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BIANCHI V COSMOLOGICAL MODEL WITH STRANGE QUARK MATTER IN $f(R)$ GRAVITY

P. P. Khade and S. M. Warbhe*

Department of Mathematics,
Vidya Bharati Mahavidyalya, Maharashtra, INDIA

E-mail : mathsvbmvpk@gmail.com

*Department of Mathematics
Vidya Bharati Mahavidyalya,
Amravati - 444601, Maharashtra, INDIA

E-mail : mudeshital26@gmail.com

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Abstract: In this research, we have studied behaviours of quark matter and strange quark matter which exist in $f(R)$ gravity in presence of Bianchi type universe. In order to obtain a deterministic solution, we have considered the scale factor which is a combination of two factors: the one is the usual power law expansion and the second one is an exponential function. We investigate exact $f(R)$ functions for Bianchi V as the contribution of strange quark and quark matter. Effect of the curvature function $f(R)$ is also observed on dynamical parameters. As per the observation both the pressure and density depend on $f(R)$ gravity and the bag constant. Hence both p_q and ρ_q remain positive. Then $\rho_{sq} \to \infty$ as $t \to 0$, and $\rho_{sq} \to 3Bc$ as $t \to \infty$. the statefinder parameters r, s converge to $1, 0$ in the distant future and found that our model aligns with the ΛCDM model both at present and in the future. Finally we discussed our physical solutions.

Keywords and Phrases: $f(R)$ Gravity, Quark matter, Strange quark matter, Bianchi V space time.

2020 Mathematics Subject Classification: 83C05, 83C15, 85A40.

1. Introduction

The f(R) gravity theory is favored over other modified theories due to its cosmological significance, particularly in the context of f(R) models that incorporate higher order curvature invariants as functions of R. These models offer solutions to issues such as dark matter and the hierarchy problem in high energy physics. The transition from deceleration to acceleration in the universe is effectively explained by f(R) theory. f(R) gravity is obtained on replacement of R by f(R) in the standard Einstein's Hilbert action, where f(R) is a general function of the Ricci scalar. Nojiri and Odintsov [1] have proposed that f(R) gravity allows for a unification of early-time inflation and late-time acceleration. Various researchers, including Paul[2], Chakrabarti [3], Banerjee [4], Shamir [5], Katore [6], Shaikh [7], and Reddy [8], have explored different aspects of $f(R)$ gravity models, investigating FRW models, spherically symmetric vacuum collapse, Bianchi type space-times, and cosmological models within this framework.

Various applications of $f(R)$ theories to cosmology and gravity proposed by De Felice [9]. $f(R)$ gravity theory has been explored extensively in various studies by researchers such as Capozziello [10], Carroll [11], Chiba [12], Pawar [13], Özdemir [14], Nazar[15], and Odintsov[16]. By considering different functions within this theory, a diverse range of phenomena can be generated. This gravitational framework offers a natural alternative to dark energy and aids in explaining the evolution of the universe. Carroll et al. [16] demonstrated how $f(R)$ gravity can account for the cosmic acceleration observed in the late stages of the universe. By defining a class of theories based on an arbitrary function of $R, f(R)$ gravity serves as a fundamental example of alternative or extended gravitational theories. These extended theories have emerged from the foundational concepts explored by Einstein and Hilbert, allowing for modifications to the gravitational aspect of the equations rather than solely focusing on refining calculations related to matter content like inflation, dark energy, dark matter, large-scale structure, and quantum gravity.

The second approach involves modifying general relativity (GR) and is known as modified gravitation theories. Among these theories, the $f(R)$ theory of gravity is considered the simplest and most direct extension of Einstein's general theory of relativity. In this theory, the higher-order curvature invariants are functions of the Ricci scalar. Nojiri and Odintsov [17] developed a general formulation for reconstructing the modified f(R) gravity for any given FRW metric. Capozziello et al. [18] examined the exact solutions of cosmological models within the context of $f(R)$ gravity. Nojiri and Odintsov [18] proposed and demonstrated that the $f(R)$ gravity model can unify early-time inflation with late-time cosmic acceleration. In a review of $f(R)$ theories, Felice and Tsujikawa [19] discussed various topics such as inflation, dark energy, cosmological perturbations, local gravity constraints, and spherically symmetric solutions in weak and strong gravitational backgrounds. Tripathy and Mishra [20] described the dynamics of anisotropic LRS Bianchi type-I models in $f(R)$ gravity and obtained solutions to the field equations. Agrawal et al. [21] recently explored the Bianchi type-I cosmological model in $f(R)$ gravity with perfect fluid and discussed the behavior of gravitational baryogenesis using two different forms of the Ricci scalar. Aditya and Reddy [22] showed that the universe achieves isotropy at late times by investigating the LRS Bianchi type-I spacetimes with cosmic strings in the $f(R)$ gravity. In $f(R)$ gravity, Santhi et al. [23] have recently studied the LRS Bianchi type-I bulk viscous string cosmological model, which shows that the realistic energy conditions are satisfied. Moreover, Bertolami et al. [24] have presented a generalization of $f(R)$ modified theories of gravity. Sharif and Shamir [25] have obtained constant curvature solution in $f(R)$ theory of gravity.

Yılmaz et al. [26] have studied the $f(R)$ quark and strange quark matter for Bianchi type I and V space time.Nagpal, et al. [27] presented FLRW cosmological models with quark and strange quark matters in $f(R,T)$ gravity.Özdemir,et al. [28] have studied Anisotropic universe models with magnetized strange quark matter in $f(R)$ gravity theory.Aktaş, et al. [29] explored Magnetized strange quark matter solutions in $f(R,T)$ gravity with cosmological constant. Singh and Beesham [30] presented LRS Bianchi I model with strange quark matter and Λ (t) in $f(R,T)$ gravity. Sahoo, et al. [31] have studied Magnetized strange quark model with Big Rip singularity in $f(R,T)$ gravity. Dixit, et al. [32] Observational constraint in Kantowski–Sachs $f(R)$ gravity model with strange quark matter. Aygün, et al. [33] have studied Strange quark matter solutions for Marder's universe in $f(R,T)$ gravity with Λ. Sahoo, et al.[34] have explored Magnetized strange quark matter in $f(R,T)$ gravity with bilinear and special form of time varying deceleration parameter. Singh, et al. [35] have discussed Plane Symmetric Cosmological Model with Strange Quark Matter in $f(R,T)$ Gravity.

The work is organized as follows. In the introduction, an Bianchi-V (BV) space-time model with SQM in the presence of a bag constant and $f(R)$ gravity is presented. In Section 2, The Formalism of $f(R)$ Theory of Gravity in the presence of quark matter(QM) and strange quark matter(SQM) are calculated. In Section 3, Solutions of quark matter in $f(R)$ gravity and the behavior of SQM is explored.The State-finder diagnostic approach to these models is discussed in Section 4. In Section 5, we discuss the jerk parameter which deals with the physical and geometrical behaviour of the universe. Discussion and Conclusion are made in Section 6.

2. The Formalism of $f(R)$ Theory of Gravity

According to Sharif and Shamir [36-37] the action of $f(R)$ gravity is given by

$$S = \int \sqrt{-g}[\frac{1}{16\pi G}f(R) + L_m]d^4x \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar and Lm is the matter Lagrangian. It should be noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein-Hilbert action.
The corresponding field equations in $f(R) gravity$ is given by

$$F(R)R_{ij} - \frac{1}{2}f(R)g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}\Box F(R) = kT_{ij} \tag{2}$$

where $F(R) = \frac{df(R)}{dR}$, $\Box = \nabla^i\nabla_j$
If we contract Eq. (2), we get

$$F(R) - 2f(R) + 3\Box F(R) = T \tag{3}$$

From Eqs. (2) and (3), we get gravitational field equations as follows

$$R_{ij} - \frac{1}{F(R)}[\nabla_i\nabla_j F(R) + T_{ij}] = \frac{1}{4F(R)}[F(R)R - \Box F(R) - T]g_{ij} \tag{4}$$

Anisotropic and homogenous,Bianchi V space-time, is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2mx}dy^2 - C^2e^{-2mx}dz^2 \tag{5}$$

where A, $B$and C are functions of cosmic time t. $m \neq 0$ is an arbitrary constant. Due to experimental results have shown that quark matter behaves like a nearly perfect fluid [38-40], the energy momentum tensor of quark matter is considered to be

$$T_{ij}^{(Quark)} = (\rho + p)u_iu_j - pg_{ij} \tag{6}$$

or

$$T_j^{(Quark)i} = diag(\rho, -p, -p, -p) \tag{7}$$

where $\rho = \rho_q +$ B, $p = p_q - B$ and $u^i = u_i = \delta_0^i$ is the four velocity in the comoving coordinates. Since quark matter behaves nearly perfect fluid, we will use the following equation of state for quark matter $p = \epsilon\rho$ Also it has been purposed the following linear equation of state for strange quark matter; $p = \epsilon(\rho - \rho_0)$, where ρ_0 denotes the energy density at zero pressure and ϵ is a constant.[41, 42] When

$\epsilon = \frac{1}{3}$ and $\rho_0 = 4B_c$, above linear equation of state is reduced to the following $p = \frac{\rho - 4B_c}{3}$. where B_c is the bag constant.
From Eqs. (4), (5) and (7) we get

$$-4F[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{S}{F}] = 3(\rho + p) \tag{8}$$

$$4F[\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{F}}{AF} - \frac{2m^2}{A^2} - \frac{S}{F} = \rho + p \tag{9}$$

$$4F[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{F}}{BF} - \frac{2m^2}{A^2} - \frac{S}{F} = \rho + p \tag{10}$$

$$4F[\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{F}}{CF} - \frac{2m^2}{A^2} - \frac{S}{F} = \rho + p \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{12}$$

where

$$S = \frac{F}{2}[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{1}{2}(1+\theta)\frac{\ddot{F}}{F}] \tag{13}$$

where a dot represents a derivative with respect to time.

3. Solutions of quark matter in $f(R)$ gravity

After solving Eqs. (8)–(11), we obtain metric potentials for Bianchi V space time as follows

$$A = a \tag{14}$$

$$B = (Da)\ exp[X \int \frac{dt}{a^3 F}] \tag{15}$$

$$C = (D^{-1}a)\ exp[-X \int \frac{dt}{a^3 F}] \tag{16}$$

where D and X are constants of integration.
The average scale factor is defined by

$$a = (ABC)^{\frac{1}{3}} \tag{17}$$

The physical quantities, namely the volume scale factor (V), generalized mean Hubble's parameter (H), expansion scalar (θ), shear scalar (σ^2), mean anisotropy parameter ($\triangle$) have the following expressions for the metric (5)

$$V = a^3 = (ABC) \tag{18}$$

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}] \tag{19}$$

$$\theta=3H=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{20}$$

$$\sigma^2=\frac{\theta^2}{3}-[\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}] \tag{21}$$

$$\triangle=6(\frac{\sigma}{\theta})^2 \tag{22}$$

Inserting Eqs. (15)–(17) into Eqs. (3), (14) and (22) we have the following solutions as,

$$f(R)=-2\rho+RF+2\frac{\dot{V}}{V}F(\frac{\dot{V}}{3V}+\frac{\dot{F}}{F})-\frac{2x^2}{FV^2}-\frac{6m^2F}{V^2/3} \tag{23}$$

$$R=2[\frac{1}{3}(\frac{\dot{V}}{V})^2-\frac{\ddot{V}}{V}]-\frac{2x^2}{F^2V^2}+\frac{6m^2}{V^2/3} \tag{24}$$

$$\sigma^2=\frac{X^2}{F^2V^2} \tag{25}$$

From Eqs. (21) and (20), we have

$$\frac{\sigma}{\theta}=\frac{X}{\dot{V}F} \tag{26}$$

As the field equations are highly nonlinear, an extra condition is needed to solve the system completely. After thoroughly reviewing refs [43-46],we considered the scale factor which is a combination of two factors: the one is the usual power law expansion and the second one is an exponential function.

$$a(t)=t^{\alpha_1}e^{\alpha_2 t} \tag{27}$$

where α_1, α_2 are positive constants. Describing a transition in the cosmic evolution from deceleration to acceleration can be achieved by utilizing the hybrid scale factor. The dynamics of the universe in its early stages are primarily governed by power law behavior, while in later phases, the exponential factor becomes dominant. When the parameter α_1 is set to zero, the exponential law is regained, and whenα_2 is zero, the scale factor simplifies to the power law. The Hubble parameter and deceleration parameter for the hybrid scale factor can be calculated as

$$H=\alpha_2+\frac{\alpha_1}{t} \tag{28}$$

$$q = -1 + \frac{\alpha_1}{(\alpha_2 t + \alpha_1)^2} \tag{29}$$

It can be observed from the formula for the deceleration parameter that during the early stages of cosmic development, the deceleration parameter is calculated as $q = -1 + \frac{1}{\alpha_1}$, while in the later phases of cosmic evolution, it tends towards -1. To achieve a transition in the deceleration parameter from positive values in the early universe to negative values in the late universe, it is necessary to fine-tune the parameter α_1 within the interval of 0 ¡ α_1 ¡ 1.One can explore various cosmological matters related to a transitioning Universe that exhibits deceleration in its early stages and acceleration in its later phases by utilizing a hypothesized dynamics using the hybrid scale factor. The parameter α_1 is constrained within the specific range of 0 ¡ α_1 ¡ 1. we also assume another relation between F and scale factor a.Recently, in the context of $f(R)$ theory Uddin et al. [47],
we assume the power law relation as

$$F = la^n, \tag{30}$$

where n - is an arbitrary integer and where l is proportionality constant. Inserting equation (27) and (30) in (14)-(16), we get

$$A = t^{\alpha_1} e^{\alpha_2 t} \tag{31}$$

$$B = Dt^{\alpha_1} e^{\alpha_2 t} \exp[\frac{X}{l} \int \frac{dt}{(t^{\alpha_1} e^{\alpha_2 t})^{n+3}}] \tag{32}$$

$$C = D^{-1} t^{\alpha_1} e^{\alpha_2 t} \; exp[\frac{X}{l} \int \frac{dt}{(t^{\alpha_1} e^{\alpha_2 t})^{n+3}}] \tag{33}$$

At the initial epoch, the, metric potentials vanish. Thus, the model has an initial singularity. As t $\rightarrow\infty$, the scale factors diverge to infinity. Hence, there will be big rip at least as far in the future since the metric potentials tend to infinity at t $\rightarrow \infty$.

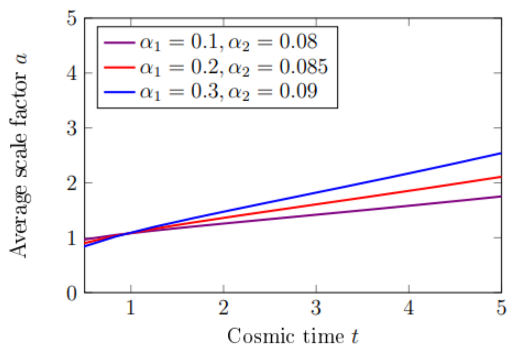

Figure 1: Average scale factor vs. Cosmic time

From the figure 1, we observed that the average scale factor or cosmic scale factor increases with time increases. The expansion scalar yields

$$\theta = 3(\alpha_2 + \frac{\alpha_1}{t}) \tag{34}$$

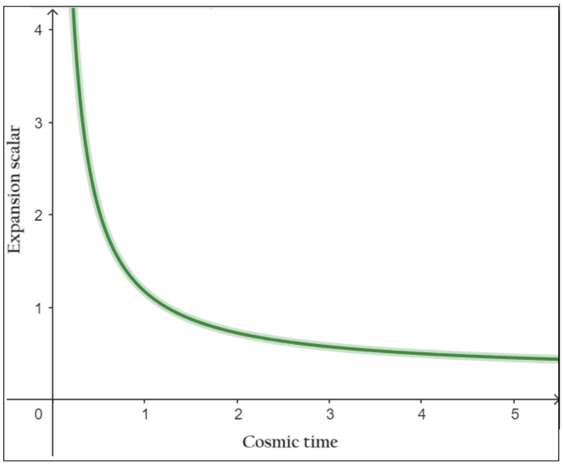

Figure 2: Expansion scalar vs. Cosmic time

The shear scalar is obtained as

$$\sigma^2 = \frac{X^2}{l^2\left(t^{(2n+6)\alpha_1} e^{(2n+6)\alpha_2 t}\right)} \tag{35}$$

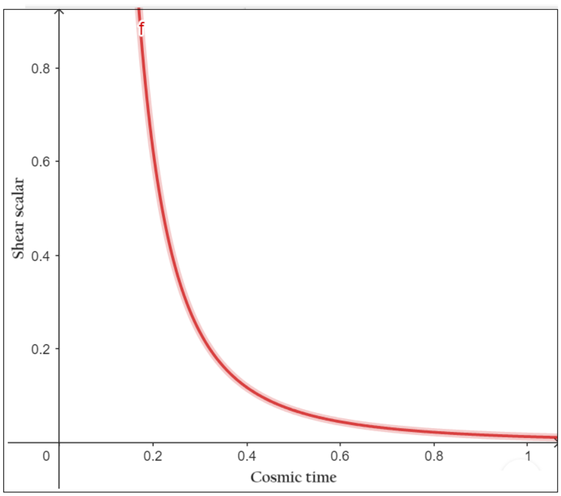

Figure 3: Shear scalar vs. Cosmic time

As specified in figures 2 and 3, it is clear that at the initial epoch, the expansion and shear scalar diverge.Equation (34) shows that the expansion scalar is infinite at the beginning, i.e. $t = 0$. It suggest that Universe starts evolving with zero volume and infinite expansion rate at $t = 0$,which is the Big-Bang scenario. The shear scalar is also infinite at the initial epoch and becomes zero as time increases as depicted in figure 3.the derived model may represent the inflationary era in the early Universe and very late times of the Universe. The deceleration parameter is negative, i.e.the Universe is fast that is in agreement with the current observations of SNe Ia and CMB.
The directional Hubble parameters are given by

$$H_1 = \frac{\alpha_2 t + \alpha_1}{t} \tag{36}$$

$$H_2 = \frac{X}{l t^{(n+3)\alpha_1} e^{(n+3)\alpha_2 t}} + \frac{\alpha_2 t + \alpha_1}{t} \tag{37}$$

$$H_3 = \frac{-X}{l t^{(n+3)\alpha_1} e^{(n+3)\alpha_2 t}} + \frac{\alpha_2 t + \alpha_1}{t} \tag{38}$$

We observed that Hubble parameter is a decreasing function of time.
The anisotropy parameter of the expansion is

$$\triangle = \frac{2X^2 t^2}{3l^2(\alpha_2 t + \alpha_1)^2 t^{(2n+6)\alpha_1} e^{(2n+6)\alpha_2 t}} \tag{39}$$

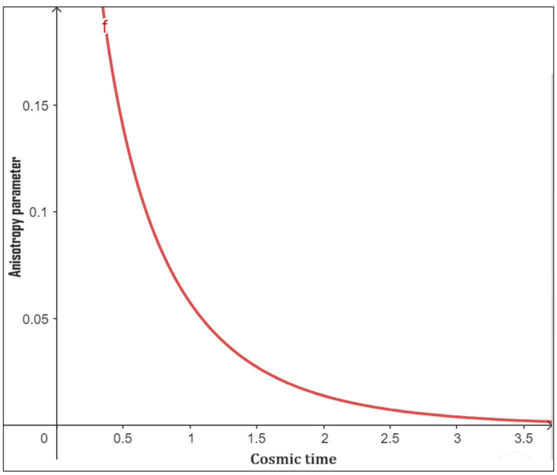

Figure 4: Anisotropy parameter vs. Cosmic time

The mean anisotropic parameter decreases to null exponentially with an increase in time (figure 4). Thus, the space approaches isotropy in this model.

$$F = lt^{\alpha_1 n}e^{\alpha_2 nt} \tag{40}$$

If we use Eqs. (31)–(33) and (40) into Eqs. (8)–(11) we get

$$\begin{aligned}\rho + p = lt^{\alpha_1 n}e^{\alpha_2 nt}[&\frac{(n-6)(\alpha_2 t+\alpha_1)^2}{t^2} + \frac{5\alpha_1}{t^2} - \frac{3X^2}{l^2 t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2 t}}\\ &-\frac{3[\alpha_2 t+\alpha_1+t][-\alpha_1 n+n^2(\alpha_2 t+\alpha_1)^2]}{2t^3} - \frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2 t}}]\end{aligned} \tag{41}$$

From Eqs. (41) and (7) we have the following dynamical variables of quark matter as

$$\begin{aligned}p = \frac{lt^{\alpha_1 n}e^{\alpha_2 nt}\epsilon}{(\epsilon+1)}\{&\frac{(n-6)(\alpha_2 t+\alpha_1)^2}{t^2} + \frac{5\alpha_1}{t^2} - \frac{3X^2}{l^2 t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2 t}}\\ &-\frac{3[\alpha_2 t+\alpha_1+t][-\alpha_1 n+n^2(\alpha_2 t+\alpha_1)^2]}{2t^3} - \frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2 t}}\}\end{aligned} \tag{42}$$

$$\begin{aligned}\rho = \frac{lt^{\alpha_1 n}e^{\alpha_2 nt}}{(\epsilon+1)}\{&\frac{(n-6)(\alpha_2 t+\alpha_1)^2}{t^2} + \frac{5\alpha_1}{t^2} - \frac{3X^2}{l^2 t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2 t}}\\ &-\frac{3[\alpha_2 t+\alpha_1+t][-\alpha_1 n+n^2(\alpha_2 t+\alpha_1)^2]}{2t^3} - \frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2 t}}\}\end{aligned} \tag{43}$$

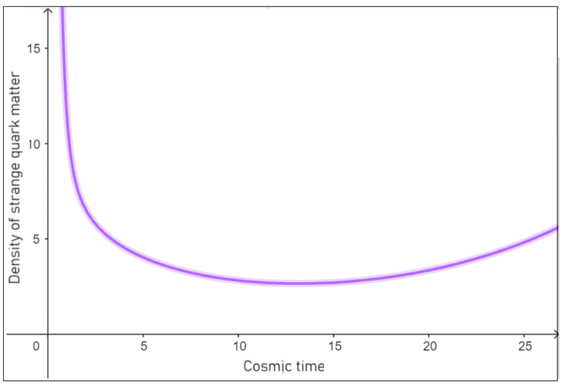

Figure 5: Density of strange quark matter vs. cosmic time

using Eqs. (27), (40) and (43) into Eq. (23) we have the following f (R)function for quark matter

$$f(R)=\frac{-2}{(1+\epsilon)}[\frac{(n-6)(\alpha_2t+\alpha_1)^2}{t^2}+\frac{5\alpha_1}{t^2}-\frac{3X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}$$
$$-\frac{3[\alpha_2t+\alpha_1+t][-\alpha_1n+n^2(\alpha_2t+\alpha_1)^2]}{2t^3}-\frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2t}}]$$
$$+2lt^{n\alpha_1}e^{n\alpha_2t}[\frac{3\alpha_1}{t^2}-\frac{6(\alpha_2t+\alpha_1)^2}{t^2}-\frac{X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}+\frac{3m^2}{t^{2\alpha_1}e^{2\alpha_2t}}]$$
$$+6l(n+1)t^{n\alpha_1}e^{n\alpha_2t}\frac{(\alpha_2t+\alpha_1)^2}{t^2}-\frac{2X^2}{l}t^{(n+6)\alpha_1}e^{(n+6)\alpha_2t}-6lm^2t^{(n-2)\alpha_1}e^{(n-2)\alpha_2t} \tag{44}$$

$$R=2[\frac{3\alpha_1}{t^2}-\frac{6(\alpha_2t+\alpha_1)^2}{t^2}-\frac{X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}+\frac{3m^2}{t^{2\alpha_1}e^{2\alpha_2t}}] \tag{45}$$

If the parameter ϵ¡-1, it indicates that quark matter could exhibit characteristics similar to phantom dark energy. This leads us to the inference that quark matter might serve as a potential origin of early dark energy, attributed to its negative pressure that results in the violation of energy conservation principles.

$$p=\frac{lt^{\alpha_1n}e^{\alpha_2nt}\epsilon}{(\epsilon+1)}\{\frac{(n-6)(\alpha_2t+\alpha_1)^2}{t^2}+\frac{5\alpha_1}{t^2}-\frac{3X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}$$
$$-\frac{3[\alpha_2t+\alpha_1+t][-\alpha_1n+n^2(\alpha_2t+\alpha_1)^2]}{2t^3}-\frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2t}}\}-\frac{\epsilon\rho_0}{(\epsilon+1)} \tag{46}$$

$$\rho=\frac{lt^{\alpha_1n}e^{\alpha_2nt}}{(\epsilon+1)}\{\frac{(n-6)(\alpha_2t+\alpha_1)^2}{t^2}+\frac{5\alpha_1}{t^2}-\frac{3X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}$$
$$-\frac{3[\alpha_2t+\alpha_1+t][-\alpha_1n+n^2(\alpha_2t+\alpha_1)^2]}{2t^3}-\frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2t}}\}+\frac{\rho_0}{(\epsilon+1)} \tag{47}$$

Inserting $\epsilon=\frac{1}{3}$ and $\rho_0=4B$ into Eqs. (42) and (43) we get the following strange quark matter solutions in the Bag Mode.
The strange quark matter solutions in the case of linear equation of state is given by,

$$p=\frac{lt^{\alpha_1n}e^{\alpha_2nt}}{4}\{\frac{(n-6)(\alpha_2t+\alpha_1)^2}{t^2}+\frac{5\alpha_1}{t^2}-\frac{3X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}$$
$$-\frac{3[\alpha_2t+\alpha_1+t][-\alpha_1n+n^2(\alpha_2t+\alpha_1)^2]}{2t^3}-\frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2t}}\}-B_c \tag{48}$$

$$\rho=\frac{3lt^{\alpha_1n}e^{\alpha_2nt}}{4}\{\frac{(n-6)(\alpha_2t+\alpha_1)^2}{t^2}+\frac{5\alpha_1}{t^2}-\frac{3X^2}{l^2t^{(2n+6)\alpha_1}e^{(2n+6)\alpha_2t}}$$
$$-\frac{3[\alpha_2t+\alpha_1+t][-\alpha_1n+n^2(\alpha_2t+\alpha_1)^2]}{2t^3}-\frac{2m^2}{t^{2\alpha_1}e^{2\alpha_2t}}\}+3B_c \tag{49}$$

4. Statefinder diagnostic

Sahni et al. [48] introduced a diagnostic proposal that makes use of the parameter pair r, s, the so-called 'statefinder'. The statefinder pair r, s is defined as

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, s = \frac{r-1}{3(q-0.5)} \tag{50}$$

We utilize the statefinder diagnostic method to analyze the behavior of our model in comparison to the ΛCDM model. We then derive the expressions for the parameters r, s specific to our model is

$$r = 1 - \frac{3\alpha_1}{(\alpha_2 t + \alpha_1)^2} + \frac{2\alpha_1}{(\alpha_2 t + \alpha_1)^3} \tag{51}$$

$$s = \frac{2\alpha_1[2 - 3(\alpha_2 t + \alpha_1)]}{3(\alpha_2 t + \alpha_1)(2\alpha_1 - 3(\alpha_2 t + \alpha_1)^2)} \tag{52}$$

In our model, the statefinder parameters r, s converge to $1, 0$ in the distant future, and their current values are also r, s. This indicates that our model aligns with the ΛCDM model both at present and in the future.

5. Jerk parameter

The cosmic jerk parameter, as defined by Visser [49], is a dimensionless parameter that involves the third derivative of the average scale factor with respect to cosmic time.

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{53}$$

Flat ΛCDM models have a constant jerk

$$j(t) = 1 \tag{54}$$

In our model, the jerk parameter yields

$$j(t) = 1 - \frac{3\alpha_1}{(\alpha_2 t + \alpha_1)^2} + \frac{2\alpha_1}{(\alpha_2 t + \alpha_1)^3} \tag{55}$$

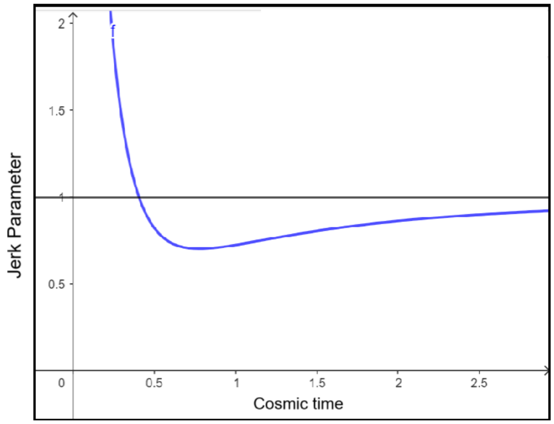

Figure 6: Jerk parameter vs. Cosmic time

According to the figure (5), we have represented the jerk parameter as a function of cosmic time. Initially, the jerk parameter decreases as cosmic time increases until $t = 7$, after which it starts to increase. The figure shows that the jerk parameter consistently remains positive throughout the entire history of the universe. For both small and large cosmic time values, the jerk parameter takes on larger values compared to those of the ΛCDM model. However, it is worth noting from the figure that the current model approaches closer to the ΛCDM model.

6. Discussion and Conclusion

In this review, we have examined how the hybrid scale factor plays a crucial role in determining the realistic cosmic dynamics. The hybrid scale factor represents a middle ground between the expansion behaviors dictated by power law and exponential laws. It consists of two components: one that mimics the power law expansion and is predominant during the initial stages of cosmic evolution, and another that mirrors exponential expansion and becomes dominant in later stages, aligning the model more closely with the widely accepted ΛCDM model as time progresses.In this context, the hybrid scale factor can replicate a deceleration parameter that starts with positive values in the early stages and transitions to negative values at later times. Nonetheless, the hybrid scale factor requires adjustment of two parameters to ensure the creation of realistic models that accurately depict cosmic evolution patterns. Using eqs (34) and (35), it can be observed that $\frac{\sigma^2}{\theta^2}$ tends to zero as $t \to \infty$ which implies that the fluid behaves like isotropic DE. The deceleration parameter q passes from positive value to negative value as time increases and it converges to -1 as $t \to \infty$ i.e., the model transits from early decelerating phase to current accelerating phase. Therefore, our model is in a good agreement with the recent observations in the context of phase transition of the universe.Furthermore, our findings suggest that the relationship between the pressure and energy density of quark and strange quark matter is inversely correlated over time.They are infinitely large at $t = 0$ and as $t \to \infty$, p_q,ρ_q p, ρ decreases with time $t = 15$ and again it approaches to ∞. For any physically realistic cosmological model, the energy density must be positive, meaning that the weak energy density condition (WEC) ought to be satisfied. It is clear from (48)–(49) that both the pressure and density depend on f(R) gravity and the bag constant.Hence both p_q and ρ_q remain positive. Then $\rho_{sq} \to \infty$ as $t \to 0$, and $\rho_{sq} \to 3B_c$ as $t \to \infty$. Also we notice that the bag constant dominates at late times, and the energy density of the SQM becomes constant. Similarly $p_{sq}t \to \infty$ as $t \to 0$, and $p_{sq} \to -B_c$ as $t \to \infty$. It is shown that, the hybrid scale factor can be good alternative in providing interesting and viable models.

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