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A NOVEL CLASS OF CONTINUITY VIA (Λ,δS)-CLOSED SETS

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Abstract: In this paper, we introduce the (Λ,δS) - continuity via (Λ,δS) - closed sets and the theorems based on them are discussed with counterexamples. Moreover, we entitle the Quasi (Λ,δS) continuity, Perfect (Λ,δS) - continuity, Totally (Λ,δS) - continuity, Strongly (Λ,δS) - continuity, Contra (Λ,δS) - continuity by applying (Λ,δS) - closed sets.

Keywords and Phrases: (Λ,δS)-closed set, (Λ,δS) - continuity, Quasi (Λ,δS) continuity, Perfect (Λ,δS) - continuity, Totally (Λ,δS) - continuity, Strongly (Λ,δS) - continuity, Contra (Λ,δS) - continuity.

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1. Introduction

In topology and its applications, the concept of a closed set is fundamental. Many researchers have defined classes of closed sets (see [1, 2, 5, 10, 11]); through them, new definitions of compactness and continuity have been found, see [10, 13]. From this point of view, we have defined a new class of open sets, namely (Λ,δS) - closed sets. Park. et al. (1997) introduce the notion of δ − semi-open sets, which are stronger than semi-open sets but weaker than δ − open sets. Georgiou (2004) developed the theory on generalization of δ − closed sets which is named as (Λ,δ) − closed sets using Λ − operator in terms of δ. In 2014, Binod Chandra Tripathy introduced the concept of generalized b-closed sets with respect to an ideal in bitopological spaces, which is the extension of the concepts of generalized b-closed sets. In 2013, Binod Chandra Tripathy et. al, introduced the concept of weakly b-continuous functions in bitopological spaces as a generalization of b-continuous functions and studied several properties of these functions. In 2011, Binod Chandra Tripathy et. al introduced the notion of b-locally open sets, bLO_* sets,bLO_{**} sets in bitopological spaces and obtain several characterizations and some properties of these sets.

Similarly we have introduced the concept of (Λ,δS) -closed sets were introduced in [13]. By using (Λ,δS) - closed sets in this article we introduced (Λ,δS) continuity (resp., (Λ,δS) – irresoluteness, Quasi (Λ,δS) – irresoluteness, Completely (Λ,δS) – irresoluteness) concepts are defined using (Λ,δS) – closed and some of their properties are analyzed, and we extend various results, properties concerning these concepts.

2. Preliminary

Throughout this paper (P,σ),(Q,τ) and (R,η) (simply P,Q,R) always mean topological space. The closure (resp., interior) will be denoted by Cl(R) (resp., Int(R)). Let R be a subset of P.

Definition 1. *A subset R of a topological space (P, σ) is called δ-open if R is the union of regular open sets. The complement of δ-open is called δ-closed.*

Lemma 1. *The intersection of arbitrary collection of δ - semiclosed sets in (P,σ) is δ - semiclosed.*

Definition 2. *Let R be a subset of a topological space (P.σ). Then $\Lambda_{\delta S}$(R) [also called $Ker_{\delta S}$(R)] is defined as follows:*

$$\Lambda_{\delta S}(\mathrm{R}) = \cap \{ \mathrm{U} \in \delta\mathrm{SO}(\mathrm{P},\sigma) \ / \ \mathrm{R} \subseteq \mathrm{U} \}.$$

Definition 3. *A subset R a topological space (P,σ) is known as (Λ,δS) - set if R = $\Lambda_{\delta S}$(R).*

Definition 4. *A subset R of a topological space (P,σ) is called $\lambda_g^{\delta S}$ - closed sets if $sCl_\delta(R) \subseteq U$ whenever $R \subseteq U$ and U is (Λ,δS)-open in P. The family of all $\lambda_g^{\delta S}$ - closed sets of (P,σ) is denoted by $\lambda_g^{\delta S}C$(P,σ).*

Definition 5. *Continuous : If $f^{-1}(V)$ is a closed set in (P,σ) for every closed set V in (Q,τ).*

Definition 6. *Semi - continuous : If $f^{-1}(V)$ is a semi - closed set in (P,σ) for every closed set V in (Q,τ).*

Definition 7. *δ - continuous : If $f^{-1}(V)$ in (P,σ) for every δ - closed set V in*

(Q,τ).

Definition 8. *δ - semi continuous : if $f^{-1}(V)$ is a δ-semi closed set in (P,σ) for every closed set V in (Q,τ).*

Definition 9. *δgs - continuous : if $f^{-1}(V)$ is a δgs-closed set in (P,σ) for every closed set V in (Q,τ).*

Definition 10. *$g\delta s$ - continuous : if $f^{-1}(V)$ is a $g\delta s$-closed set in (P,σ) for every closed set V in (Q,τ).*

Definition 11. *sg - continuous : if $f^{-1}(V)$ is a sg-closed set in (P,σ)for every closed set V in (Q,τ).*

Definition 12. *gs - continuous : if $f^{-1}(V)$ is a gs-closed set in (P,σ)for every closed set V in (Q,τ).*

Definition 13. *δg - continuous : if $f^{-1}(V)$ is a δg -closed set in (P,σ)for every closed set V in (Q,τ).*

Definition 14. *λ_g^δ - continuous : A function $f : (P,\sigma) \to (Q,\tau)$ is said to be λ_g^δ - continuous if the inverse image of every open set in (Q,τ) is λ_g^δ - open in (P,σ).*

Definition 15. *Super - continuous : if $f^{-1}(V)$ is a δ-closed set of (P,σ) for every closed set V of(Q,τ).*

Definition 16. *Contra - continuous : if $f^{-1}(V)$ is a δ-closed set of (P,σ) for every closed set V of(Q,τ).*

Definition 17. *Strongly - continuous : if $f^{-1}(V)$ is a closed in (P,σ) for every open subset V in (Q,τ).*

Definition 18. *Totally - continuous : if $f^{-1}(V)$ is a clopen in (P,σ) for every open subset V in (Q,τ).*

Definition 19. *α - continuous : if $f^{-1}(V)$ is a α - closed set in (P,σ) for every closed set V in (Q,τ).*

Lemma 2. *In a topological space (P,σ) the following properties hold.*

a) Every δ-semiclosed set is (Λ,δS)-closed set.

b) Every δ-open set is δ-semiopen set.

c) Every δ-semiclosed set is $\lambda_g^{\delta S}$ - closed set.

d) Every δ-closed set is $\lambda_g^{\delta S}$ - closed set.

e) Every regular closed set is $\lambda_g^{\delta S}$ - closed set.

f) Every $\lambda_g^{\delta S}$ - closed set is δgs-closed set.

g) Every $\lambda_g^{\delta S}$ - closed set is $g\delta s$-closed set.

3. (Λ,δS)-continuous function

Definition 20. *A map $f : (P,\sigma)(Q,\tau)$ is called (Λ,δS)-continuous function if the inverse image (V) of each open set V in (Q,τ) is (Λ,δS)–open in (P,σ).*

Theorem 1.

a) Every δ-semicontinuous function is (Λ,δS) continuous function.

b) Every super continuous function is (Λ,δS) continuous function.

Proof. a) Let f : (P,σ) → (Q,τ) be δ-semicontinuous. Then for V ∈ τ, f^{-1}(V) is δ-semiopen. But by Lemma 1(a), V is (Λ,δS) – open. Therefore f is (Λ,δS) – continuous.
b) Let f : (P,σ) → (Q,τ) be super continuous . Then for V ∈ τ, f^{-1}(V) is δ-open. But by Lemma 1(b), f^{-1}(V) is δ-semiopen. Therefore f is δ -semicontinuous. By(a), f is (Λ,δS) continuous.

Example 1. a) Let P = Q= {p, q, r, s} and σ {P, ∅, {p}, {r}, {p, q}, {p, r}, {p, q, r}, {p, r, s} }, τ = {Q, ∅,{p, q, r} }. Let a function f : (P,σ) → (Q,τ) be defined by f(p) = f(q) = f(r) = s, f(s) = r. Then f is (Λ,δS) continuous but not a δ-semicontinuous function.
b) Let P = Q = {p,q,r,s} and σ = {P,∅,{p},{q},{p,q} }, τ = {Q,∅, {p},{q},{p,q},{p,r} }. Let a function f : (P,σ) → (Q,τ) defined be an identity. Then f is (Λ,δS) continuous but not a super continuous function.

Definition 21. *A function f : (P,σ) → (Q,τ) is called*
(Λ,δS) – irresolute *if f^{-1}(V) is a (Λ,δS) – open subset of P for every (Λ,δS) - open subset V of Q.*
Quasi - (Λ,δS) – irresolute *if f^{-1}(V) is a (Λ,δS) – open subset of P for every δ-semiopen subset of Q.*

Theorem 2. *For a function f : (P,σ) → (Q,τ), the following statements are equivalent:*

a) f is (Λ,δS) – irresolute;

b) f^{-1}(B) is a (Λ,δS) – closed subset of P for every (Λ,δS) – closed subset B of Q;

c) For each $x \in P$ and for each (Λ,δS) – open set V of Q containing f(x) there exists a (Λ,δS) – open set U of P containing x and f(U) ⊆ V;

d) f((Λ,δS)Cl(A)) ⊂ [f((Λ,δS)Cl(A)] for each subset A of P;

e) [f^{-1} (Λ,δS)Cl(A)] ⊂ f^{-1} (Λ,δS)Cl(A) for each subset B of Q;

Proof. Claim (a)⇔(b)

Assume that f is (Λ,δS) – irresolute. Let W be any (Λ,δS)-closed subset of Q. Then Q – W is (Λ,δS)-open. Since f is (Λ,δS) – irresolute, f^{-1}(Q – W) is (Λ,δS)-open But f^{-1}(Q – W) = f^{-1}(Q) - f^{-1}(W) = P - f^{-1}(W) is (Λ,δS) – open. therefore f^{-1}(W) is (Λ,δS) – closed. Conversely, Let us assume W be a (Λ,δS) – open set of Q. Then Q – W is (Λ,δS) – closed in Q. Since f is (Λ,δS) – irresolute inverse image of each (Λ,δS) – closed set in Q is (Λ,δS) – closed in P, then f^{-1}(Q – W) is (Λ,δS) – closed in P (i.e) f^{-1}(Q) - f^{-1}(W) is (Λ,δS) – closed in P (i.e) P - f^{-1}(W) is (Λ,δS) – closed in P. Hence f^{-1}(W) is (Λ,δS) – open in P. ⇒ f is (Λ,δS) – irresolute.

Claim (b)⇔(c)

Let x ∈ P and V be any (Λ,δS) – open set of Q such that f(x) ∈ V. Then Q – V is (Λ,δS) – closed set and f^{-1}(Q – V) is (Λ,δS) – closed in P. As f(x) ∉ Q – V, x ∉ f^{-1}(Q – V) = P - f^{-1}(V) ⇒ x ∈ f^{-1}(V) which is (Λ,δS) open in P. Let U = f^{-1}(V). Then x ∈ U and f(U) ⊆ V Thus U is the required (Λ,δS) – open set such that x ∈ U and f(U) ⊆ V. Which implies (C) Conversely, Let W be (Λ,δS) closed in Q. Thus let V = Q – W then V be a (Λ,δS)-open set of Q and x ∈ f^{-1}(V). Then f(x) ∈ v By condition (c) there exists a (Λ,δS) – open set U_x in P such that x ∈ U_x and f(U_x) ⊆ V therefore x ∈ U_x ⊆ f^{-1}(V) Hence f^{-1}(V) = ∪ U_x/ x ∈ f^{-1}(V). we have f^{-1}(V) is (Λ,δS) – open in P Hence P - f^{-1}(V) = f^{-1}(Q – V) = f^{-1})(W). Which implies (b).

Claim (b)⇔(d)

Let A be any subset of P. Since ((Λ,δS)Cl(f(A))) is (Λ,δS) – closed in Q. Then by using (b) f^{-1} ((Λ,δS)Cl(f(A))) is (Λ,δS) – closed in P. Now A ⊆ f^{-1} (f(A)) ⊆ f^{-1} ((Λ,δS) Cl(f(A)) Now f((Λ,δS)Cl(A)) ⊆ f(f^{-1}((Λ,δS)(Cl(f(A))))) ⊆ (Λ,δS)Cl(f(A)) therefore f((Λ,δS)Cl(A)) ⊆ (Λ,δS)Cl(f(A)) Conversely, Let V be a (Λ,δS)-Closed subset of Q. Then f^{-1}(V) ⊆ P. By (d), f((Λ,δS)Cl(f^{-1}))) ⊆ (Λ,δS)Cl(f(f^{-1}(V))) ⊆ (Λ,δS)Cl(V) = V f^{-1}(f((Λ,δS)Cl(f^{-1}(V)))) ⊆ f^{-1}(V) ⇒ (Λ,δS)Cl(f^{-1}(V)) ⊆ f^{-1}(V) f^{-1}(V) ⊆ ((Λ,δS)Clf^{-1}(V)) ⇒ f^{-1}(V) = (Λ,δS)Cl(f^{-1}) Hence f^{-1}(V) is a (Λ,δS) closed subset of P for every (Λ,δS) closed subset of Q.

Claim (d)⇔(e)

Let B be a subset of Q. Then f^{-1}(B) is a subset of P. By (d) f((Λ,δS)Cl(f^{-1}(B))) ⊆ (Λ,δS)Cl(f(f^{-1}(B)) ⊆ (Λ,δS)Cl(B) Hence (Λ,δS)Cl(f^{-1}(B)) ⊆ f^{-1}((Λ,δS)Cl(B))

Conversely, Let A be a subset of Q. Take B = f(A) By(e) (Λ,δS)Cl(A) ⊆ f^{-1}(Λ,δS) Cl(f(A)). Hence f((Λ,δS)Cl(A)) ⊆ (Λ,δS)Cl(f(A)) Hence the proof.

Theorem 3. *For a function f : (P,σ) → (Q,τ), the following statements are equivalent:*

a) f is quasi-(Λ,δS)-irresolute:

b) f^{-1}(B) is a (Λ,δS) – closed subset of P for every δ-semiclosed subset B of Q.

c) for each x ∈ P and for each δ – semiopen set V of Q containing f(x) there exists a (Λ,δS) – open set U of P containing x and f(U) ⊆ V.

Proof. Obvious.

Theorem 4. *For a function f : (P,σ) → (Q,τ), the following statements are true*

a) If the map f is (Λ,δS) – irresolute, then the map f is quasi - (Λ,δS) – irresolute.

b) If the map f is δ – semicontinuous, then then map f is (Λ,δS) – continuous.

c) If the map f is δ – semiclosed, then the map f is (Λ,δS) – irresolute.

Proof. a) Let V be any δ – semiclosed subset of Q.
By Lemma 1, we get V is (Λ,δS) – closed subset of Q. Since f is (Λ,δS) – irresolute, f^{-1}(V) is (Λ,δS) – closed in P. ⇒ f is quasi (Λ,δS) – irresolute.
b) Let V be any closed subset of Q. Since f is δ – semicontinuous, f^{-1}(V) is δ – semiclosed in P. we have f^{-1}(V) is (Λ,δS) – closed in P. therefore f is (Λ,δS) – continuous.
c) Let V be a δ – semiclosed subset of Q. Since f is δ – semi-irresolute, f^{-1}(V) is δ – semiclosed in P. we have, f^{-1}(V) is (Λ,δS) – closed in P. therefore f is quasi (Λ,δS) – irresolute.

Remark 1. *Composition of two (Λ,δS) – continuous functions need not be (Λ,δS) – continuous as shown by the following example.*

Example 2. Let P = Q = {p,q,r,s}, and R = {p,q,r}. then σ = {P, ∅, {p }, {q}, {p,q}, {p, r}} Q = {Q, ∅, {p }, {q}, {p,q } } and R = {R, ∅, {r}}. Let the function f : (P,σ) → (Q,τ) and g :(Q,τ) → (R,η) be defined by f(p) = s,f(q) = r, f(r) = p, f(s) = q and g(p) = p, g(q) = q, g(r) = r, g(s) = s then Composition (Λ,δS) – continuous functions need not be (Λ,δS) – continuous.

Theorem 5. *Let f : (P,σ) → (Q,τ)and g : (Q,τ) → (R,η) be two function hold:*

a) *If f is (Λ,δS) – continuous and g is continuous then* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – continuous.*

b) *If f is quasi - (Λ,δS) – irresolute and g is δ – semicontinuous then,* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – continuous.*

c) *If f is (Λ,δS) – irresolute and g is super – continuous then,* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – continuous.*

d) *If f is (Λ,δS) – irresolute and g is completely – continuous, then* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – continuous.*

e) *If f is (Λ,δS) – irresolute and g is (Λ,δS) – continuous, then,* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – continuous.*

Proof. a) Let V be any closed subset of R. Since g is continuous, g^{-1}(V) is closed subset of Q. Since f is (Λ,δS) – continuous,
$f^{-1}(g^{-1}(\mathrm{V})) = (g_\circ f)^{-1}(\mathrm{V})$ is a (Λ,δS)-closed in P.
therefore $g_\circ f$ is (Λ,δS) – continuous.
b) Let V be any closed subset of R. Since g is δ - semicontinuous, g^{-1}(V) is δ – semiclosed subset of Q. Since f is quasi - (Λ,δS) – continuous,
$f^{-1}(g^{-1}(\mathrm{V})) = (g_\circ f)^{-1}(\mathrm{V})$ is a (Λ,δS)-closed in P.
therefore $g_\circ f$ is (Λ,δS) – continuous.
c) Since every super continuous function is (Λ,δS) – continuous, therefore we have g is (Λ,δS)-continuous. Let V be any closed subset of R. Since g is (Λ,δS) – continuous, g^{-1}(V) is (Λ,δS) – closed subset of Q. Since f is (Λ,δS) – irresolute.
$f^{-1}(g^{-1}(\mathrm{V})) = (g_\circ f)^{-1}(\mathrm{V})$ is a (Λ,δS) – closed in P.
therefore $g_\circ f$ is (Λ,δS) – continuous.
d) Since every completely continuous function is super – continuous
Therefore we have g is super – continuous. Hence from (c), the result follows.
e) Let V be closed subset of R. Since g is (Λ,δS) - continuous, g^{-1}(V) is(Λ,δS) - closed subset of Q. Since f is (Λ,δS) – irresolute,
$f^{-1}(g^{-1}(\mathrm{V})) = (g_\circ f)^{-1}(\mathrm{V})$ is a (Λ,δS)-closed in P
therefore $g_\circ f$ is (Λ,δS) – irresolute.

Theorem 6. *Let f : (P,σ) → (Q,τ)and g : (Q,τ) → (R,η) be two functions. Then the following hold:*

a) *If f is (Λ,δS) – irresolute and g is (Λ,δS) – irresolute, then* $g_\circ f$*: (P,σ) → (R,η) is (Λ,δS) – irresolute.*

b) If f is quasi - (Λ,δS) – irresolute and g is δ – semi-irresolute, then $g_{\circ}f$: (P,σ) → (R,η) is quasi-(Λ,δS) – irresolute.

c) If f is (Λ,δS) – irresolute and g is quasi-(Λ,δS) – irresolute, then $g_{\circ}f$: (P,σ) → (R,η) is quasi-(Λ,δS) – irresolute.

Proof. a) Let V be a (Λ,δS) – closed subset of R. Since g is (Λ,δS) – irresolute, g^{-1}(V) is (Λ,δS) – closed subset of Q. Since f is (Λ,δS) – irresolute,
f^{-1} (g^{-1} (V)) = $(g_{\circ}f)^{-1}$(V) is a (Λ,δS)-closed in P.
$g_{\circ}f$ is (Λ,δS) – irresolute.
b) Let V be a δ – semiclosed subset of R. Since g is δ – semi-irresolute, g^{-1}(V) is δ – semiclosed subset of Q. Since f is quasi - (Λ,δS) – irresolute,
f^{-1} (g^{-1} (V)) = $(g_{\circ}f)^{-1}$(V) is a (Λ,δS)-closed in P.
$g_{\circ}f$ is quasi (Λ,δS) – irresolute.
c) Let V be a δ – semiclosed subset of R. Since g is quasi - (Λ,δS) – irresolute, g^{-1}(V) is (Λ,δS) – closed subset of Q. Since f is (Λ,δS) – irresolute,
f^{-1} (g^{-1} (V)) = $(g_{\circ}f)^{-1}$(V) is a (Λ,δS)-closed in P.
$g_{\circ}f$ is quasi - (Λ,δS) – irresolute.

Definition 22. *A function f : (P,σ) → (Q,τ) is said to be a completely (Λ,δS) – irresolute function if the inverse image of every (Λ,δS) – open subset of Q is regular open in P.*

Example 3. Let P = {p,q,r,s}, Q = {p,q,r} and σ = {P,∅,{p},{q,r}}, τ = {Q,∅,{p},{q},{p,q},{p,r}}. Let a function f : (P,σ) → (Q,τ) be defined by f(p) = q, f(q) = p, f(r) = r. Then f is (Λ,δS) continuous but not a δ-semicontinuous function.

Theorem 7. *Let f : (P,σ) → (Q,τ)*

a) f is completely (Λ,δS) – irresolute.

b) The inverse image of every (Λ,δS) – closed subset of Y is regular closed in P.

Proof. Obvious.

Remark 2. *It is clear that every strongly continuous function is completely (Λ,δS) – irresolute. However the converse is not true by the following example.*

Example 4. Same as Example 4.

Theorem 8. *Every Completely (Λ,δS) – irresolute function is*

a) (Λ,δS) – irresolute.

b) δ – semi-irresolute.

c) Quasi - (Λ,δS) – irresolute.

d) R – map.

e) Almost δ – continuous.

Proof: a) The result follows from the fact that every regular open set is (Λ,δS) – open.
b) Let f : (P,σ) → (Q,τ) be a completely (Λ,δS) – irresolute function and V be an δ - semiopen in Q. By Lemma 1, we have V is be (Λ,δS) – open in Q. Since f is completely be (Λ,δS) – irresolute, f^{-1}(V) is regular open in P. Since every regular open set is δ-semiopen, f^{-1}(V) is δS-open in P. ⇒ f is δ– semiirresolute.
c) Let f : (P,σ) → (Q,τ) be a completely (Λ,δS) – irresolute function and V be an δ – semiopen in P. By Lemma 1, V is (Λ,δS) – open in Q. Since f is completely (Λ,δS) – irresolute, f^{-1}(V) is regular open in P. Since every regular open set is δ – semiopen, f^{-1}(V) is δ – semiopen in P. By Lemma 1, f^{-1}(V) is (Λ,δS) – open in P. ⇒ f is quasi (Λ,δS) – irresolute.
d) Let f : (P,σ) → (Q,τ) be a completely (Λ,δS) – irresolute function and V be a regular open set in Q. Since every regular open set is δ – semiopen. V is δ – semiopen in Q. By Lemma 1, V is (Λ,δS) open in Q. Since f is completely (Λ,δS) – irresolute, f^{-1} (V) is regular open in P. ⇒ f is R – map.
e) Let f : (P,σ) → (Q,τ) be a completely (Λ,δS) – irresolute function and V be a regular open set in Q. Since every regular open set is δ – semiopen. V is δ – semiopen in Q. By Lemma 1, V is (Λ,δS) open in Q. Since f is completely (Λ,δS) – irresolute, f^{-1} (V) is regular open in P. ⇒ f^{-1} (V) is δ – semiopen in P. Hence f is almost (Λ,δS) – continuous.

Remark 3. *The converse of the above theorem is not true as shown by the following examples.*

Example 5.

a) Let P = {p,q,r,s}, Q = {p,q,r,s} and σ = {X, ∅, {p}, {r}, {p,q}, {p,r}, {p,q,r}, {p,r,s}}, τ = {X, ∅, {p}, {r}, {p,q}, {p,r}, {p,q,r}, {p,r,s}}. Let a function f : (P,σ) → (Q,τ) be defined an identity function. Then f is (Λ,δS) continuous but not a δ-semicontinuous function.

b) Same as (a)

c) Let P = {p,q,r,s}, Q = {p,q,r,s} and σ = {X, $\emptyset$, {p}, {r}, {p,q}, {p,r}, {p,q,r}, {p,r,s}}, τ = {X, $\emptyset$, {p}, {r}, {p,q}, {p,r}, {p,q,r}, {p,r,s}}. Let a function f : (P,σ) $\rightarrow$ (Q,τ) be defined by f(p) = q, f(q) = p, f(r) = r, f(s) = r. Then f is (Λ,δS) irresolute but not a Complete (Λ,δS) irresolute

d) Same as (c).

Definition 23. *A space (P,σ) is said to be (Λ,δ) – space if every (Λ,δ) – closed subset of P is δ – semiclosed in P.*

Theorem 9. *Let f : (P,σ) $\rightarrow$ (Q,τ) be a completely α – irresolute function where Q is a (Λ,δS) – space then f is completely (Λ,δS) – irresolute.*

Proof. Let V be a (Λ,δS) – closed subset of Q. Since Q is a (Λ,δS) – space, V is δ – semiclosed, V is α – closed in Q. Since every δ – semiclosed set is α – closed, V is α – closed in Q. Now f being completely α – irresolute implies f^{-1}(V) is regular closed in P. therefore f is completely (Λ,δS) – irresolute.

Theorem 10. *Let f : (P,σ) $\rightarrow$ (Q,τ)and g : (Q,τ) $\rightarrow$ (R,η) be two functions. Then the following hold:*

a) If f is completely (Λ,δS) – irresolute and g is (Λ,δS) – continuous, then $g_\circ f$ is completely continuous.

b) If f is completely (Λ,δS) – irresolute and g is (Λ,δS) – irresolute, then $g_\circ f$ is completely (Λ,δS) – irresolute.

c) If f is almost δ – semi-continuous and g is completely (Λ,δS) – irresolute, then $g_\circ f$ is (Λ,δS) – irresolute.

d) If f is completely continuous and g is completely (Λ,δS) – irresolute, then $g_\circ f$ is completely (Λ,δS) – irresolute.

e) If f is a R – map and g is completely (Λ,δS) – irresolute, then $g_\circ f$ completely (Λ,δS) – irresolute.

f) If f is completely (Λ,δS) – irresolute and g is a R – map, then $g_\circ f$ is almost δ – semi-continuous

g) If f is almost δ – semi-continuous and g is completely (Λ,δS) – irresolute, then $g_\circ f$ is δ – semi-irresolute.

Proof. a) Let V be an open set in R. Since g is (Λ,δS) – continuous. g^{-1}(V) is (Λ,δS) – open in Q. Since f is completely (Λ,δS) – irresolute. $f^{-1}(g^{-1}$(V)) =

$g_\circ f^{-1}$(V) is regular open in P. Hence $g_\circ f$ is completely continuous.
b) Let V be an (Λ,δS) – open set in R. Since g is (Λ,δS) – irresolute, g^{-1}(V) is (Λ,δS) – open in Q. Since f is completely (Λ,δS) – irresolute. $f^{-1}(g^{-1}(\text{V})) = g_\circ f^{-1}(\text{V})$ is regular open in P. Hence $g_\circ f$ is completely (Λ,δS) – irresolute.
c) Let V be a (Λ,δS) - open set in R. Since g is completely (Λ,δS) – irresolute, g^{-1}(V) is regular open in Q. Since f is almost δ – semi continuous, $f^{-1}(g^{-1}(\text{V})) = g_\circ f^{-1}(\text{V})$ is δ – semiopen in P. By Lemma 1, $g_\circ f^{-1}$(V) is (Λ,δS) – open in P $\Rightarrow$ $g_\circ f$ is (Λ,δS) – irresolute.
d) Let V be an (Λ,δS) – open set in R. Since g is completely (Λ,δS) – irresolute, g^{-1}(V) is regular open in Q. Since every regular open is open we have g^{-1}(V) is open in P. Since f is completely continuous. $f^{-1}(g^{-1}(\text{V})) = g_\circ f^{-1}(\text{V})$ is regular open in P. $\Rightarrow$ $g_\circ f$ is completely (Λ,δS) – irresolute.
e) Let V be an (Λ,δS) – open in R. Since g is completely (Λ,δS) – irresolute, g^{-1}(V) is regular open in Q. Since f is a R – map, $f^{-1}(g^{-1}(\text{V})) = g_\circ f^{-1}(\text{V})$ is regular open in P. Hence $g_\circ f$ is completely (Λ,δS) – irresolute.
f) Let V be an regular open set in Q. Since g is a R-map, g^{-1}(V) is regular open in Q. Since every regular open is δ-semiopen, g^{-1}(V) is δ-semiopen. Using Lemma 1, g^{-1}(V) is (Λ,δS) – open Since f is completely (Λ,δS) – irresolute, $f^{-1}(g^{-1}(\text{V}))$ $= g_\circ f^{-1}(\text{V})$ is regular open in P. $g_\circ f^{-1}$(V) is δ-semiopen in P. $\Rightarrow$ $g_\circ f$ is almost δ-semicontinuous.
g) Let V be a δ – semi open set in R. Using Lemma 1, V is (Λ,δS)–open set. Since g is completely (Λ,δS) – irresolute, g^{-1}(V) is regular open in Q. Since f is almost δ – semi continuous, $f^{-1}(g^{-1}(\text{V})) = g_\circ f^{-1}(\text{V})$ is δ – semiopen in P. Hence $g_\circ f$ is δ – semi irresolute.

Theorem 11. *If f : (P,σ) → (Q,τ) is a surjective, δ^* - semiclosed function and g : (Q,τ) → (R,η) is a function such that $g_\circ f$: (P,σ) → (R,η) is completely (Λ,δS) – irresolute, then g is (Λ,δS) – irresolute.*

Proof. Let V be a (Λ,δS) – closed set in R. Since $g_\circ f$ is completely (Λ,δS) – irresolute. $g_\circ f^{-1}(\text{V}) = f^{-1}(g^{-1}(\text{V}))$ is regular closed in P. Since every regular closed set is δ – semiclosed, $f^{-1}(g^{-1}(\text{V}))$ is δ – semiclosed in P. Now f is δ^* - semiclosed and surjective implies f($f^{-1}(g^{-1}(\text{V}))$) = g^{-1}(V) is a (Λ,δS) – closed in Q. Thus g is a (Λ,δS) – irresolute.

Theorem 12. *From the above result we have the following diagram where A → B represents A implies but not conversely.*

a) Completely (Λ,δS) – irresolute

b) Almost δ – semi-continuous

c) δ – semi-irresolute

d) Quasi - (Λ,δS) – irresolute

e) (Λ,δS) – irresolute

f) Strongly continuous

Lemma 3. *Let S be an open subset of a topological space (P,σ). Then the following hold:*

a) If U is regular open in P, then so is $U \cap S$ in the subspace (S, σ_S).

b) If $B \subset S$ is regular open in (S, σ_S) there exists a regular open set U in (P,σ) such that $B = U \cap S$.

Theorem 13. *If f : (P,σ) $\rightarrow$ (Q,τ) is completely (Λ,δS) – irresolute and A is any open subset in P, then the restriction $f|_A : A \rightarrow Q$ is completely (Λ,δS) – irresolute.*
Proof. Let V be any (Λ,δS) – open subset of Q. Since f is completely (Λ,δS) – irresolute, f^{-1}(V) is regular open in P. Since A is open in P, by Lemma 1. $(f|_A)^{-1}$(V) = A $\cap$ f^{-1}(V) is regular open in A. And so $f|_A$ is completely (Λ,δS) – irresolute.

Lemma 4. *Let Q be a preopen subset of a topological space (P,σ). Then $Q \cap C$ is regular open in Q for every open subset U of P.*

Theorem 14. *If f : (P,σ) $\rightarrow$ (Q,τ) is completely (Λ,δS) – irresolute and A is any preopen subset in P, then the restriction $f|_A : A \rightarrow Q$ is completely (Λ,δS) – irresolute.*
Proof. Let V be any (Λ,δS) – open subset of Q. Since f is completely (Λ,δS) – irresolute, f^{-1}(V) is regular open in P. Since A is preopen in P, and by using Lemma 1 $(f|_A)^{-1}$(V) = A $\cap$ f^{-1}(V) is regular open in A And so $f|_A$ is completely (Λ,δS) – irresolute.

Theorem 15. *A topological space (P,σ) is connected if every completely (Λ,δS) – irresolute function from a space P into any T_0 – space Y is constant.*
Proof. Suppose P is not connected and every completely (Λ,δS) – irresolute function from a space P into Q is constant. Since P is not connected, there exist a proper non empty clopen subset A of P. Let Q = {p,q} and τ = {Q,$\emptyset$,{p},{q}} be a topology for Q. Let f: P $\rightarrow$ Q be a function such that f(A) = {p} and f(P – A) = {q}. Then f is non-constant completely (Λ,δS) – irresolute function such that Q is T_0, which is a contradiction. Hence P must be connected.

Definition 24. *A topological space (P,σ) is said to be*

a) *(Λ,δS) – connected if P cannot be written as a disjoint union of two nonempty (Λ,δS) – open subsets in P.*

b) *r – connected if P cannot be written as a disjoint union of two nonempty regular open subsets in P.*

c) *hyperconnected if every open subset of P is dense.*

Theorem 16. *If f : (P,σ) → (Q,τ) is completely (Λ,δS) – irresolute surjection and P is r-connected, then Q is (Λ,δS) – connected.*

Proof. Suppose Q is not (Λ,δS) – connected. Then Q = A ∪ B where A and B are disjoint nonempty (Λ,δS) – open subsets in Q. Since f is completely (Λ,δS) – irresolute surjection, f^{-1}(A) and f^{-1}(B) are regular open sets in P such that X = $f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A) \cap f^{-1}(B) = \emptyset$ which shows that P is not r-connected. Which is a contradiction. Hence Q is (Λ,δS) – connected.

Theorem 17. *Completely (Λ,δS)-connected image of hyperconnected space is (Λ,δS)-connected.*

Proof. Let f : (P,σ) → (Q,τ) be completely (Λ,δS)-irresolute function such that P is hyperconnected. Assume that B is proper (Λ,δS)-clopen subset of Q. Then A = f^{-1}(B) is both regular open and regular closed set in P as f is completely (Λ,δS) - irresolute. Therefore $A^{-} \neq$ P. This clearly contradicts the fact that P is hyperconnected . Thus Q is (Λ,δS) – connected.

Definition 24. *A topological space (P,σ) is said to be*

a) *(Λ,δS) - T_1 if every pair of distinct points x and y, there exists (Λ,δS) – open sets G and H containing x and y respectively such that $y \notin U$ and $x \notin V$.*

b) *(Λ,δS) - T_2 if for every pair of distinct points x and y there exists joint (Λ,δS) – open sets G and H containing x and y respectively.*

c) *r - T_1 if for every pair of disjoint points x and y, there exists r-open sets G and H connecting x and y respectively such that $x \notin H$ and $y \notin G$.*

Theorem 18. *If f : (P,σ) → (Q,τ) be completely (Λ,δS)-irresolute injective function and Q is (Λ,δS) - T_1, then P is r - T_1.*

Proof. Since Q is (Λ,δS) - T_1, for p ≠ q in P, there exist (Λ,δS) – open sets V and W such that f(q) ∉ V, f(P) ∉ W. Since f is completely (Λ,δS) – irresolute, f^{-1} (V) and f^{-1} (W) are regular open sets in P such that p ∈ f^{-1} (V) , q ∈ f^{-1} (W), p ∉ f^{-1} (W), q ∉ f^{-1} (V). This shows that P is r - T_1.

Theorem 19. *If f : (P,σ) → (Q,τ) be completely (Λ,δS)-irresolute injective function and Q is (Λ,δS) - T_2, then P is r - T_2.*

Proof. Since f is injective, f(x) ≠ f(y) for x,y ∈ P and x ≠ y. Since Q is (Λ,δS) - T_2 there exists (Λ,δS) – open sets G and H in Q such that f(x) ∈ G, f(y) ∈ H and G ∩ H = ∅. Let U = f^{-1} (G) and V = f^{-1} (H). Since f is completely (Λ,δS) – irresolute, U and V are regular open in P. Also x ∈ f^{-1} (G) = U, y ∈ f^{-1} (H) = V and U ∩ V = f^{-1} (G) ∩ f^{-1} (H) = f^{-1} (G ∩ H) = ∅ Hence P is (Λ,δS) - T_2.

4. Conclusion

In this paper, we have displayed the notions of (Λ,δS)-open sets and (Λ,δS)-closed sets and discussed their master properties. Then, we introduced some continuous function of (Λ,δS)-open sets. In addition, we have defined the so-called Quasi (Λ,δS) continuity, Perfect (Λ,δS) - continuity, Totally (Λ,δS) - continuity, Strongly (Λ,δS) - continuity, Contra (Λ,δS) - continuity via (Λ,δS)-open sets and the theorems based on them are discussed with counterexamples.

In future we will intruducing our set in Fuzzy topological space using fuzzy δ which introduced by Tripathy B.C. [15, 16, 21].

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