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NEW SOFT EXPERT METRIC APPLICATIONS

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Abstract: Motivated by the study of shabir and Naz [15] on soft topological spaces, we introduce the new propositions of soft expert metric space, soft expert closed sphere, soft expert closed set and studied some properties of these concepts. Which are fundamental for further researches on soft expert metric spaces.

Keywords and Phrases: Soft expert set, soft expert point, soft expert open set, soft expert closed set, soft expert metric spaces and so on.

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1. Introduction and Definitions

There are Uncertainties in many Complicated Problems in the fields of engineering, physics, Computer Science, Medical science, Social Science and economics. These problems can not be solved by Classical Methods. For Solving these Problems Molodtsov [13] Introduced the concept of soft set. It is a type of mathematical tool that helps to solve problems dealing with uncertain data. The notion of Soft topological Spaces was introduced by Shabir and Naz [15] on an initial Universe with a fixed set of Parameters and they investigated Some Properties of soft topological Spaces. Maji et al. [10, 11] introduced several application of soft sets in decision making Problems. Following this soft, Metric spaces [2, 4, 5, 6], neighborhood properties of Soft topological spaces were studied in [14]. Further the subject of Soft expert metric spaces has also not been studied yet. For this reason, using the definition of soft expert Set given by [1]. Ulucay, Sahin and Olgan [17] introduce a notion of soft expert metric and Soft expert metric space based on soft expert Point of soft expert Sets, notions of Soft expert real number and some properties of Soft expert metric spaces. In this paper, we continue investigating the Property of soft expert metric Space with Propositions based on soft expert closed sphere with soft expert Point in soft expert metric spaces.

2. Preliminary Theorems

In this section we recall some basic notions in soft experts Set theory [1] Let U be Universe, E a Set of Parameters and X a set of experts. Let O be a set opinions. $Z = E \times O \times X$ and $A \subseteq Z$.

Definition 2.1. ([1]) *A pair (F, A) is called a soft expert set over U, Where F is mapping given by $F : A \to P(U)$, where $P(U)$ denotes the Power Set of U. We assume in this paper, two-Valued opinions only in set O that is $O = \{0 = disagree, 1 = agree\}$. But multi - Valued opinions may be assumed as well.*

Definition 2.2. ([1]) *For two soft expert sets (F, A) and (G, B) over U, $(F.A)$ is called a soft expert subset of (G, B) if*

1. $A \subseteq B$,
2. $\forall \varepsilon \in B,\ G(\varepsilon) \subseteq F(\varepsilon)$.

This relationship is denoted by $(F, A)\tilde{\subseteq}(G, B)$. In this case (G, B) is called a soft expert superset of (F, A).

Definition 2.3. ([1]) *Two soft expert sets (F, A) and (G, B) over a common universe U, are said to be equal if (F, A) is a soft expert subsets of (G, B) and $(G.B)$ is a soft expert subset of (F, A).*

Definition 2.4. ([1]) *Let E be a set of parameters and X a set of experts. The NOT set of $Z = E \times O \times X$ denoted by IZ, is defined by*

$$IZ = \{(I_{e_i,x_j,o_k}), \forall i, j, k\}$$

Where, Ie_i is not e_i.

Theorem 2.5. ([1]) *The complement of a soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, IA)$, where $F^c : IA \to P(U)$ is mapping given by*

$$F(\alpha)^c = U - F(I_\alpha), \quad \forall \alpha \in IA.$$

Definition 2.6. ([1]) *An agree soft expert set* $(F, A)_1$ *over* U *is a soft expert subset of* (F, A) *defined as follow:*

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Definition 2.7. ([1]) *An disagree soft expert set* $(F, A)_0$ *over* U *is a soft expert subset of* (F, A) *defined as follow:*

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

Definition 2.8. ([1]) *The union of two soft expert sets* (F, A) *and* (G, B) *over* U *denoted by* $''(F, A)\widetilde{\cup}(G, B)''$, *is the soft expert set* (H, C), *where* $C = A \cup B$ *and*

$$\forall e \in C, H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We express it as $(F, A)\widetilde{\cup}(G, B) = (H, C)$.
The following definition of intersection of two soft expert sets is given as that of the bi intersection in.

Definition 2.9. ([1]) *The intersection of two soft expert sets* (F, A) *and* (G, B) *over* U *denoted by* $''(F, A)\widetilde{\cap}(G, B)''$, *is the soft expert set* (H, C), *where* $C = A \cap B \forall e \in C$, *and*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cap G(e), & \text{if } e \in A \cap B \end{cases}$$

We express it as $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 2.10. ([1]) *If* (F, A) *and* (G, B) *are two soft expert sets over* U *then* (F, A) *AND* (G, B) *denoted by* $(F, A) \wedge (G, B)$, *is defined by* $(F, A) \wedge (G, B) = (H, A \times B)$, *where* $H(\alpha, \beta) = F(\alpha)\widetilde{\cap}G(\beta); \forall(\alpha, \beta) \in A \times B$.

Definition 2.11. ([1]) *If* (F, A) *and* (G, B) *are two soft expert sets over* U *then* (F, A) *OR* (G, B) *denoted by* $(F, A) \vee (G, B)$ *is defined by* $(F, A) \vee (G, B) = (O, A \times B)$, *where* $O(\alpha, \beta) = F(\alpha)\widetilde{\cup}G(\beta); \forall(\alpha, \beta) \in A \times B$.

3. Soft Expert Real Number, Soft Expert Point and Soft Expert Metric Spaces

Definition 3.1. *Let* R *be the set of real numbers and* $B(R)$ *be collection of all non-empties bounded subsets of* R *and* E *a set parameters and* X *a set of expert. Let*

O be a set opinions, $Z = E \times O \times X$ and $A \subseteq Z$. Then a mapping $F : A \to B(R)$ is called a soft expert real set. It is denoted by (F, A). If specifically (F, A) is a singleton soft expert set, then identifying (F, A) with the corresponding soft expert element, it will be called a soft expert real number and denoted $\widetilde{k}, \widetilde{m}, \widetilde{n}$such that $\widetilde{k}(e) = e, \forall e \in A$ etc. For example $\bar{1}$ soft expert real number $\bar{1}(e) = 1 \forall e \in A$.

Definition 3.2. [17] *For two soft expert real numbers;*

1. $\widetilde{k} \widetilde{\leq} \widetilde{m}$ if $\widetilde{k}(a) \widetilde{\leq} \widetilde{m}(a) \forall a \in A$;
2. $\widetilde{k} \widetilde{\geq} \widetilde{m}$ if $\widetilde{k}(a) \widetilde{\geq} \widetilde{m}(a) \forall a \in A$;
3. $\widetilde{k} \widetilde{<} \widetilde{m}$ if $\widetilde{k}(a) \widetilde{<} \widetilde{m}(a) \forall a \in A$;
4. $\widetilde{k} \widetilde{>} \widetilde{m}$ if $\widetilde{k}(a) \widetilde{>} \widetilde{m}(a) \forall a \in A$.

Definition 3.3. [17] *Soft expert set (P, A) over U is said to be soft expert point if there is exactly one $e \in A$, such that $P(a) = \{\widetilde{x}\}$ for some $\widetilde{x} \in U$ and $P(a') = \o \forall a' \in A\{a\}$. It will be denoted by $\widetilde{x}_a$.*

Definition 3.4. [17] *Two soft expert points $\widetilde{x}_a$,$\widetilde{y}_a$ are said to be equal if $a' = a$ and $P(a) = P(a')$ i.e. $\widetilde{x} = \widetilde{y}$. Thus $\widetilde{x}_a = \widetilde{y}_a \Leftrightarrow \widetilde{x} \neq \widetilde{y}$, or $a' \neq a$.*

Theorem 3.5. [17] *The union of any collection of soft expert points can be considered as soft expert set and every soft expert set can express as union of all soft expert points belonging to it.*

$$(F, A) = \bigcup_{\widetilde{x}_a \in (F,A)} \widetilde{x}_a$$

Let $\widetilde{U}$ be absolute soft expert set i.e. $F(a) = U$, $a \in A$, where, $(F, A) = \widetilde{U}$ and $SEP\widetilde{U}$ be collection of all soft expert point $\widetilde{U}$ and $R(A)^*$ denoted the set of all non-negative soft expert real numbers.

Definition 3.6. [17] *A mapping $\widetilde{d} : SEP(\widetilde{U}) \times SEP(\widetilde{U}) \to R(A)^*$ is said to be a soft expert metric on the soft expert set $\widetilde{U}$ if $\widetilde{d}$ satisfies the following conditions:*

SEM 1 $\widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) \widetilde{\geq} \bar{0} \ \ \forall \widetilde{x}_{a^1}, \widetilde{y}_{a^2} \in \widetilde{U}$,

SEM 2 $\widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) = \bar{0} \ \Leftrightarrow \widetilde{x}_{a^1} = \widetilde{y}_{a^2}$,

SEM 3 $\widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) = \widetilde{d}(\widetilde{y}_{a^2}, \widetilde{x}_{a^1}) \ \ \forall \widetilde{x}_{a^1}, \widetilde{y}_{a^2} \in \widetilde{U}$,

SEM 4 $\forall \widetilde{x}_{a^1}, \widetilde{y}_{a^2}, \widetilde{z}_{a^3} \in \widetilde{U}, \widetilde{d}(\widetilde{x}_{a^1}, \widetilde{z}_{a^3}) \widetilde{\leq} \widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) + \widetilde{d}(\widetilde{y}_{a^2}, \widetilde{z}_{a^3})$.

The soft expert set $\widetilde{U}$ with soft expert metric $\widetilde{d}$ on $\widetilde{U}$ is called a soft expert metric space and denoted by $(\widetilde{U}, \widetilde{d}, A)$.

Definition 3.7. [17] *Let $(F, A)(\neq) \in s(\widetilde{U})$, then the collection of all soft expert elements of (F, A) will be denoted by $SEE(F, A)$. For a collection B of soft expert elements of $\widetilde{U}$, the soft expert set generated by B is denoted by $SES(B)$.*

Example 3.8. [17] Let $\widetilde{U}$ is being a non- empty set and E a set of parameters and X a set of experts. Let O be a set opinions, $Z = E \times O \times X$ and $A \subseteq Z$. Let $\widetilde{U}$ be the absolute soft expert set i.e. $F(r) = \widetilde{U}, \forall r \in A$, where $(F, A) = \widetilde{U}$. We define $\forall \widetilde{x}_{a^1}, \widetilde{y}_{a^2} \in U$, $\widetilde{d} : SEP(\widetilde{U}) \times SEP(\widetilde{U}) \to R(A)^*$ by

$$\widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) = \begin{cases} \bar{0}, & \widetilde{x}_{a^1} = \widetilde{y}_{a^2} \\ \bar{1}, & \widetilde{x}_{a^1} \neq \widetilde{y}_{a^2} \end{cases}$$

Normally $\widetilde{d}$ satisfies all the soft expert metric axioms. So, $\widetilde{d}$ is soft expert metric on the soft expert set $\widetilde{U}$ and $(\widetilde{U}, \widetilde{d}, A)$. $\widetilde{d}$ is called the discrete soft expert metric on the soft expert set $\widetilde{U}$ and $(\widetilde{U}, \widetilde{d}, A)$ is said to be the discrete soft expert metric space.

Definition 3.9. [17] *Let $(\widetilde{U}, \widetilde{d}, A)$ be a soft expert metric space and $\widetilde{\varepsilon}$ be a non-negative soft expert real number*

$$D(\widetilde{x}_{a^1}, \widetilde{\varepsilon}) = \{\widetilde{y}_{a^2} \in \widetilde{U} : \widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) \widetilde{<} \widetilde{\varepsilon}\} \widetilde{\subseteq} SEP(\widetilde{U})$$

is called the soft expert open ball with center $\widetilde{x}_{a^1}$ an radius $\widetilde{\varepsilon}$ and

$$D(\widetilde{x}_{a^1}, \widetilde{\varepsilon}) = \{\widetilde{y}_{a^2} \in \widetilde{U} : \widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) \widetilde{\leq} \widetilde{\varepsilon}\} \widetilde{\subseteq} SEP(\widetilde{U})$$

is called the soft expert closed ball with center $\widetilde{x}_{a^1}$, an radius $\widetilde{\varepsilon}$.

Definition 3.10. [17] *Let $(\widetilde{U}, \widetilde{d}, A)$ be a soft expert metric space and (F, B) be a soft expert subset of $\widetilde{U}$.(F, B) is called a soft expert open subset of $\widetilde{U}$ and $\widetilde{\varepsilon}$ non-negative soft expert real number if; $D(\widetilde{x}_a, \widetilde{\varepsilon}) \widetilde{\subseteq} (F, B) \forall \widetilde{x}_a \in (F, B)$.*

Definition 3.11. [17] *Let $(\widetilde{U}, \widetilde{d}, A)$ be a soft expert metric space. A soft expert set $(G.R) \widetilde{\subseteq} \widetilde{U}$ is said to be "soft expert closed in $\widetilde{U}$ with respect to $\widetilde{d}$" if its complement $(G, R)^c$ is soft expert open set in $(\widetilde{U}, \widetilde{d}, A)$.*

Theorem 3.12. [17] *In a soft expert metric space every soft expert open ball is a soft expert open set.*

4. Main Theorems

In this Section We presented different definition and applications of soft Expert distance between two Soft Expert Point Using Soft Expert metric of $\widetilde{d}$

Proposition 4.1. *Let $(\widetilde{x}_{a^1}), (\widetilde{y}_{a^2}) \widetilde{\in} \widetilde{U}$ and $\widetilde{r} \in \widetilde{R}(A)^*$. We can define a soft Expert function for Soft Expert sets as the following*

$$\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = \begin{cases} 0, & \widetilde{x}_{a^1} = \widetilde{y}_{a^2} \\ \widetilde{r}, & \widetilde{r} \geq 0, \widetilde{x}_{a^1} \neq \widetilde{y}_{a^2} \end{cases}$$

This defined $\widetilde{d}$ Soft Expert function is soft expert metric on $\widetilde{U}$.
Proof. Let $(\widetilde{x}_{a^1}), (\widetilde{y}_{a^2}) \widetilde{\in} \widetilde{U}$

(i) If $\widetilde{r} = \widetilde{o}$ then, $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0$ and if $\widetilde{r} \geq \widetilde{o}$ then, $\widetilde{x}_{a^1} \neq \widetilde{y}_{a^2}$ and $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 1 > 0$.

(ii) If $\widetilde{r} = \widetilde{o}$ then $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0$ and conversely if $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0$ then, $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $\widetilde{r} = \widetilde{o}$.

(iii) If $\widetilde{r} = \widetilde{o}$ then, $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0 = \widetilde{d}((\widetilde{y}_{a^2}), (\widetilde{x}_{a^1}))$. If $\widetilde{r} \geq \widetilde{o}$ then, $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = \widetilde{r} = \widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2}))$ then the soft expert symmetry property is observed.

(iv) Let $\widetilde{r} = \widetilde{o}$ then, $\widetilde{x}_{a^1} = \widetilde{y}_{a^2}$ and $(\widetilde{x}_{a^1}), (\widetilde{y}_{a^2}), (\widetilde{z}_{a^3}) \widetilde{\in} \widetilde{U}$ from the axiom (i), $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0$, $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{z}_{a^3})) = 0$ and $\widetilde{d}((\widetilde{z}_{a^3}), (\widetilde{y}_{a^2})) = 0$ and then, $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) \leq \widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{z}_{a^3})) + \widetilde{d}((\widetilde{z}_{a^3}), (\widetilde{y}_{a^2}))$ on the other hand if $\widetilde{r} \geq \widetilde{o}$ then $\widetilde{x}_{a^1} \neq \widetilde{y}_{a^2}$.

Hence $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = \widetilde{r}$, $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{z}_{a^3})) = \widetilde{r}$ and $\widetilde{d}((\widetilde{z}_{a^3}), (\widetilde{y}_{a^2})) = \widetilde{r}$. So that $\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) \leq \widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{z}_{a^3})) + \widetilde{d}((\widetilde{z}_{a^3}), (\widetilde{y}_{a^2}))$ and the soft expert triangular inequality is satisfied.

Definition 4.2. *The soft expert metric space $(\widetilde{U}, \widetilde{d})$ defined in proposition [4.1] is called as soft expert discrete metric.*

Proposition 4.3. *Let $(\widetilde{U}, \widetilde{d})$ be a soft expert metric space for all $(\widetilde{x}_{a^1}), (\widetilde{y}_{a^2}) \widetilde{\in} \widetilde{U}$, $\widetilde{r} \in \widetilde{R}(A)^*$., the soft expert function $\widetilde{d_E}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = \min\{\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})), \widetilde{r}\}$ is a soft expert metric on $\widetilde{U}$.*
Proof. We need to prove that $\widetilde{d_E}$ satisfies the soft expert metric space axioms

(i) $\widetilde{d_E}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) \geq 0$ is obvius
$\widetilde{d_E}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})) = 0 \Leftrightarrow \min\{\widetilde{d}((\widetilde{x}_{a^1}), (\widetilde{y}_{a^2})), r\} = 0$

$\Leftrightarrow \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = 0 \Leftrightarrow (\widetilde{x}_{a^1}) = (\widetilde{y}_{a^2})$. The converge is also true. Then (i) is satisfied.

(ii) If $\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))$ then
$\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \widetilde{d_E}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))$. If
$\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \widetilde{r}$ then $\widetilde{d_E}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) = \widetilde{r}$
Hence, $\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \widetilde{d_E}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))$

(iii) For all $(\widetilde{x}_{a^1})$, $(\widetilde{y}_{a^2})$, $(\widetilde{z}_{a^3})\widetilde{\in}\widetilde{U}$
If $\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \widetilde{r}$ or $\widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2})) = \widetilde{r}$ then
$\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) + \widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2})) \geq \widetilde{r} = \widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))$.
If $\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) < \widetilde{r}$ Ve $\widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2})) < \widetilde{r}$
Then $\widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) = \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))$ Ve $\widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2})) = \widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))$ consequently
$\widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2})) = \min\{\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}),\widetilde{r})\} \leq$
$\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) \leq \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) + \widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))$
$= \widetilde{d_E}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) + \widetilde{d_E}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))$ is satisfied.
Hence, $(\widetilde{U},\widetilde{d_E})$ is a soft expert space

Example 4.4. In a soft expert metric space $(\widetilde{U},\widetilde{d})$ prove that

$$|\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) - \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))| \leq \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))\ \forall\ \widetilde{x}_{a^1},\widetilde{y}_{a^2},\widetilde{z}_{a^3}\widetilde{\in}\widetilde{U}$$

Solution. : Let $\widetilde{x}_{a^1},\widetilde{y}_{a^2},\widetilde{z}_{a^3}$ be may three soft expert points in $\widetilde{U}$, then by triangle Inequality, we have

$$\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) \leq \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) + \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))$$

$$\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) - \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3})) \leq \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) \tag{1}$$

or
Again by triangle inequality

$$\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3})) \leq \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) + \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))$$

$$\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3})) - \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) \leq \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))$$

$\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))$ by symmetry $\widetilde{d}$

$$-\{\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) - \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))\} \leq \widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) \tag{2}$$

It follows from (1) and (2) that

$$|\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))-\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))|\leq\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))$$

$$\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))\geq|\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))-\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))|$$

Theorem 4.5. *Let $(\widetilde{U},\widetilde{d})$ be any soft expert metric space and Let $\widetilde{E}$ be a soft expert positive number, then there exists a soft expert metric $\widetilde{d}^*$ for $\widetilde{U}$ such that the soft expert metric space $(\widetilde{U},\widetilde{d}^*)$ is bounded with $\delta(\widetilde{U})\leq\widetilde{E}$*

Proof. We define $\widetilde{d}^*$ by

$$\widetilde{d}^*((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))=\frac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}$$

Whenever $\widetilde{x}_{a^1},\widetilde{y}_{a^2}\in\widetilde{U}$

Then $\widetilde{d}^*$ is a soft expert metric for $\widetilde{U}$ as shown below:

$[M_1]$

$\widetilde{d}^*((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))=0$

$\Leftrightarrow\dfrac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}=0$

$\Leftrightarrow\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))=0$

$\Leftrightarrow\widetilde{x}_{a^1}=\widetilde{y}_{a^2}$

$[M_2]$

$\widetilde{d}^*((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))=\dfrac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}$

$=\dfrac{\widetilde{E}\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))}{1+\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))}=\widetilde{d}^*((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1}))$ by $[M_2]$ for $\widetilde{d}$

$[M_3]$

Let $\widetilde{x}_{a^1},\widetilde{y}_{a^2},\widetilde{z}_{a^3}\widetilde{\in}\widetilde{U}$ then

$\widetilde{d}^*((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))=\dfrac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}$

$=\widetilde{E}-\dfrac{\widetilde{E}}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}$

$\leq\widetilde{E}-\dfrac{\widetilde{E}}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}$

$\leq\dfrac{\widetilde{E}[\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))]}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}$

$$= \frac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))} + \frac{\widetilde{E}\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}$$

$$\leq \frac{\widetilde{E}\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))}{1+\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3}))} + \frac{\widetilde{E}\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}{1+\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))}$$

$\widetilde{\mathbf{d}}^*((\widetilde{x}_{a^1}),(\widetilde{z}_{a^3})) + \widetilde{\mathbf{d}}^*((\widetilde{z}_{a^3}),(\widetilde{y}_{a^2}))$

Also $\widetilde{\mathbf{d}}^*((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2})) = \dfrac{\widetilde{E}}{1+\dfrac{1}{\widetilde{d}((\widetilde{x}_{a^1}),(\widetilde{y}_{a^2}))}}$

$\leq \widetilde{E}\ \forall\ \widetilde{x}_{a^1}, \widetilde{y}_{a^2} \in \widetilde{U}$

Hence $\widetilde{\mathbf{d}}^*$ is a bounded soft expert metric for $\widetilde{U}$ with $\widetilde{\delta}(\widetilde{U}) \leq \widetilde{E}$.

4.6. Proposition of soft expert closed sphere

Proposition 4.6.1. *In a soft expert metric space every soft expert closed sphere is soft expert closed set.*

Proof. Let $(\widetilde{U}, \widetilde{d})$ be a soft expert metric space and $\widetilde{D}[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]$ be a soft expert closed sphere in $\widetilde{U}$, so

$$D[\widetilde{x}_{a^2}, \widetilde{\varepsilon}] = \{\widetilde{y}_{a^2} \widetilde{\in} \widetilde{U} : \widetilde{d}(\widetilde{x}_{a^1}, \widetilde{y}_{a^2}) \widetilde{\leq} \widetilde{\varepsilon}\} \widetilde{\subseteq} SEP(\widetilde{U})$$

We shall show that $\widetilde{D}[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]^c$ is soft expert open.
So suppose $\widetilde{D}[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]^c = \widetilde{\emptyset}$
Let $\widetilde{y}_{a^2} \widetilde{\in} D[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]^c$
Then $\widetilde{y}_{a^2} \widetilde{\notin} D[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]$ and so
$\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) - \widetilde{\varepsilon} = \widetilde{\rho}$, say is a non negative soft expert real number.
Now we take a soft expert open sphere of radius $(\widetilde{\rho})$ centered at $(\widetilde{y}_{a^2})$ and show that
$\widetilde{D}[\widetilde{x}_{a^2}, \widetilde{\varepsilon}]^c$ is open for this we have to show that $D[\widetilde{y}_{a^2}, \widetilde{\rho}] \widetilde{\subseteq} D[\widetilde{x}_{a^1}, \widetilde{\varepsilon}]^c$
Let $(\widetilde{z}_{a^3}) \widetilde{\in} D[\widetilde{y}_{a^2}, \widetilde{\rho}]$ then
$\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3})) \widetilde{\leq} \widetilde{\rho}$
Now
$\widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) \widetilde{\leq} \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3})) + \widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{x}_{a^1}))$
$\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{x}_{a^1})) \widetilde{\geq} \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) - \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{z}_{a^3}))$
$\widetilde{\geq}\ \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) - \widetilde{d}((\widetilde{y}_{a^2}),(\widetilde{x}_{a^1})) - \widetilde{\varepsilon}$
$\widetilde{d}((\widetilde{z}_{a^3}),(\widetilde{x}_{a^1})) \widetilde{\geq} \widetilde{\varepsilon}$.
Hence, $(\widetilde{z}_{a^3}) \widetilde{\in} D[\widetilde{x}_{a^1}, \widetilde{\varepsilon}]^c$
Thus we have to show that
$(\widetilde{z}_{a^3}) \widetilde{\in} D[\widetilde{y}_{a^2}, \widetilde{\rho}]^c$

$(\widetilde{z}_{a^3})\widetilde{\in}D[\widetilde{x}_{a^1},\widetilde{\varepsilon}]^c$
Therefore $D[\widetilde{y}_{a^2},\widetilde{\rho}]\widetilde{\subseteq}D[\widetilde{x}_{a^1},\widetilde{\varepsilon}]^c$
It follows that $D[\widetilde{x}_{a^1},\widetilde{\varepsilon}]^c$ is a soft expert open.
Therefore $D[\widetilde{x}_{a^1},\widetilde{\varepsilon}]$ is soft expert closed.

Proposition 4.6.2. *In soft expert metric space, the intersection of arbitrary collection of soft expert closed sets is soft expert closed.*
Proof. Suppose $(\widetilde{U},\widetilde{d})$ be a soft expert metric space and Let $\{F_{\widetilde{\lambda}} : \widetilde{\lambda} \in \widetilde{\wedge}\}$ be an arbitrary collection of soft expert closed subset of $\widetilde{U}$. Then we have to show that $\bigcap\limits_{\widetilde{\lambda}\in\widetilde{\wedge}} h\widetilde{\lambda}$ is also soft expert closed set,
Where $\widetilde{\wedge}$ is a soft expert index set.
$F\widetilde{\lambda}$ is a closed.
$\Rightarrow F^c_{\widetilde{\lambda}}$ is open.
$\Rightarrow \bigcup\limits_{\widetilde{\lambda}\in\widetilde{\wedge}} F^c_{\widetilde{\lambda}}$ is open.
$\Rightarrow \left(\bigcap\limits_{\widetilde{\lambda}\in\widetilde{\wedge}} F_{\widetilde{\lambda}}\right)^c$ is open. $\Rightarrow \bigcap\limits_{\widetilde{\lambda}\in\widetilde{\wedge}} F_{\widetilde{\lambda}}$ is closed.

5. Conclusion

In this Paper, motivated by the study of shabir and Naz [15] on soft topological spaces, we introduce the new propositions of soft expert closed sphere and soft expert metric spaces are studied.

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References

[1] Alkhazaleh, S. and Salleh A. R., Soft expert sets, Advances in Decision Sciences, vol. 2011, Article ID 757868, (2011), 12 pages.

[2] Bayramov, S., Gunduz(Aras) C., Soft locally compact and soft paracompact spaces, Journal of Mathematics and System Science, in press.

[3] Chen, D., Tsang E. C. C., Yeung D. S., and Wang X., The parameterization reduction of soft sets and its applications, Computers Mathematics with Applications, 49(5-6) (2005), 757-763.

[4] Das, Sujoy and Samanta, S. K., Soft real sets, soft real numbers and their properties, J. Fuzzy Math., 20(3) (2012), 551-576.

[5] Das, Sujoy and Samanta, S. K., Soft metric, Annals of Fuzzy Mathematics and Informatics, 6(1) (2013), 77-94.

[6] Das, Sujoy and Samanta, S. K., On soft metric spaces, J. Fuzzy Math, in press.

[7] Gregori, V. and Salvador R., Some properties of fuzzy metric spaces, Fuzzy Sets and Systems, 115(3) (2000), 485-489.

[8] Kaleva, O. and Seppo S., On fuzzy metric spaces, Fuzzy Sets and Systems 12(3) (1984), 215-229.

[9] Kramosil, I. and Jiri Michálek, Fuzzy metrics and statistical metric spaces, Kybernetika, 11(5) (1975), 336-344.

[10] Maji, P. K., Biswas, R. and Roy A. R., Soft set theory, Computers Mathematics with Applications, 45(4-5) (2003), 555-562.

[11] Maji, P. K., Roy, A. R., and Biswas R., An application of soft sets in a decision making problem, Computers Mathematics with Applications, 44(8-9) (2002), 1077-1083.

[12] Mashhour, Ali S., and Nehad N. Morsi, Fuzzy metric neighbourhood spaces, Fuzzy Sets and Systems, 45(3) (1992), 367-388.

[13] Molodtsov D., Soft set theory-first results, Computers Mathematics with Applications, 37(4-5) (1999), 19-31.

[14] Nazmul, Sk, and Samanta S. K., Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform, 6 (2013), 1-15.

[15] Shabir M. and Naz M., On soft topological Spaces, Computer and Mathematics with Applications, 61 (2011), 1786-1799.

[16] Sharma Sushil, On fuzzy metric space, Southeast Asian Bulletin of Mathematics, 26(1) (2003), 133-145.

[17] Ulucay V., Sahim M., Necati olgun and Adem on Soft Expert Metric Spaces, Malaysian Journal of Mathematical sciences, 10(2) (2016), 221-231.

[18] Zahedi Khameneh A., Kılıçman A. and Salleh A. R., Fuzzy soft product topology, Annals of Fuzzy Mathematics and Information, 7(6) (2014), 935-947.

[19] Zahedi Khameneh A., Kılıçman A. and Salleh A. R., Fuzzy soft boundary, Annals of Fuzzy Mathematics and Information, 8(5) (2014), 687-703.

[20] Zahedi Khameneh A., Kılıçman A. and Salleh A. R., An Adjustable Method for Data Ranking based on Fuzzy Soft Sets, Indian Journal of Science and Technology, DOI: 10.17485/ijst/2015/v8i21/72587.