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SOME NEW SETS IN NANO SEMI-LOCAL FUNCTIONS

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Abstract: Ongoing research focuses on utilizing the existing generalized classes in nano semi-local functions within the framework of ideals. As part of this effort, we have successfully constructed and introduced new concepts and notions related to specific sets that address and handle semi-ideals within the space $\mathbb{N}^X_\star$. These new sets aim to expand the understanding and application of semi-ideal structures in this context.

Additionally, further investigations are being carried out to explore semi-ideal nano topological spaces by leveraging the generalized classes already established in $\mathbb{N}^X_\star$. This research seeks to delve deeper into the properties, behavior, and potential applications of these spaces. Through this, we aim to develop a more comprehensive framework that integrates semi-ideal concepts into nano topology, thereby enriching the theoretical and practical dimensions of the field.

Keywords and Phrases: $\mathbb{sI}^n_t$-set, $\mathbb{sI}^n_{t_\alpha}$-set, $\mathbb{sI}^n_{\mathcal{R}}$-set, $\mathbb{sI}^n_{\mathcal{R}_\alpha}$-set, $\mathbb{sI}^n_{t\#}$-set, $\mathbb{sI}^n_{t^\#_\alpha}$-set, $\mathbb{sI}^n_{\mathcal{R}\#}$-set, $\mathbb{sI}^n_{\mathcal{R}^\#_\alpha}$-set and $\mathbb{sI}^n_{\mathcal{SR}}$-set.

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1. Introduction and Preliminaries

R. Vaidyanathaswamy [17, 18] and K. Kuratowski [3] independently proposed the concept of ideals in this study. Lower approximation, upper approximation, and boundary are examined by Z. Pawlak [7], and M. Lellis Thivagar et al. [4] presented the ideas about weaker forms of $\mathbb{N}^X$. I. Rajasekaran (2023) recently introduced nano semi-local functions.

The following symbol is used by us throughout this paper: nano-open, nano-closed (resp. n-$\mathcal{OS}$, n-$\mathcal{CS}$) and $(U, \mathcal{N})$ we denote by $\mathbb{N}^X$, for an ideal nano topological spaces $(U, \mathbb{I}, \mathcal{N})$ we denote by $\mathbb{N}^X_\star$.

In the current work, Further research is being conducted using the existing generalised classes in nano semi-local functions in ideal, and we have constructed and introduced the notions of some new sets that look at and deal with semi-ideal nano topological spaces. Further research is being carried out in semi-ideal nano topological spaces using the created generalised classes of $\mathbb{N}^X_\star$.

Definition 1.1. *Let U be a non-empty finite set of objects called the universe and $\Re$ be an equivalence relation on U termed as indiscernibility relation. Then U is divided into disjoint equivalence classes.That is $L_\Re(X), U_\Re(X), B_\Re(X)\}$ where $L_\Re(X) = \bigcup_{x\in U}\{\Re(x) : \Re(x) \subseteq X\}$ and $U_\Re(X) = \bigcup_{x\in U}\{\Re(x) : \Re(x) \cap X \neq \phi\}$ and $B_\Re(X) = U_\Re(X) - L_\Re(X)$*

Definition 1.2. *Let U be the universe and $\Re$ be an equivalence relation on U and $\tau_{\Re(X)} = \{U, \phi, L_\Re(X), U_\Re(X), B_\Re(X)\}$, where $X \subseteq U$ and $\tau_{\Re(X)}$ satisfies the following conditions*

1. *U and $\phi \in \tau_{\Re(X)}$;*
2. *The collection of elements of any sub-collection of $\tau_{\Re(X)}$ is in $\tau_{\Re(X)}$;*
3. *The intersection of the elements of any finite sub-collection of $\tau_{\Re(X)}$ in $\tau_{\Re(X)}$. Thus $\tau_{\Re(X)}$ forms a topology on U.(U, $\tau_{\Re(X)}$) is called a nano topological space.*

Example 1.3. Let U $= \{10, 20, 50, 70\}$ with $U/\Re = \{\{10\}, \{50\}, \{20, 70\}\}$ and $X = \{10, 20\}$. The nano topology is $\tau_{\Re(X)} = \{\phi, U, \{50\}, \{10, 50\}, \{20, 50, 70\}\}$

Abbreviation	Description
$\tau_R(X)$ or $\mathcal{N}$	Nano Topology
$A^\star(U,\mathcal{N})$	Nano local function
$A^\star(U,\mathcal{N}_{\mathbb{S}})$	Nano semi-local function
n-$\mathcal{OS}$	Nano open set
n-$\mathcal{CS}$	Nano closed set
${}_{\mathbb{S}}\mathbb{I}^n_t$-set	Nano t-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set	Nano t_α-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set	Nano $\mathcal{R}$-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set	Nano $\mathcal{R}_\alpha$-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{t^\#}$-set	Nano $t^\#$-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set	Nano $t^\#_\alpha$-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#}$-set	Nano $\mathcal{R}^\#$-$\mathbb{I}_{\mathbb{S}}$-set
${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set	Nano $\mathcal{R}^\#_\alpha$-$\mathbb{I}_{\mathbb{S}}$-set

2. Some new nano sets in via semi-local functions

Definition 2.1. *A subset T in $\mathbb{N}^X_\star$ is referred to as nano*

1. ${}_{\mathbb{S}}\mathbb{I}^n_t$*-set if* $I_n(T) = I_n(C^\star_n(T))$,
2. ${}_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$*-set if* $I_n(T) = I_n(C^\star_n(I_n(T)))$,
3. ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$*-set if* $T = J_1 \cap J_2$*, where* J_1 *is* n-$\mathcal{OS}$ *and* J_2 *is* ${}_{\mathbb{S}}\mathbb{I}^n_t$*-set,*
4. ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$*-set if* $T = J_1 \cap J_2$*, where* J_1 *is* n-$\mathcal{OS}$ *and* J_2 *is* ${}_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$*-set.*

Example 2.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{c, d\}$. Then $\mathcal{N} = \{\phi, \{d\}, \{a, c\}, \{a, c, d\}, U\}$ and $\mathbb{I} = \{\phi, \{c\}\}$.

1. ${}_{\mathbb{S}}\mathbb{I}^n_t$-set $= \{\phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, U\}$.
2. ${}_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set $= \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$ $\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U\}$.
3. ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set $=\{\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\},$ $\{b, c, d\}, U\}$.
4. ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set $= \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$ $\{a, b, c\}, \{a, b, d\}, \{a, c, d\}\{b, c, d\}, U\}$.

Remark 2.3. *In space* $\mathbb{N}^X_\star$,

1. *if* E *is* n-$\mathcal{OS} \Longrightarrow E$ *is* ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$*-set.*

2. *if* E *is* ${}_{\mathbb{S}}\mathbb{I}^n_t$*-set* $\Longrightarrow E$ *is* ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$*-set.*

Remark 2.4. *The converses of Remark 2.3 are not true, as demonstrated in the examples below.*

Example 2.5. Let $U = \{2, 4, 6, 8\}$ with $U/R = \{\{4\}, \{8\}, \{2, 6\}\}$ and $X = \{6, 8\}$. Then $\mathcal{N} = \{\phi, \{8\}, \{2, 6\}, \{2, 6, 8\}, U\}$ and $\mathbb{I} = \{\phi, \{6\}\}$.

1. $\{4\}$ is not n-$\mathcal{OS}$ but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.

2. $\{2, 6, 8\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_t$-set but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.

Proposition 2.6. *Let* T *and* T_1 *be subsets of* $\mathbb{N}^X_\star$. *If* T *and* T_1 *are* ${}_{\mathbb{S}}\mathbb{I}^n_t$*-sets, then* $T \cap T_1$ *is* ${}_{\mathbb{S}}\mathbb{I}^n_t$*-set.*

Proof. Let T and T_1 be ${}_{\mathbb{S}}\mathbb{I}^n_t$-sets. Then there is $I_n(T \cap T_1) \subseteq I_n(C^\star_n(T \cap T_1)) \subseteq I_n(C^\star_n(T) \cap C^\star_n(T_1)) = I_n(C^\star_n(T)) \cap I_n(C^\star_n(T_1)) = I_n(T) \cap I_n(T_1) = I_n(T \cap T_1)$. Then $I_n(T \cap T_1) = I_n(C^\star_n(T \cap T_1))$ and hence $T \cap T_1$ is a ${}_{\mathbb{S}}\mathbb{I}^n_t$-set.

Example 2.7. In the above Example 2.5, $\{2, 6\}$ and $\{6, 8\}$ is ${}_{\mathbb{S}}\mathbb{I}^n_t$-set. But $\{2, 6\} \cap \{6, 8\} = \{6\}$ is ${}_{\mathbb{S}}\mathbb{I}^n_t$-set.

Proposition 2.8. *The next characteristics are identical for a* T *subset of a* $\mathbb{N}^X_\star$*:*

1. E *is* n-$\mathcal{OS}$,

2. E *is* ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS}$ & ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$*-set.*

Proof. (1) $\Longrightarrow$ (2): Let E be n-$\mathcal{OS}$. Then $E = I_n(E) \subseteq I_n(C^\star_n(E))$ and E is ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS}$. Also by Remark 2.3, E is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.

(2) $\Longrightarrow$ (1): Given E is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set. So $E = D_1 \cap D_2$ where D_1 is n-$\mathcal{OS}$ and $I_n(D_2) = I_n(C_n(D_2))$. Then $E \subseteq D_1 = I_n(D_1)$. Also, E is ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS} \Longrightarrow E \subseteq I_n(C_n(E)) \subseteq I_n(C^\star_n(D_2)) = I_n(D_2)$ by assuming. Thus $E \subseteq I_n(D_1) \cap I_n(D_2) = I_n(D_1 \cap D_2) = I_n(E)$ as well as E is n-$\mathcal{OS}$.

Remark 2.9. *In space* $\mathbb{N}^X_\star$, *the families of* ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS}$ *and* ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$*-set are independent.*

Example 2.10. In the above Example 2.5,

1. $\{2, 8\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set but ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS}$.

2. $\{4\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_p$-$\mathcal{OS}$ but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.

Remark 2.11. *In space* $\mathbb{N}^X_\star$,

1. *if E is n-$\mathcal{OS} \Longrightarrow E$ is $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.*
2. *if E is $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set $\Longrightarrow E$ is $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.*

The diagram depicts these connections..

$$\begin{array}{ccccc} {}_{\mathbb{S}}\mathbb{I}^n_t\textbf{-set} & & & & {}_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}\textbf{-set} \\ \downarrow & & & & \downarrow \\ {}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}\textbf{-set} & \longleftarrow & n\text{-}\mathcal{OS} & \longrightarrow & {}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}\textbf{-set} \end{array}$$

The converse of the figure is untrue, as seen in the next Example.

Example 2.12. In the above Example 2.5,

1. $\{4\}$ is not n-$\mathcal{OS}$ but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.
2. $\{2, 6, 8\}$ is not t-nI-set but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.
3. $\{2\}$ is not n-$\mathcal{OS}$ but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.
4. $\{2, 6, 8\}$ is not $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.

Proposition 2.13. *If T_1 and T_2 are $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-sets in $\mathbb{N}^X_\star$, then $T_1 \cap T_2$ is $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set.*

Proof. Let T_1 and T_2 be $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-sets. Then there is $I_n(T_1 \cap T_2) \subseteq I_n(C^\star_n(I_n(T_1 \cap T_2))) \subseteq I_n[C^\star_n(I_n(T_1)) \cap C^\star_n(I_n(T_2))] = I_n(C^\star_n(I_n(T_1))) \cap I_n(C^\star_n(I_n(T_2))) = I_n(T_1) \cap I_n(T_2) = I_n(T_1 \cap T_2)$. Then $I_n(T_1 \cap T_2) = I_n(C^\star_n(I_n(T_1 \cap T_2)))$ and hence $T_1 \cap T_2$ is a $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set.

Example 2.14. In the above Example 2.5, $\{4, 6\}$ and $\{2, 4\}$ is $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set. But $\{4, 6\} \cap \{2, 4\} = \{4\}$ is $_{\mathbb{S}}\mathbb{I}^n_{t_\alpha}$-set.

Proposition 2.15. *The next characteristics are identical for a E subset of a $\mathbb{N}^X_\star$:*

1. *E is n-$\mathcal{OS}$.*
2. *E is $_{\mathbb{S}}\mathbb{I}^n_\alpha$-$\mathcal{OS}$ and $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.*

Proof. (1) $\Longrightarrow$ (2): Let E be n-$\mathcal{OS}$. Then $E = I_n(E) \subseteq C^\star_n(I_n(E))$ and $E = I_n(E) \subseteq I_n(C^\star_n(I_n(E)))$. Therefore E is $_{\mathbb{S}}\mathbb{I}^n_\alpha$-$\mathcal{OS}$. Also by (1) of Remark 2.11, E is a $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set.

(2) $\Longrightarrow$ (1): Given E is a $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}_\alpha}$-set. So $E = D_1 \cap D_2$ where D_1 is n-$\mathcal{OS}$ and $I_n(D_2) = I_n(C^\star_n(I_n(D_2)))$. Then $E \subseteq D_1 = I_n(D_1)$. Also E is $_{\mathbb{S}}\mathbb{I}^n_\alpha$-$\mathcal{OS}$ implies $E \subseteq I_n(C^\star_n(I_n(E))) \subseteq I_n(C^\star_n(I_n(D_2))) = I_n(D_2)$ by assumption. Thus $E \subseteq I_n(D_1) \cap I_n(D_2) = I_n(D_1 \cap D_2) = I_n(E)$ and E is n-$\mathcal{OS}$.

3. On a few novel kinds of Semi-ideal Nano sets

Definition 3.1. *A subset E in $\mathbb{N}^X_\star$ is referred to as nano*

1. *$_{\mathbb{S}}\mathbb{I}^n_{t\#}$-set if $I_n(E) = C^\star_n(I_n(E))$.*

2. *$_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set if $I_n(E) = C^\star_n(I_n(C^\star_n(E)))$.*

3. *$_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set if $E = L \cap J$, where L is n-$\mathcal{OS}$ and J is $_{\mathbb{S}}\mathbb{I}^n_{t\#}$-set.*

4. *$_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set if $E = L \cap J$, where L is n-$\mathcal{OS}$ and J is $_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set.*

5. *$_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set if $E = L \cap J$, where L is n-$\mathcal{OS}$ and J is $_{\mathbb{S}}\mathbb{I}^n_t$-set and $I_n(C^\star_n(J)) = C^\star_n(I_n(J))$.*

Example 3.2. Let $U = \{2, 3, 5, 7\}$ with $U/R = \{\{3\}, \{7\}, \{2, 5\}\}$ and $X = \{5, 7\}$. Then $\mathcal{N} = \{\phi, \{7\}, \{2, 5\}, \{2, 5, 7\}, U\}$ and $\mathbb{I} = \{\phi, \{5\}\}$.

1. $_{\mathbb{S}}\mathbb{I}^n_{t\#}$-set $= \{\phi, \{2\}, \{3\}, \{5\}, \{7\}, \{2, 3\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{2, 3, 5\}, \{2, 3, 7\}, \{3, 5, 7\}, U\}$.

2. $_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set $= \{\phi, \{3\}, \{5\}, \{7\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{3, 5, 7\}, U\}$.

3. $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set $= \{\phi, \{2\}, \{3\}, \{5\}, \{7\}, \{2, 3\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{2, 3, 5\}, \{2, 3, 7\}, \{2, 5, 7\}, \{3, 5, 7\}, U\}$.

4. $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set $= \{\phi, \{3\}, \{5\}, \{7\}, \{2, 5\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{2, 3, 5\}, \{2, 5, 7\}, \{3, 5, 7\}, U\}$.

5. $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set $= \{\phi, \{3\}, \{5\}, \{7\}, \{2, 5\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{3, 5, 7\}, U\}$.

Remark 3.3. *In space $\mathbb{N}^X_\star$,*

1. *if E is n-$\mathcal{OS} \Longrightarrow E$ is $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set.*

2. *if E is $_{\mathbb{S}}\mathbb{I}^n_{t\#}$-set $\Longrightarrow E$ is $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set.*

Remark 3.4. *As illustrated in the next two examples, the opposite in every portion of the Remark 3.3 is not need to be true.*

Example 3.5. In the above Example 2.5,

1. $\{2\}$ is not n-$\mathcal{OS}$ but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set.

2. $\{2, 6, 8\}$ is not $_{\mathbb{S}}\mathbb{I}^n_{t\#}$-set but $_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}\#}$-set.

Proposition 3.6. *If L and T are $\mathbb{sI}^n_{t\#}$-sets in $\mathbb{N}^X_\star$, then $L \cap T$ is $\mathbb{sI}^n_{t\#}$-set.*
Proof. Let L and T be $\mathbb{sI}^n_{t\#}$-sets. $I_n(L \cap T) \subseteq I_n(L \cap T) \subseteq C^\star_n(I_n(L \cap T)) = C^\star_n(I_n(L) \cap I_n(T)) \subseteq C^\star_n(I_n(L)) \cap C^\star_n(I_n(T)) = I_n(L) \cap I_n(T)$ (by guess) $= I_n(L \cap T)$. Thus $I_n(L \cap T) = C^\star_n(I_n(L \cap T))$ and hence $L \cap T$ is $\mathbb{sI}^n_{t\#}$-set.

Theorem 3.7. *The next characteristics are identical for a T subset of a $\mathbb{N}^X_\star$:*

1. *T is n-$\mathcal{OS}$,*

2. *T is $\mathbb{sI}^n_s$-$\mathcal{OS}$ & $\mathbb{sI}^n_{\mathcal{R}\#}$-set.*

Proof. (2)$\Longleftarrow$(1): (2) is followed by Remark 3.3 of 3.3 and (1) of [13].

(1)$\Longleftarrow$(2): Given that T is $\mathbb{sI}^n_{\mathcal{R}\#}$-set. So $T = T_1 \cap T_2$ where T_1 is n-$\mathcal{OS}$ and $I_n(T_2) = C^\star_n(I_n(T_2))$. Then $T \subseteq T_1 = I_n(T_1)$. Also T is $\mathbb{sI}^n_s$-$\mathcal{OS}$ implies $T \subseteq C^\star_n(I_n(T)) \subseteq C^\star_n(I_n(T_2)) = I_n(T_2)$ by guess. Thus $T \subseteq I_n(T_1) \cap I_n(T_2) = I_n(T_1 \cap T_2) = I_n(T_1)$ and so T is n-$\mathcal{OS}$.

Remark 3.8. *In space $\mathbb{N}^X_\star$, the families of $\mathbb{sI}^n_s$-$\mathcal{OS}$ and $\mathbb{sI}^n_{\mathcal{R}\#}$-set are independent.*

Example 3.9. In the above Example 2.5,

1. $\{2, 4, 6\}$ is not $\mathbb{sI}^n_{\mathcal{R}\#}$-set but $\mathbb{sI}^n_s$-$\mathcal{OS}$.

2. $\{4\}$ is not $\mathbb{sI}^n_s$-$\mathcal{OS}$ but $\mathbb{sI}^n_{\mathcal{R}\#}$-set.

Remark 3.10. *In space $\mathbb{N}^X_\star$,*

1. *if T is n-$\mathcal{OS} \Longrightarrow T$ is $\mathbb{sI}^n_{\mathcal{R}^\#_\alpha}$-set.*

2. *if T is $\mathbb{sI}^n_{t^\#_\alpha}$-set $\Longrightarrow T$ is $\mathbb{sI}^n_{\mathcal{R}^\#_\alpha}$-set.*

Remark 3.11. *As illustrated in the next two examples, the opposite in every portion of the Remark 3.10 is not need to be true.*

Example 3.12. In the above Example 2.5,

1. $\{4\}$ is not n-$\mathcal{OS}$ but $\mathbb{sI}^n_{\mathcal{R}^\#_\alpha}$-set.

2. $\{2, 6\}$ is not $\mathbb{sI}^n_{t^\#_\alpha}$-set but $\mathbb{sI}^n_{\mathcal{R}^\#_\alpha}$-set.

Proposition 3.13. *If L and T are $\mathbb{sI}^n_{t^\#_\alpha}$-sets in $\mathbb{N}^X_\star$, then $L \cap T$ is $\mathbb{sI}^n_{t^\#_\alpha}$-set.*
Proof. Let L and T be $\mathbb{sI}^n_{t^\#_\alpha}$-sets. $I_n(L \cap T) \subseteq I_n(L \cap T) \subseteq I_n(C^\star_n(L \cap T)) \subseteq C^\star_n(I_n(C^\star_n(L \cap T))) \subseteq C^\star_n(I_n(C^\star_n(L))) \cap C^\star_n(I_n(C^\star_n(T))) = I_n(L) \cap I_n(T)$ (by guess)

$= I_n(L \cap T)$. Then $I_n(L \cap T) = C_n^\star(I_n(C_n^\star(L \cap T))$ and hence $L \cap T$ is ${}_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set.

Remark 3.14. *In space $\mathbb{N}^X_\star$, the families of ${}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathcal{OS}$ and ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set are independent.*

Example 3.15. In the above Example 2.5,

1. $\{2\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set but ${}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathcal{OS}$.

2. $\{4\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathcal{OS}$ but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set.

Theorem 3.16. *Regarding a subset L in $\mathbb{N}^X_\star$, the following attributes are interchangeable.*

1. *L is n-$\mathcal{OS}$;*

2. *T is ${}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathcal{OS}$ and ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set.*

Proof. (2)$\Longleftarrow$(1): (2) is followed by Remark 3.3 of [13] and (1) of Remark 3.10.

(1)$\Longleftarrow$(2): Given that L is a ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}^\#_\alpha}$-set. Thus $L = T_1 \cap T_2$ where T_1 is n-$\mathcal{OS}$ and T_2 is ${}_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set. Then $L \subseteq T_1 = I_n(T_1)$. Also L is ${}_{\mathbb{S}}\mathbb{I}^n_\beta$-$\mathcal{OS}$ implies $L \subseteq C_n^\star(I_n(C_n^\star(T_1))) \subseteq C_n^\star(I_n(C_n^\star(T_2))) = I_n(T_2)$ since T_2 is ${}_{\mathbb{S}}\mathbb{I}^n_{t^\#_\alpha}$-set. Thus $L \subseteq I_n(T_1) \cap I_n(T_2) = I_n(T_1 \cap T_2) = I_n(L)$ as well as L is n-$\mathcal{OS}$.

Remark 3.17. *For a subset E in $\mathbb{N}^X_\star$, the next relations are true:*

1. *E is n-$\mathcal{OS} \Longrightarrow E$ is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set.*

2. *E is ${}_{\mathbb{S}}\mathbb{I}^n_t$-set with $I_n(C_n^\star(E)) = C_n^\star(I_n(E)) \Longrightarrow E$ is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set.*

Proof. Since the Definition of ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set, the proof follows directly.

Remark 3.18. *As illustrated in the following example, the converses of Remark 3.17(1) are not true.*

Example 3.19. In Example 2.5, $\{4\}$ is not n-$\mathcal{OS}$ but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set.

Proposition 3.20. *In a space $\mathbb{N}^X_\star$, if E is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set $\Longrightarrow E$ is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.*
Proof. Proof is provided by the fact that ${}_{\mathbb{S}}\mathbb{I}^n_t$-set E with $I_n(C_n^\star(E)) = C_n^\star(I_n(E))$ is ${}_{\mathbb{S}}\mathbb{I}^n_t$-set, that is a ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set (Definition 3.1 by (3)).

Remark 3.21. *As illustrated in the following examples, the opposite of Proposition 3.20 does not have to be true.*

Example 3.22. In the above Example 2.5, $\{2, 4, 6\}$ is not ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set but ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{R}}$-set.

Theorem 3.23. *For a subset L in $\mathbb{N}^X_\star$, the next attributes are interchangeable:*

1. *L is n-$\mathcal{OS}$*

2. *L is ${}_{\mathbb{S}}\mathbb{I}^n_b$-$\mathcal{OS}$ and ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set.*

Proof. (1)$\Longrightarrow$(2): (2) is followed by Remark 3.3 of [13] and (1) of Remark 3.17.

(2)$\Longrightarrow$(1): Given T is ${}_{\mathbb{S}}\mathbb{I}^n_{\mathcal{SR}}$-set. So $L = T_1 \cap T_2$ where T_1 is n-$\mathcal{OS}$ and T_2 is ${}_{\mathbb{S}}\mathbb{I}^n_t$-set with $I_n(C^\star_n(T_2)) = C^\star_n(I_n(T_2))$. Then $L \subseteq T_1 = I_n(T_2)$. Also L is ${}_{\mathbb{S}}\mathbb{I}^n_b$-$\mathcal{OS}$ implies $L \subseteq I_n(C^\star_n(T_1)) \cup C^\star_n(I_n(T_2)) \subseteq I_n(C^\star_n(T_2)) \cup C^\star_n(I_n(T_2)) = I_n(L)$ by guess. Thus $L \subseteq I_n(T_1) \cap I_n(T_2) = I_n(T_1 \cap T_2) = I_n(L)$ as well as L is n-$\mathcal{OS}$.

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