{\theta_2},\zeta):(\tilde{\theta_2},\zeta)\supseteq(\tilde{\theta_1},\zeta)$ *where* $(\tilde{\theta_2},\zeta)$ *is* $FH_yScs\}$*.*

Definition 2.13. [5] *Let* (Θ,Υ,τ) *be a* FH_ySts *over* Θ *and* $(\tilde{\theta_1},\zeta)$ *be a* FH_ySs *in* Θ*. Then,* $(\tilde{\theta_1},\zeta)$ *is called the* FH_yS *semiopen set (briefly,* $FH_yS\mathcal{S}os$*) if* $(\tilde{\theta_1},\zeta)\subseteq FH_yScl(int(\tilde{\theta_1},\zeta))$*.*

A FH_ySs $(\tilde{\theta_1},\zeta)$ *is called a* FH_yS *semiclosed set (briefly,* $FH_yS\mathcal{S}cs$*) if its complement* $(\tilde{\theta_1},\zeta)^c$ *is a* $FH_yS\mathcal{S}os$*.*

Definition 2.14. [6] *Let* (Θ,Υ,τ) *be a* FH_ySts *over* Θ*. An* FH_ySs $(\tilde{\theta_1},\zeta)$ *is said* *to be a FH_yS regular open set (briefly, FH_ySros) if $(\tilde{\theta_1}, \zeta) = FH_ySint(FH_yScl(\tilde{\theta_1}, \zeta))$. The complement of FH_ySros is called a FH_yS regular closed set (briefly, FH_ySrcs) in Θ.*

Definition 2.15. [6] *Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs on Θ. Then the FH_yS*

(i) δ-interior (briefly, FH_ySint) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta int(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a FH_ySros in $\Theta\}$

(ii) δ-closure (briefly, FH_yScl) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta cl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \supseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a FH_ySrcs in $\Theta\}$

Definition 2.16. [6] *Let (Θ, Υ, τ) be a FH_ySts over Θ. An FH_ySs $(\tilde{\theta_1}, \zeta)$ is said to be a FH_yS*

(i) semi-regular if $(\tilde{\theta_1}, \zeta)$ is both $FH_yS\mathcal{S}os$ and $FH_yS\mathcal{S}cs$.

(ii) pre open set (briefly, $FH_yS\mathcal{P}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint(FH_yScl(\tilde{\theta_1}, \zeta)$

(iii) δ-open set (briefly, $FH_yS\delta os$) if $(\tilde{\theta_1}, \zeta) = FH_yS\delta int(\tilde{\theta_1}, \zeta)$

(iv) δ-pre open set (briefly, $FH_yS\delta\mathcal{P}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint(FH_yS\delta cl(\tilde{\theta_1}, \zeta))$

(v) δ-semi open set (briefly, $FH_yS\delta\mathcal{S}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl(FH_yS\delta int(\tilde{\theta_1}, \zeta))$

(vi) e-open set (briefly, FH_ySeos) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl(FH_yS\delta\ int(\tilde{\theta_1}, \zeta)) \cup FH_ySint$ $(FH_yS\delta\ cl(\tilde{\theta_1}, \zeta))$.

(vii) δ α-open set (briefly, $FH_yS\delta\alpha os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint(FH_yScl(FH_yS\delta int(\tilde{\theta_1}, \zeta)))$.

(viii) e^*-open set (briefly, FH_ySe^*os) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl(FH_ySint(FH_yS\delta cl(\tilde{\theta_1}, \zeta)))$.

The complement of $FH_yS\delta os$ (resp. $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, FH_yS eos, $FH_yS\delta\alpha os$ & FH_ySe^*os) is called a $FH_yS\delta$ (resp. FH_yS pre, $FH_yS\delta$ pre, $FH_yS\delta$ semi, FH_ySe, $FH_yS\delta\alpha$ & FH_ySe^*) closed set (briefly, $FH_yS\delta cs$ (resp. $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$, FH_ySeos, $FH_yS\delta\alpha cs$ & FH_ySe^*cs)) in Θ.

The family of all $FH_yS\delta os$ (resp. $FH_yS\delta cs$, FH_ySros, FH_ySrcs, $FH_yS\mathcal{P}os$, $FH_yS\mathcal{P}cs$ $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}os$, $FH_yS\delta\mathcal{S}cs$, FH_ySeos, FH_ySecs, $FH_yS\delta\alpha os$, $FH_yS\delta\alpha cs$, FH_ySe^*os & FH_ySe^*cs) of Θ is denoted by $FH_yS\ \delta OS(\Theta)$ (resp. $FH_yS\delta CS(\Theta)$, $FH_ySrOS(\Theta)$, $FH_ySrOS(\Theta)$, $FH_yS\mathcal{P}OS(\Theta)$, $FH_yS\mathcal{P}CS(\Theta)$, $FH_yS\delta\mathcal{P}OS(\Theta)$, $FH_yS\ \delta\mathcal{P}CS(\Theta)$, $FH_yS\delta\mathcal{S}OS(\Theta)$, $FH_yS\delta\mathcal{S}CS(\Theta)$, $FH_ySeOS(\Theta)$, $FH_ySeCS(\Theta)$, $FH_yS\delta\alpha OS(\Theta)$, $FH_yS\delta\alpha CS(\Theta)$, $FH_ySe^*OS(\Theta)$ & $FH_ySe^*CS(\Theta)$).

Definition 2.17. [6] *Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs on Θ. Then the FH_yS*

(i) δ-pre (resp. δ-semi) interior (briefly, $FH_yS\delta\mathcal{P}int$ (resp. $FH_yS\delta\mathcal{S}int$)) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta\mathcal{P}int(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a $FH_yS\delta\mathcal{P}os$ (resp. $FH_yS\delta\mathcal{S}os$) in $\Theta\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $FH_yS\delta\mathcal{P}cl$ (resp. $FH_yS\delta\mathcal{S}cl$)) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta\mathcal{P}cl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \supseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a $FH_yS\delta\mathcal{P}cs$ (resp. $FH_yS\delta\mathcal{S}cs$) in $\Theta\}$

(iii) e interior (briefly, $FH_ySeint(\tilde{\theta_1}, \zeta)$ is defined by $FH_ySeint(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{L}, \zeta) : (\tilde{L}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ & $(\tilde{L}, \zeta)$ is a FH_ySeos in $\Theta\}$.

(iv) e closure (briefly, $FH_ySecl(\tilde{\theta_1}, \zeta)$ is defined by $FH_ySecl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{L}, \zeta) : (\tilde{\theta_1}, \zeta) \subseteq (\tilde{L}, \zeta)$ & $(\tilde{\theta_1}, \zeta)$ is a FH_ySecs in $\Theta\}$.

Definition 2.18. [6] *Consider any two FH_ySts (Θ, L, τ) and (Ω, M, σ). A map $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is called as FH_yS*

(i) continuous (resp. semi-continuous, pre-continuous, δ continuous, δ semi continuous, δ pre continuous, e continuous, $\delta\alpha$ continuous & e^* continuous) (briefly, FH_ySCts, $FH_yS\mathcal{S}Cts$, $FH_yS\mathcal{P}Cts$, $FH_yS\delta Cts$, $FH_yS\delta\mathcal{S}Cts$, $FH_yS\delta\mathcal{P}Cts$, FH_ySeCts, $FH_yS\delta\alpha Cts$ & FH_ySe^*Cts) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_ySos (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_yS\delta os$,$FH_yS\delta\mathcal{S}os$, $FH_yS\delta\mathcal{P}os$, FH_ySeos, $FH_yS\delta\alpha os$ & FH_ySe^*os) in (Θ, L, τ).

(ii) e-irresolute (resp. irresolute, δ irresolute, $\mathcal{P}$ irresolute, $\delta\mathcal{P}$ irresolute, $\delta\mathcal{S}$ irresolute, $\delta\alpha$ irresolute, e^* irresolute) (briefly, FH_ySeIrr (resp. FH_ySIrr , $FH_yS\delta Irr$, $FH_yS\mathcal{P}Irr$, $FH_yS\delta\mathcal{P}Irr$, $FH_yS\delta\mathcal{S}Irr$, $FH_yS\delta\alpha Irr$, FH_ySe^*Irr)) if the inverse image of every FH_ySeos (resp. $FH_yS\mathcal{S}os$, $FH_yS\delta os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\alpha os$ & FH_ySe^*os) in (Ω, M, σ) is a FH_ySeos (resp. $FH_yS\mathcal{S}os$, $FH_yS\delta os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\alpha os$ & FH_ySe^*os) in (Θ, L, τ).

Definition 2.19. [6] *Consider any two FH_ySts (Θ, L, τ) and (Ω, M, σ). A map $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is called as FH_yS open (resp. semi-open, pre-open, δ open, δ semi open, δ pre open, e open, $\delta\alpha$ open & e^*) map (briefly, FH_ySO, $FH_yS\mathcal{S}O$, $FH_yS\mathcal{P}O$, $FH_yS\delta O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\mathcal{P}O$, FH_ySeO, $FH_yS\delta\alpha O$ &*

FH_ySe^*O) if the image of each FH_ySos in (Θ, L, τ) is a FH_ySos (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_yS\delta os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\delta\mathcal{P}os$, FH_ySeos, $FH_yS\delta\alpha os$ & FH_ySe^*os) in (Ω, M, σ).

Definition 2.20. [6] *A mapping $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is FH_yS e-closed (resp. closed, δ closed, δ-semi closed, δ-pre closed & e^*-closed) (briefly, FH_ySeC (resp.FH_ySC, $FH_yS\delta C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\mathcal{P}C$ & FH_ySe^*C)) if the image of every FH_yScs of (Θ, L, τ) is FH_ySec (resp. FH_ySc, $FH_yS\delta c$, $FH_yS\delta\mathcal{S}c$, $FH_yS\delta\mathcal{P}c$ & FH_ySe^*c) set in (Ω, M, σ).*

Definition 2.21. [18] *Consider two neutrosophic hypersoft sets $(\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ over Θ. The cotangent similarity measure for these two sets based on the cotangent function is given by*

$$S_{Ct}((\tilde{\theta_1}, \zeta), (\tilde{\theta_2}, \zeta) = \frac{1}{n}\sum_{i=1}^{n} cot[\frac{\pi}{4} + \frac{\pi}{12}(|\mu^i_{\theta_1} - \mu^i_{\theta_2}| \vee |\sigma^i_{\theta_1} - \sigma^i_{\theta_2}| \vee |\nu^i_{\theta_1} - \nu^i_{\theta_2}|)]$$

where $\vee$ denotes the maximum operator.

3. More on Homeomorphisms in Fuzzy Hypersoft Topological Spaces

In this section, various forms of FH_yS homeomorphisms are introduced and their related properties are discussed.

Definition 3.1. *A bijection $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is called a FH_yS e-homeomorphism (resp. homeomorphism, $\mathcal{S}$ homeomorphism, δ homeomorphism, $\mathcal{P}$ homeomorphism, $\delta\mathcal{P}$ homeomorphism, $\delta\mathcal{S}$ homeomorphism, $\delta\alpha$ homeomorphism and e^* homeomorphism) (briefly, FH_ySeHom (resp. FH_ySHom, $FH_yS\mathcal{S}Hom$, $FH_yS\delta Hom$, $FH_yS\mathcal{P}Hom$, $FH_yS\delta\mathcal{P}Hom$, $FH_yS\delta\mathcal{S}Hom$, $FH_yS\delta\alpha Hom$ & FH_ySe^* Hom)) if $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeCts (resp. FH_ySCts, $FH_yS\mathcal{S}Cts$, $FH_yS\delta Cts$, $FH_yS\mathcal{P}Cts$, $FH_yS\delta\mathcal{P}Cts$, $FH_yS\delta\mathcal{S}Cts$, $FH_yS\delta\alpha Cts$ & FH_ySe^*Cts) mappings.*

Theorem 3.1. *Each FH_ySHom is a FH_ySeHom (resp. $FH_yS\mathcal{S}Hom$, $FH_yS\delta Hom$, $FH_yS\mathcal{P}Hom$, $FH_yS\delta\mathcal{P}Hom$, $FH_yS\delta\mathcal{S}Hom$, $FH_yS\delta\alpha Hom$ & FH_ySe^*Hom). But not conversely.*

Proof.

(i) Let $\mathfrak{h}$ be FH_ySHom. Then by the hypothesis, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySCts. But every FH_ySCts function is FH_ySeCts because each FH_ySos is FH_ySeos [7]. Hence, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeCts. Therefore, $\mathfrak{h}$ is a FH_ySeHom.

(ii) Let $\mathfrak{h}$ be FH_ySHom. Then by the hypothesis, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySCts. But every FH_ySCts function is $FH_yS\delta Cts$ because each $FH_yS\delta os$ is FH_ySos [7]. Hence, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FH_yS\delta Cts$. Therefore, $\mathfrak{h}$ is a $FH_yS\delta Hom$.

The other cases are similar.

Example 3.1. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\phi_1, \phi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon'_1 \times \Upsilon'_2$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2\}, \Upsilon_2 = \{b_1, b_2\}\ \Upsilon'_1 = \{c_1, c_2\}, \Upsilon'_2 = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_2, b_1), \{\frac{\chi_1}{0.4}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.4}, \frac{\chi_2}{0.2}\} \rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_2, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1)\}$ is FH_ySts.

Let the $FH_ySs's$ $(\tilde{\psi_1}, \zeta_2)$ and $(\tilde{\psi_2}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\phi_1}{0.3}, \frac{\phi_2}{0.5}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\phi_1}{0.5}, \frac{\phi_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{\psi_2}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\phi_1}{0.3}, \frac{\phi_2}{0.4}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\phi_1}{0.2}, \frac{\phi_2}{0.4}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_2)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Theta, L) \to (\Omega, M)$ be a FH_yS mapping as follows:

$$\omega(\chi_1) = \phi_2, \omega(\chi_2) = \phi_1,$$
$$\nu(a_2, b_1) = (c_2, d_1), \nu(a_1, b_1) = (c_2, d_1),\ \nu(a_1, b_2) = (c_1, d_2),\ \nu(a_2, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{\psi_1}, \zeta_2) = (\tilde{\theta_2}, \zeta_2), \mathfrak{h}^{-1}(\tilde{\psi_2}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$$

Then $\mathfrak{h}$ is FH_ySCts. Then $\mathfrak{h}$ is FH_ySeHom but not FH_ySHom because $(\tilde{\psi_1}, \zeta_2)$ is FH_ySos in Ω and $\mathfrak{h}^{-1}(\tilde{\psi_1}, \zeta_2) = (\tilde{\theta_2}, \zeta_2)$ is FH_ySeos but not FH_ySos in Θ.

Remark 3.1. *The diagram shows FH_ySHom's in FHS_yts.*

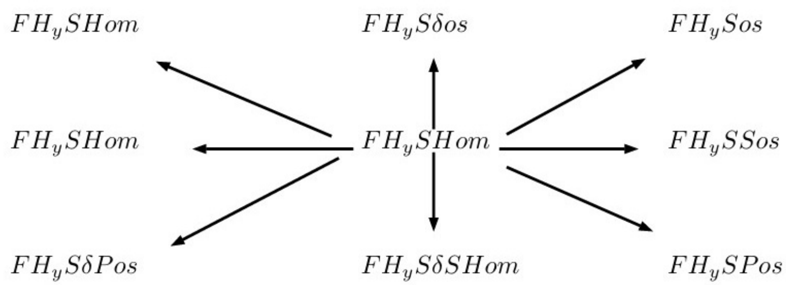

Figure 1. FH_yS Homeomorphisms in FHS_yts

Theorem 3.2. *Let $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ be a bijective mapping. If $\mathfrak{h}$ is FH_ySeCts (resp. FH_ySCts, $FH_yS\mathcal{S}Cts$, $FH_yS\delta Cts$, $FH_yS\mathcal{P}Cts$, $FH_yS\delta\mathcal{P}Cts$, $FH_yS\delta\mathcal{S}Cts$, $FH_yS\delta\alpha Cts$, & FH_ySe^*Cts), then the followings statements are equivalent:*

(i) *$\mathfrak{h}$ is a FH_ySeC (resp. FH_ySC, $FH_yS\mathcal{S}C$, $FH_yS\delta C$, $FH_yS\mathcal{P}C$, $FH_yS\delta\mathcal{P}C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\alpha C$ & FH_ySe^*C) mapping.*

(ii) *$\mathfrak{h}$ is a FH_ySeO (resp. FH_ySO, $FH_yS\mathcal{S}O$, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$ & FH_ySe^*O) mapping.*

(iii) *$\mathfrak{h}^{-1}$ is a FH_ySeHom (resp. FH_ySHom, $FH_yS\mathcal{S}Hom$, $FH_yS\delta Hom$, $FH_yS\mathcal{P}Hom$, $FH_yS\delta\mathcal{P}Hom$, $FH_yS\delta\mathcal{S}Hom$, $FH_yS\delta\alpha Hom$ & FH_ySe^*Hom).*

Proof. (i) $\Rightarrow$ (ii) : Assume that $\mathfrak{h}$ is a bijective mapping and a FH_ySeC mapping. Hence, $\mathfrak{h}^{-1}$ is a FH_ySeCts mapping. We know that each FH_ySos in (Θ, L, τ) is a FH_ySeos in (Ω, M, σ). Hence, $\mathfrak{h}$ is a FH_ySeO mapping.

(ii) $\Rightarrow$ (iii) : Let $\mathfrak{h}$ be a bijective and FH_ySO mapping. Further, $\mathfrak{h}^{-1}$ is a FH_ySeCts mapping. Hence, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeCts. Therefore, $\mathfrak{h}$ is a FH_ySeHom.

(iii) $\Rightarrow$ (i): Let $\mathfrak{h}$ be a FH_ySeHom. Then $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeCts. Since each FH_yScs in (Θ, L, τ) is a FH_ySecs in (Ω, M, σ), $\mathfrak{h}$ is a FH_ySeC mapping.

The other cases are similar.

Definition 3.2. *A FH_ySts (Θ, L, τ) is said to be a $FH_yS\ eT_{\frac{1}{2}}$-space if every FH_ySecs is FH_ySc in (Θ, L, τ).*

Theorem 3.3. *Let $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ be a FH_ySeHom. Then $\mathfrak{h}$ is a FH_ySHom if (Θ, L, τ) and (Ω, M, σ) are $FH_ySeT_{\frac{1}{2}}$-space.*

Proof. Assume that $(\tilde{\theta_2}, \zeta)$ is a FH_yScs in (Ω, M, σ). Then $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$ is a FH_ySecs in (Θ, L, τ). Since (Θ, L, τ) is a $FH_ySeT_{\frac{1}{2}}$-space, $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$ is a FH_yScs in (Θ, L, τ). Therefore, $\mathfrak{h}$ is FH_ySCts. By hypothesis, $\mathfrak{h}^{-1}$ is FH_ySeCts. Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in (Θ, L, τ). Then, $(\mathfrak{h}^{-1})^{-1}(\tilde{\theta_1}, \zeta) = \mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_yScs in (Ω, M, σ), by presumption. Since (Ω, M, σ) is a $FH_ySeT_{\frac{1}{2}}$-space, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_yScs in (Ω, M, σ). Hence, $\mathfrak{h}^{-1}$ is FH_ySCts. Hence, $\mathfrak{h}$ is a FH_ySHom.

Theorem 3.4. *Let $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ be a FH_ySts. Then the following are equivalent if (Ω, M, σ) is a $FH_ySeT_{\frac{1}{2}}$-space:*

(i) *$\mathfrak{h}$ is FH_ySeC mapping.*

(ii) *If $(\tilde{\theta_1}, \zeta)$ is a FH_ySos in (Θ, L, τ), then $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySeos in (Ω, M, σ).*

(iii) $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq FH_yScl(FH_ySint(\mathfrak{h}(\tilde{\theta_1}, \zeta)))$ *for every* FH_ySs $(\tilde{\theta_1}, \zeta)$ *in* (Θ, L, τ).

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySs in (Θ, L, τ). Then, $FH_ySint(\tilde{\theta_1}, \zeta)$ is a FH_ySos in (Θ, L, τ). Then, $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta))$ is a FH_ySeos in (Ω, M, σ). Since (Ω, M, σ) is a $FH_ySeT_{\frac{1}{2}}$-space, $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta))$ is a FH_ySos in (Ω, M, σ). Therefore, $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) = FH_ySint(\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta))) \subseteq FH_yScl(FH_ySint(\mathfrak{h}(\tilde{\theta_1}, \zeta)))$.

(iii) $\Rightarrow$ (i): Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in (Θ, L, τ). Then, $(\tilde{\theta_1}, \zeta)^c$ is a FH_ySos in (Θ, L, τ). From, $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)^c) \subseteq FH_yScl(FH_ySint(\mathfrak{h}(\tilde{\theta_1}, \zeta)^c))$, $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c) \subseteq FH_yScl(FH_ySint(\mathfrak{h}(\tilde{\theta_1}, \zeta)^c))$. Therefore, $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c)$ is FH_ySeos in (Ω, M, σ). Therefore, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_ySecs in (Θ, L, τ). Hence, $\mathfrak{h}$ is a FH_ySC mapping.

Theorem 3.5. *Let* $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ *and* $\mathfrak{g} : (\Omega, M, \sigma) \to (P, \rho, Q)$ *be* FH_ySeC, *where* (Θ, L, τ) *and* (P, ρ, Q) *are two* FH_ySts*'s and* (Ω, M, σ) *a* $FH_ySeT_{\frac{1}{2}}$*-space, then the composition* $\mathfrak{g} \circ \mathfrak{h}$ *is* FH_ySeC.

Proof. Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in (Θ, L, τ). Since $\mathfrak{h}$ is FH_ySec and $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_ySecs in (Ω, M, σ), by assumption, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_yScs in (Ω, M, σ). Since $\mathfrak{g}$ is FH_ySec, then $\mathfrak{g}(\mathfrak{h}(\tilde{\theta_1}, \zeta))$ is FH_ySec in (P, ρ, Q) and $\mathfrak{g}(\mathfrak{h}(\tilde{\theta_1}, \zeta)) = (\mathfrak{g} \circ \mathfrak{h})(\tilde{\theta_1}, \zeta)$. Therefore, $\mathfrak{g} \circ \mathfrak{h}$ is FH_ySeC.

Theorem 3.6. *Let* $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ *and* $\mathfrak{g} : (\Omega, M, \sigma) \to (P, \rho, Q)$ *be two* FH_ySts*'s, then the following hold:*

(i) If $\mathfrak{g} \circ \mathfrak{h}$ *is* FH_ySeO *(resp.*FH_ySO, $FH_yS\mathcal{S}O$, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$ *&* FH_ySe^*O*) and* $\mathfrak{h}$ *is* FH_ySCts*, then* $\mathfrak{g}$ *is* FH_ySeO *(resp.* FH_ySO, $FH_yS\mathcal{S}O$, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$ *&* FH_ySe^*O*).*

(ii) If $\mathfrak{g}\circ\mathfrak{h}$ *is* FH_ySO *and* $\mathfrak{g}$ *is* FH_ySeCts *(resp.* FH_ySCts, $FH_yS\mathcal{S}Cts$, $FH_yS\delta Cts$, $FH_yS\mathcal{P}Cts$, $FH_yS\delta\mathcal{P}Cts$, $FH_yS\delta\mathcal{S}Cts$, $FH_yS\delta\alpha Cts$ *&* FH_ySe^*Cts*), then* $\mathfrak{h}$ *is* FH_ySeO *(resp.* FH_ySO, $FH_yS\mathcal{S}O$, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$ *&* FH_ySe^*O*).*

Proof. Obvious.

4. More on C Homeomorphisms in Fuzzy Hypersoft Topological Spaces

In this section, various forms of FH_yS C homeomorphisms are introduced and their related properties are discussed.

Definition 4.1. *A bijection* $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ *is called a* FH_ySe*-C homeomorphism (resp. C homeomorphism,* δ*-C homeomorphism,* $\mathcal{P}$*-C homeomorphism,* $\delta\mathcal{P}$*-C homeomorphism,* $\delta\mathcal{S}$*-C homeomorphism,* $\delta\alpha$*-C homeomorphism* *and e^*-C homeomorphism) (briefly, $FH_ySeCHom$ (resp. FH_ySCHom, FH_yS $\delta CHom$, FH_yS $\mathcal{P}CHom$, FH_yS $\delta\mathcal{P}CHom$, FH_yS $\delta\mathcal{S}CHom$, FH_yS $\delta\alpha CHom$ & FH_ySe^*CHom)) if $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeIrr (resp. FH_ySIrr, $FH_yS\delta Irr$, $FH_yS\mathcal{P}Irr$, $FH_yS\delta\mathcal{P}Irr$, $FH_yS\delta\mathcal{S}Irr$, $FH_yS\delta\alpha Irr$ & FH_ySe^*Irr) mappings.*

Theorem 4.1. *Each $FH_ySeCHom$ (resp. FH_ySCHom, $FH_yS\delta$ $CHom$, FH_yS $\mathcal{P}$ $CHom$, FH_yS $\delta\mathcal{P}CHom$, $FH_yS\delta\mathcal{S}CHom$, $FH_yS\delta\alpha CHom$ & FH_ySe^*CHom) is a FH_ySeHom (resp. FH_ySHom, $FH_yS\delta Hom$, $FH_yS\mathcal{P}Hom$, $FH_yS\delta\mathcal{P}Hom$, $FH_yS\delta\mathcal{S}Hom$, $FH_yS\delta\alpha Hom$ & FH_ySe^*Hom). But not conversely.*

Proof. Let us assume that $(\tilde{\theta_2}, \zeta)$ is a FH_yScs in (Ω, M, σ). This shows that $(\tilde{\theta_2}, \zeta)$ is a FH_ySecs in (Ω, M, σ). By assumption, $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$ is a FH_ySecs in (Θ, L, τ). Hence, $\mathfrak{h}$ is a FH_ySeCts mapping. Hence, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are FH_ySeCts mappings. Hence $\mathfrak{h}$ is a FH_ySeHom.

The other cases are similar.

Example 4.1. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\phi_1, \phi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon'_1 \times \Upsilon'_2$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2\}, \Upsilon_2 = \{b_1, b_2\}\ \Upsilon'_1 = \{c_1, c_2\}, \Upsilon'_2 = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_2, b_1), \{\frac{\chi_1}{0.4}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.4}, \frac{\chi_2}{0.2}\}\rangle \end{Bmatrix}$$
$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_2, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1)\}$ is FH_ySts.

Let the $FH_ySs's$ $(\tilde{\psi_1}, \zeta_1)$ and $(\tilde{\psi_2}, \zeta_2)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\phi_1}{0.3}, \frac{\phi_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\phi_1}{0.2}, \frac{\phi_2}{0.4}\}\rangle \end{Bmatrix}$$
$$(\tilde{\psi_2}, \zeta_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\phi_1}{0.2}, \frac{\phi_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\phi_1}{0.5}, \frac{\phi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_2)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Theta, L) \to (\Omega, M)$ be a FH_yS mapping as follows:

$$\omega(\chi_1) = \phi_2, \omega(\chi_2) = \phi_1,$$
$$\nu(a_2, b_1) = (c_2, d_1), \nu(a_1, b_1) = (c_2, d_1),\ \nu(a_1, b_2) = (c_1, d_2),\ \nu(a_2, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_2}, \zeta_1), \mathfrak{h}^{-1}(\tilde{\psi_2}, \zeta_2) = (\tilde{\theta_1}, \zeta_2)$$

Here $\mathfrak{h}$ is FH_ySCts. Then $\mathfrak{h}$ is FH_ySeHom because $(\tilde{\theta_1}, \zeta_1)$ is FH_ySos in Θ and $\mathfrak{h}(\tilde{\theta_1}, \zeta_1) = (\tilde{\psi_1}, \zeta_1)$ is FH_ySeos in Ω. Also, $(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}^{-1}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is FH_ySeos in Θ. $\mathfrak{h}$ is not $FH_ySeCHom$ because $(\tilde{\theta_2}, \zeta_2)$ is FH_ySeos in Θ and $\mathfrak{h}(\tilde{\theta_2}, \zeta_2) = (\tilde{\psi_2}, \zeta_2)$ is not FH_ySeos in Ω. Then $\mathfrak{h}$ is FH_ySeHom but not $FH_ySeCHom$.

Theorem 4.2. *If* $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ *is a* $FH_ySeCHom$*, then* FH_yS $ecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) \subseteq \mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta))$ *for each* FH_ySs $(\tilde{\theta_2}, \zeta)$ *in* (Ω, M, σ).
Proof. Let $(\tilde{\theta_2}, \zeta)$ be a FH_ySs in (Ω, M, σ). Then, $FH_yScl(\tilde{\theta_2}, \zeta)$ is a FH_yScs in (Ω, M, σ) and every FH_yScs is a FH_ySecs in (Ω, M, σ). Assume $\mathfrak{h}$ is FH_ySeIrr and $\mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta))$ is a FH_ySecs in (Θ, L, τ). Then,

$$FH_yScl(\mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta))) = \mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta)).$$

$$\text{Here}, FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) \subseteq FH_ySecl(\mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta))) = \mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta)).$$

$$\text{Therefore}, FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) \subseteq \mathfrak{h}^{-1}(FH_yScl(\tilde{\theta_2}, \zeta))$$

for every FH_ySs $(\tilde{\theta_2}, \zeta)$ in (Ω, M, σ).

Theorem 4.3. *Let* $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ *be a* $FH_ySeCHom$*. Then* FH_ySecl $(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) = \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta))$ *for each* FH_ySs $(\tilde{\theta_2}, \zeta)$ *in* (Ω, M, σ).
Proof. Since $\mathfrak{h}$ is a $FH_ySeCHom$, $\mathfrak{h}$ is a FH_ySeIrr mapping. Let $(\tilde{\theta_2}, \zeta)$ be a FH_ySs in (Ω, M, σ). Clearly, $FH_ySecl(\tilde{\theta_2}, \zeta)$ is a FH_ySecs in (Ω, M, σ). Then $FH_ySecl(\tilde{\theta_2}, \zeta)$ is a FH_ySecs in (Ω, M, σ).

$$\begin{aligned}
\text{Since}, \mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta) &\subseteq \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta)),\\
\text{then}, FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) &\subseteq FH_ySecl(\mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta)))\\
&= \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta)).\\
\text{Therefore}, FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) &\subseteq \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta)).
\end{aligned}$$

Let $\mathfrak{h}$ be a F$H_ySeCHom$. $\mathfrak{h}^{-1}$ is a FH_ySeIrr mapping. Let us consider FH_ySs $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$ in (Θ, L, τ), which implies $FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))$ is a FH_ySecs in (Θ, L, τ). Hence, $FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))$ is a FH_ySecs in (Θ, L, τ). This implies that

$$(\mathfrak{h}^{-1})^{-1}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))) = \mathfrak{h}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)))$$

is a FH_ySecs in (Ω, M, σ) . This proves,

$$\begin{aligned}
(\tilde{\theta_2}, \zeta) = (\mathfrak{h}^{-1})^{-1}(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)) &\subseteq (\mathfrak{h}^{-1})^{-1}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)))\\
&= \mathfrak{h}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))).\\
\text{Therefore}, FH_ySecl(\tilde{\theta_2}, \zeta) &\subseteq FH_ySecl(\mathfrak{h}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))))\\
&= \mathfrak{h}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))),
\end{aligned}$$

since, $\mathfrak{h}^{-1}$ is a FH_ySeIrr mapping. Hence,

$$\mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2},\zeta)) \subseteq \mathfrak{h}^{-1}(\mathfrak{h}(FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)))) = FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)).$$

$$\text{That is, } \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2},\zeta)) \subseteq FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)).$$

$$\text{Hence, } FH_ySecl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)) = \mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2},\zeta)).$$

Remark 4.1. *The Theorems 4.2 and 4.3 are also true for FH_ySCHom, $FH_yS\delta CHom$, $FH_yS\mathcal{P}CHom$, $FH_yS\delta\mathcal{P}CHom$, $FH_yS\delta\mathcal{S}CHom$, $FH_yS\delta\alpha CHom$ & $FH_y\ Se^*CHom$ and their respective closure operators.*

Theorem 4.4. *If $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ and $\mathfrak{g} : (\Omega, M, \sigma) \to (P, \rho, Q)$ are $FH_ySeCHom$ (resp. FH_ySCHom, $FH_yS\delta CHom$, $FH_yS\mathcal{P}CHom$, $FH_yS\delta\mathcal{P}CHom$, $FH_yS\delta\mathcal{S}CHom$, $FH_yS\delta\alpha CHom$ & FH_ySe^*CHom)'s, then $\mathfrak{g}\circ\mathfrak{h}$ is a $FH_ySeCHom$ (resp. FH_ySCHom, $FH_yS\delta CHom$, $FH_yS\mathcal{P}CHom$, $FH_yS\delta\mathcal{P}CHom$, $FH_yS\delta\mathcal{S}CHom$, $FH_yS\delta\alpha CHom$ & FH_ySe^*CHom).*

Proof. Let $\mathfrak{h}$ and $\mathfrak{g}$ be two $FH_ySeCHom$'s. Assume $(\tilde{\theta_2},\zeta)$ is a FH_ySecs in (P,ρ,Q). Then, $\mathfrak{g}^{-1}(\tilde{\theta_2},\zeta)$ is a FH_ySecs in (Ω, M, σ). Then, by hypothesis, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{\theta_2},\zeta))$ is a FH_ySecs in (Θ, L, τ). Hence, $\mathfrak{g}\circ\mathfrak{h}$ is a FH_ySeIrr mapping. Now, let $(\tilde{\theta_1},\zeta)$ be a FH_ySecs in (Θ, L, τ). Then, by presumption, $\mathfrak{h}(\mathfrak{g})$ is a FH_ySecs in (Ω, M, σ). Then, by hypothesis, $\mathfrak{g}(\mathfrak{h}(\tilde{\theta_1},\zeta))$ is a FH_ySecs in (P,ρ,Q). This implies that $\mathfrak{g}\circ\mathfrak{h}$ is a FH_ySeIrr mapping. Hence, $\mathfrak{g}\circ\mathfrak{h}$ is a $FH_ySeCHom$.

The other cases are similar.

5. Cotangent Similarity Measure for Fuzzy Hypersoft Sets

In this section, we use cotangent functions to construct a new similarity measure for $FH_ySs's$.

Definition 5.1. *Consider two FH_ySs's $(\tilde{\theta_1},\zeta)$ and $(\tilde{\theta_2},\zeta)$ over Θ. The cotangent similarity measure for these two sets based on the cotangent function is given by*

$$S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = \frac{1}{n}\sum_{i=1}^{n} cot[\frac{\pi}{4} + \frac{\pi}{12}(|\mu^i_{\tilde{\theta_1}} - \mu^i_{\tilde{\theta_2}}|)].$$

Proposition 5.1. *The cotangent similarity measure $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$, satisfies the following properties:*

(i) $0 \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) \leq 1$.

(ii) $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_1},\zeta))$.

(iii) $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$ *iff* $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$.

(iv) *If* $(\tilde{\theta_3},\zeta)$ *is a* FH_ySs *in* Θ *and* $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$, *then* $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$ *and* $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_3},\zeta))$.

Proof. (i) Since the value of cotangent function and the membership value of FH_ySs's are in the interval $[0,1]$, the similarity measure based on the cotangent functions which is arithmetic mean of these cotangent functions, are also in $[0,1]$. Therefore, $0 \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) \leq 1$.

(ii) Proof is obvious.

(iii) For any two FH_ySs's $(\tilde{\theta_1},\zeta)$ and $(\tilde{\theta_2},\zeta)$ in Θ, if $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$, then $\mu^i_{(\tilde{\theta_1},\zeta)} = \mu^i_{(\tilde{\theta_2},\zeta)}$, for $i = 1,2,...,n$. Thus, we obtain $|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| = 0$.

And so the cotangent similarity measure $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$. Conversely, let $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$. Since $cot\frac{\pi}{4} = 1$, this implies that

$|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| = 0$.

Therefore, we obtain $\mu^i_{(\tilde{\theta_1},\zeta)} = \mu^i_{(\tilde{\theta_2},\zeta)}$, for $i = 1,2,3,...,n$. Hence, $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$.

(iv) If $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$, then $\mu^i_{(\tilde{\theta_1},\zeta)} \leq \mu^i_{(\tilde{\theta_2},\zeta)} \leq \mu^i_{(\tilde{\theta_3},\zeta)}$, for $i = 1,2,3,...,n$.

Thus, we have

$|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| \leq |\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}|$, $|\mu^i_{(\tilde{\theta_2},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}| \leq |\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}|$

Hence, $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$. Then, $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$ and

$S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_3},\zeta))$.

As the cotangent function is decreasing with the interval $[0,\frac{\pi}{4}]$, the proof is completed.

Similarly, the weighted version of cotangent similarity measure is given as

$WS_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = \frac{1}{n}\sum_{i=1}^{n} W_i cot[\frac{\pi}{4} + \frac{\pi}{12}(|\mu^i_{\tilde{\theta_1}} - \mu^i_{\tilde{\theta_2}}|)]$

where $0 \leq W_1, W_2, W_3, ..., W_n \leq 1$ with $\sum_{i=1}^{n} W_i = 1$.

6. Algorithm

In this section, the algorithm based on the proposed similarity measure is given.

As per the medical history, the various symptoms of Covid-19 are Fever, Headache, Dry Cough, Body pain, Chest pain and Difficulty in breathing. We categorize these symptoms as the distinct set of severe symptoms, most common symptom and less common symptoms.

Severe symptoms = Difficulty in breathing, Chest pain

Most common symptoms = Fever, Dry cough

Less common symptoms = Headache, Body pain

We can formulate the symptoms of the Covid-19 patients collected from the hospital records as FH_ySs's by considering the membership values as 'Covid-19' and 'No Covid-19'. The Covid-19 patients data are colleted from the hospital and they are formulated as a fuzzy hypersoft problem. Now, consider the patients visiting hospital with Covid-19 symptoms. Let us formulate those patients' symptoms as the FH_ySs's using the defined category of the symptoms. Based on the severity of the mentioned symptoms, the degree of membership and non membership values are taken in the FH_ySs's. Using the proposed cotangent similarity measure, the examination can be done by comparing the symptoms of the Covid-19 patients and the patients visiting hospital with the symptoms related to Covid-19. Thus, a decision can be made whether the patients have the possibility of suffering from Covid-19 or not.

We next give the implementation steps of the proposed algorithm based on cotangent similarity measure for FH_ySs's in which the flow chart of the proposed alogorithm is shown in the figure.

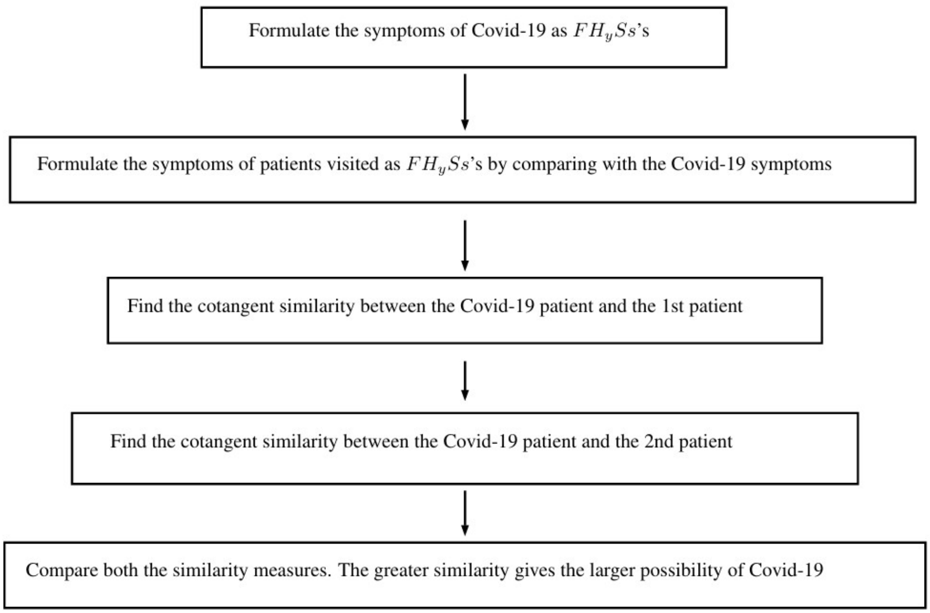

Figure 2. Flowchart of the proposed algorithm

Step 1: Formulate the symptoms of Covid-19 patients as a FH_ySs by considering the degree of relation between the Covid-19 patients and the Covid-19 symptoms.

Step 2: Formulate the symptoms of the two patients visited the hospital as FH_ySs's by considering the relation between the patients and the Covid-19 symptoms.

Step 3: Find the similarity between the symptoms of the Covid-19 patients and the 1st patient visited hospital using the proposed cotangent similarity measure.

Step 4: Find the similarity between the symptoms of the Covid-19 patients and the 2nd patient visited hospital using the proposed cotangent similarity measure.

Step 5: Compare both the similarity measures. The more the similarity, there is a higher chance for the patient to be suffering from Covid-19.

7. Application in Covid-19 Diagnosis using Cotangent Similarity Measure

Example 7.1. Consider 2 patients visiting hospital with the following symptoms: Fever, Head ache, Dry cough, Body pain, Chest pain and Difficulty in breathing. The symptoms of Covid-19 patients can be categorized as
Severe symptoms = Chest pain, Difficulty in breathing
Most common symptoms = Dry cough, Fever
Less common symptoms = Body pain, Headache
Using the FH_yS model problem, patients can be tested whether or not they are likely to be infected with Covid-19. Let Θ be the universal set $\Theta = \{\chi_1, \chi_2\} = \{\text{Covid-19, No Covid-19}\}$. The attributes are given as:

$$\Upsilon_1 = \{a_1 = \text{Chest pain},\ a_2 = \text{Difficulty in breathing}\ \}$$
$$\Upsilon_2 = \{b_1 = \text{Dry cough},\ b_2 = \text{Fever}\}$$
$$\Upsilon_3 = \{c_1 = \text{Body pain},\ c_2 = \text{Headache}\}$$

The FH_ySs's which give the degree of relation between the Covid-19 patients and the Covid-19 symptoms and between the 2 patients visited and their symptoms are defined below.

The FH_ySs $(\tilde{\theta_1}, \zeta)$ describes the evaluation of the Covid-19 patients and their symptoms as per the hospital records.

$$(\tilde{\theta_1}, \zeta) = \left\{ \begin{array}{c} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{1.0}, \frac{\chi_2}{0.2}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.2}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.1}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\} \rangle \end{array} \right\}$$

The FH_ySs's $(\tilde{\theta_2}, \zeta)$ and $(\tilde{\theta_3}, \zeta)$ describe the evaluation of the 2 patients visited and their symptoms respectively.

$$(\tilde{\theta_2}, \zeta) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.0}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.8}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\} \rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.4}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.6}, \frac{\chi_2}{0.4}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.2}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.2}\} \rangle \end{Bmatrix}$$

Using the proposed cotangent similarity measure, we get
$S_C((\tilde{\theta_1}, \zeta), (\tilde{\theta_2}, \zeta)) = 0.6661$
$S_C((\tilde{\theta_1}, \zeta), (\tilde{\theta_3}, \zeta)) = 0.9279.$
As the similarity between the Covid-19 patient and the 2nd patient is lesser than 1st patient, there is a higher chance for the 2nd patient suffering from Covid-19.

There are several similarity measures in fuzzy environment such as tangent similarity measure, cotangent similarity measure, cosine similarity measure etc. If the similarity between the two sets is more close to 1, there is a possiblity of more similarity between the given two sets. Using this concept, we have arrived for a decision in the above example. The other kind of similarities also give the same results. All the similarities can be applied in both fuzzy and neutrosophic environments depends on the membership functions. This application in fuzzy hypersoft sets accounts for noisy or incomplete data typically encountered in real-world medical settings.

While fuzzy hypersoft sets offer a powerful tool for handling uncertainty in medical diagnosis due to their ability to incorporate multiple parameters and vague information, their limitations in the medical field include: complexity in parameter selection, potential for overfitting, lack of interpretability in certain scenarios, difficulty in handling large datasets, and the need for substantial expert knowledge to accurately define parameters and membership functions; making it challenging to translate theoretical results into practical clinical applications in some cases. The fuzzy hypersoft set models can be thoroughly validated on large clinical datasets to ensure their effectiveness in real-world medical scenarios.

8. Conclusion

In this paper, FH_ySHom, $FH_yS\mathcal{S}Hom$, $FH_yS\delta Hom$, $FH_yS\mathcal{P}Hom$, $FH_yS\delta\mathcal{P}Hom$, $FH_yS\delta\mathcal{S}Hom$, $FH_yS\delta\alpha Hom$, FH_ySeHom & FH_ySe^*Hom and various forms of FH_ySCHom are introduced in FH_ySts and the properties are analyzed with the examples. Further, a cotangent similarity measure for FH_ySs's is introduced and an application in diagnosing Covid-19 using cotangent similarity measure is discussed with an example. In future, these findings can be extended to various forms of FH_yS contra continuous mapping, FH_yS contra open mapping, FH_yS contra closed mapping and FH_yS contra homeomorphism. By combining fuzzy hypersoft sets with other machine learning techniques in future, the interpretability and prediction accuracy can be enhanced.

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