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Manuscript based Nepali Indigenous Mathematics

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Abstract: The manuscript is written in an earlier form of Nagari, *Śāradā* or any different script. Mainly it was in use from the 8th to the 12th century. It was found in the northwestern part of India, such as Kashmir and neighboring regions, Kailas. The language in these manuscripts is the combination of Sanskrit and Prakrita. This has been written by a brāhmana and king of mathematicians, for the sake of Hasika, son of Vasiṣṭha, in order that it may be used by his descendants. The originality of any countries' Mathematics, mean the history of the intellectual efforts that men have made in order to understand the indigenous mathematical phenomena or which comes to the same thing, the history of analytic or scientific aspects of mathematical thought. The objective of this article is, explored the development of manuscript based mathematics in early period of Nepal.
Keywords: Manuscript, History, cuneiform, civilization, inscription, Vedas, palm leaf.

1. General Background

The manuscript is a compilation of mathematical rules and examples in verse and prose commentaries on these verses. Typically, a rule is given, with an example or examples, where each example is followed by a statement nyāsa/ sthāpanā of the example's numerical information in tabular form, then a computation that works out the example by following the rule step-by-step while quoting it, and finally a verification to confirm that the solution satisfies the problem. This is a style similar to that of Bhāskara I's commentary on the Ganita (mathematics) chapter of the Āryabhaṭīya, including the emphasis on verification that became obsolete in later works. It is preceded by a broken word rtikāvati, which is believed to be the same as the place Mārtikāvata that is mentioned by Varāhamihira. He mentions this place in his Bṛhatsaṃhitā among other locations in northwestern India, such as Takṣaśilā, Gandhāra, etc.[19] Based on this, it is believed that the work of the Bakhshālī manuscript may have been composed in that region.

The rules are algorithms and techniques for a variety of problems, such as fundamental mathematical operations, systems of linear equations, quadratic equations,

arithmetic progressions and arithmetic-geometric series, computing roots approximately, dealing with negative numbers, profit and loss, measurement such as of the fineness of gold, etc.

The manuscript was discovered in 1881 by a peasant in the village of Bakhshali, which is near Peshawar, now in Pakistan. The first research on the manuscript was done by A. F. R. Hoernlé. Its date is uncertain, and has generated considerable debate. Most scholars agree that the physical manuscript is a copy of a more ancient text, so that the dating of that ancient text is possible only based on the content. Recent scholarship dates it between the 2nd century BC and the 3rd century AD; Ian Pearce summarizes the positions: Gurjar discusses its date in detail, and concludes it can be dated no more accurately than 'between 2nd century BC and 2nd century AD'. He offers compelling evidence by way of detailed analysis of the contents of the manuscript originally carried out by R Hoernle. His evidence includes the language in which it was written ('died out' around 300 AD), discussion of currency found in several problems, and the absence of techniques known to have been developed by the 5th century. The manuscript of the mathematics was written in different leaves of birch, palm, bamboo, leather roll, cloths, and papyrus, etc.

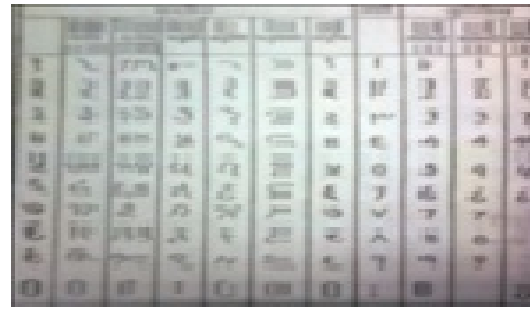
However, earlier scholars have tended to date it around 400 AD. Hayashi had suggested a possible 7th-century date, while in an early colonial estimate, G.R. Kaye had assessed it to be as late as the 12th century AD. Such late dates are quite unlikely because the language used was already dying by the 4th century; also the work does not mention integer equations and other topics which were of widespread interest after Aryabhata (476 A.D.). Today, Kaye's assessment is widely discredited. The reason why the date of the manuscript is important is that if the work indeed dates from the 3rd century or earlier, it would imply that the concept of the mathematical zero was known several centuries earlier than the work of Brahmagupta in the 7th century.

Here the question is why we do not study and review the early mathematical developments of Nepalese context? Why do we study the history of any science? Current work, so one would think, will preserve whatever is still useful of the work of preceding generations. Concepts, methods, and results that are not so preserved are Pre-sum competently not worth bothering about? Why we go back to old authors and rehearse (practice) views? Cannot old stuff be safely left to the care a few specialists who love it for its own shake? Who search our history [10,11]? Whole developments cannot be pouring down through an article. On the basis of these questions I expressed a very brief theoretical glimpse of mathematical developments in Nepal with references based on manuscripts.

Mathematics is a science of arts; it is an inductive science and as a way of

thinking. The chronology is the backbone of history [5] and Mathematics is the backbone of science and Technology. History of Mathematics is foundation or milestone of development of Mathematics. In my view Mathematical developments are the barometer of developments. The industrial mathematics is the way of changing life style of the people. The development of Mathematics is parallel to the development of human civilization. These all evidences focused on mathematical developments.

The world map of development of civilization in context of mathematics is given by a scholar in his documented paper prepared for new thinking to the coming generations that is Mathematics for 21st century [18] that supports to search mathematical developments.



The manuscript forms of mathematical written gives the early history of Mathematics. It preserves the base and its mechanization which shows it is a dynamic subject. The antique mathematics, which is prehistoric mathematics, is also the foundation for manuscript mathematics. For worldwide considerations the history of Mathematics cannot go back much earlier than the times of Thales (600 B.C). But in Nepalese context we cannot say the date exactly. But it can be said that its study has begun from very beginning of the Nepalese society, that is from Vedic age (c. 1500-700B.C.), Guru Kula. Mathematical concepts, symbols of numerals and number systems are developed from various efforts of different civilizations and communities. Actually mathematics is a nature thus their change occurs as social changes and developments. The development and changes of numerals in Bhojadeva inscription (870 A.D.), Tilochan (11^{th}century), Bamshawali manuscript, Bhoudha Manuscripts, Jain Manuscripts, Nagari (modern) in Indian Arabian and 13th century (2), 14th century (3) 14th century (4) [17] was in European is showing in the above figure.

S.P. Khatiwada coated first 43 symbols to represent the vowels and consonants, and the remaining 19 symbols represent the numerals [9]. These are the Brahmi symbols of letters of Alphabets and numerals in third century B.C. Brahmi (Lippie)

inscriptions as the sources of development of numeral system. These symbols, notations and pattern are found in various manuscripts. A manuscript preserves the originality of the creations. Here the very brief information is sharing through manuscripts based mathematics. These are explained under coming headings.

2. Mathematics In Vedic Age

In spiritually us we all accept that 33, 0000000 gods live in Kailas of Nepal. How we accept it? This is on the basis of counting and counting is the prerequisites of mathematical developments. Hence this evidence over holms to us counting begins here. On the other hand counting is innate to man [16]. Thus either accepts or not mathematics step down from Mt. Everest (?)[13] That is Tethys Sea. In Sanskrit verse in Vedanta Jyoutisha of Lagadha states that,

Yatha Shikha Mayuranam, Naganam Madayo Yatha;
Tatvada Vedangashastranam, Ganitam Murdhani Sthitam.

That is, just as branches of a peacock and jewel-stone of a snake are placed at the highest place of body (forehead), similarly position of Ganit is highest in all the branches of Vedah and Shastras. Hence that mathematic is in core/centre or top of all branches of knowledge.

In the development of Mathematics and numeral system the development of inscriptions play a major role? Since the numerals are as the symbols taken from inscriptions which represent the various numbers. The time bond of Vedic age cannot be declared, its tentative reliable period is not found anywhere but it is clear that it is the time of Vedas. The word 'Veda' has derivational meaning. That is the fountain-head and illimitable store-holstered elution that it is not to be approach from a factual standpoint but from the distend point such as the Vedas, as traditionally accepted in India as the respiratory of all knowledge [6] . The counting and measurement system was used in Vedic period. Vedas are the root of entire Sanskrit literature and it is as the base of Mathematics. "Vedic Mathematics" is the name given to the ancient system of mathematics, or, to be precise, a unique technique of calculations based on simple Rules and principles, with which any mathematical problem is it arithmetic, algebra, geometry or trigonometry can be solved, hold our breath, orally! The system is based on 16 Vedic sutras or aphorisms, which are actually word formulae describing natural ways of solving a whole range of problems. Some examples of sutras are "By one more than the one before", "All from 9 & the last from 10", and "Vertically & Crosswise". These 16 one-line formulae originally written in Sanskrit, which can be easily memorized, enables one to solve long mathematical problems quickly. For example, Application of derivatives to solve quadratic equations we have the formula, The first order derivative of the

expression $= \pm\sqrt{\text{discriminent of the equation}}$ gives the solution of the equation. Such as $5x^2 - 3x - 2 = 0$. Now proceed as above formula,
$10x - 3 = \pm\sqrt{(-3)^2 - 4 \times 5(-2)} = \pm 7 \Rightarrow x = 1 \ \& - 2/5$. Derivatives and turning points, If $f(x) = \dfrac{g(x)}{h(x)}$, then the formula defined by, $f(x) = \dfrac{g'(x)}{h'(x)}, h(x) \neq 0$ gives the values of x and then the turning points. Consider an example, $f(x) = \dfrac{x}{1+x^2} = \dfrac{g(x)}{h(x)}$ (let). As above formula $\dfrac{x}{1+x^2} = \dfrac{1}{2x} \Rightarrow x = \pm 1$, so $f(x) = 1/2$ at x=1 and -1/2 at x= -1 and hence turning points are (1,1/2), (-1,-1/2). In this way we can solve various mathematical problems based on successive derivatives, integrations by parts and factorisations etc., in very short and easy methods. These types of expressions of mathematics is found in verse form in sanskrit mathematics.

3. Some evidences of manuscripts of mathematics

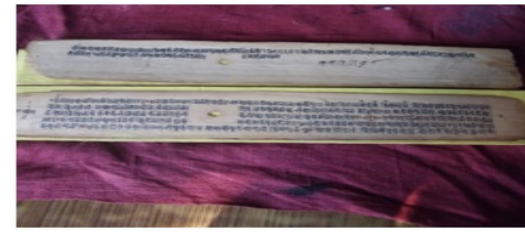
In Nepalese context various manuscripts of mathematical developments are found in different collections either in personal library or in constitutional library. Some are given as examples with their brief descriptions.

3.1 Sumati Siddhanta

Sumati siddhanta was written in (c.556 - 960 A.D) on the paper of Palm leaf (Tadapatra) as the manuscript of mathematics. The book 'Sumati Siddhanta' is available at Kaiser Library as a manuscript. It is a proud of Nepalese to note that it was written in Kathmandu valley. Latter in 1409 A.D., great astrologer Dharmapati Bardhan constructed the Sumati Siddhanta. A brief scenario of this book is found in a scientific journal 'Scientific World . The book Sumati Sidhanta' and 'Sumati Tantra give the mathematical glimpse. Sumati Tantra was used in Puran purely written Sanskrit language whereas Sumati Sidhanta in mixed language of Newari and Sanskrit. It is convention that Sumati Tantra has been written in 505 A.D. in the basis of Surya Siddhanta of Barahamihirs Panchasidhantika. It was used for prepared calendars and for future prediction in Malla Era. Astrologers usually took the help from both Sumati Tantra and Sumati Sidhanta. It is found that even in Lichchhavi Era and mid-period; these books were found useful for astrologers for their calculations [7]. For evidences,

Pant, Naya Raj (1978) has written "Sumatitantram. Dinesh Raj Pant and Deviprasad Bhandari have assisted him. In the sense of the curve sketching a three leaf photograph is showing with expressing the 1000 years old manuscript in Bhujimol script [11]. Here the initiation for this determination (1000 years old manuscript?) goes to Nepal Mathematics Center which is newly emerged institute in the field of Mathematics. Specially the script shown there are Bhujimol, Newari

and Sanskrit.

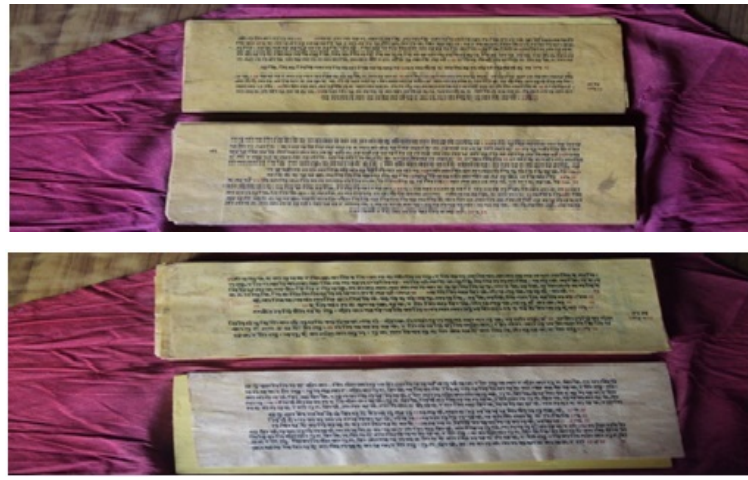
3.2 Bhaskaracharya and His Algebra

Bhaskaracarya was born in 1114 A.D. Bhaskaracharya was the greatest mathematician in his period. In the sense of contribution of Bhaskaracharya, the great nineteenth century German mathematician Weierstrass said, "A mathematician, who is also not something of poet, can never be a complete mathematician [14]. Thus Bhaskaracharyas poetic contribution in the development of Mathematics is very vast and pioneer. He has command in each field of Mathematics whose evidence is his lilavati and algebra. Their manuscript of Bhaskaracharya's algebra is preserved at Kaiser Library. It consist 55 leafs. Some pages were lost (page7-11) and some were tearing and damaged by the insects. It is written in Nagari (Nepali) (Pako Kagat) hard paper. Inside of this book various frames or tables are formed and the Devnagari and Hindu-Arabic numerals expressing the addition and subtraction of currency. It is very old so the sides of the paper are not clearly seen. Its name is Algebra but according to its expressions and concept it is as arithmetic, geometry. It also expressed the geometrical concepts. Although it is not analyses minutely. Who and when copied the manuscript of Bhaskaracharya's algebra? Thus it is under investigation. It is recorded in 359 of the library record. For evidences,


3.3 Astabakraprakararetika

It is an ancient book which was written in paper leaf (Pako Kagata). Here the subject matter is based in Vedas in Nagari lippie. The paper and letters are very

clear and attractive. Leafs are yellow in colour, some leafs have pale yellow colour. It consist 51 leafs. The recorded number to this book in the library was 256. The word 'Eakonabimshopadesyu is used in this book. This shows that the counting system was based in 20. It is seen 1912 (?) ghusadi 4, what does it mean? It is as a date, but the purpose of this date is not clear and name of the writer of this book is unknown. Few evidences of its pages are given for evidences,



4. Measurement System of Length and Volume in Nepali community

Every measurement system is based on Mathematics, such as area, volume, weight etc. Divisions of land daily uses materials are measured in various measurement systems. Masses can be measured in Gram, milligram, kilogram, Dhak (5kgs.)Dharny, Mana, Pathy and Chimti. Length and distance in amal, bitta, hand, kosh, rumal kosh, miles, kilometers, metres, centimeters, milimetres, inch, etc. Area in Ropani, Aana, Dham and Paisa; Bigaha, Katthha, Dhur; as quantity of seeds, ploughing days; Mato-muri; the measurement system for grasses, strow in Napo (the tightly tied grasses or strow in a string of length five hands), Sorai (consisting 16 tied bundles of grasses), Bitho and Bhari, Doko, etc in local area of village. It is also varies in different communities and societies and social groups. The monumental construction indicates the Nepali indigenous mathematics.

5. Inscriptions, Cuneiform in Mathematics

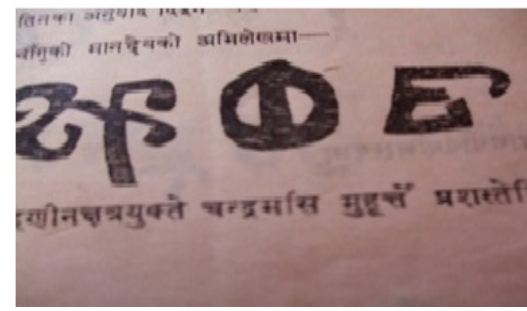
The numeral systems and different alphabetical symbols in differed situations were developed and they were gradually changes and developed for modern concept. They were not occur occasionally or accidently. Manuscripts and inscriptions are the back bone of developing the technologies and societies. Some mathematical inscriptions found in different regions of Nepal. Inscription on the pedestal of the Jaya Varma- sculpture, length 42 cm. Photograph courtesy HMG Department

of Archeology; photograph by Govindae Ghimire, taken May 7, 1992 [16] The inscriptions found in Asoka Brahmi pillar is shown as below. It consist 19 symbols.

The manuscript of Sumti Siddhanta found at Kaiser Library was 1000 thousand years old. Its date is (c556 - 960 A.D) and its writers name is still unknown. But its use was for preparing calendar. It has been consist nearly134 leafs (?)It is recorded at no.82 in the library's record.

6. Inscriptions in Changu Narayan Temple

These inscriptions were written in Lichchhavi period, in which various symbols are used for numerals. They are showing in below as photographic forms. The Exact date cannot be determined, it is on the investigation.



The hexagonal and octagonal constructions of bricks are to be seen in the compound of the Changu Narayan Temple. In which we have seen the numerals for showing the date. But the numerical symbols used their represent the numbers 286, 386... Which one is correct? This is still on investigation. It is clear that these numerals were used in Lichchhavi age. What did it represent! It shows that the numerals used in mathematical sense to representing various phenomena would have been developed in Nepal. This shows the contribution of Nepalese people. The copper plate which was used to records our ancient culture and developments. In each plate the date is given in Nepalese context.

7. The Conclusions

The manuscript of Sumati Siddhanta was 1000 thousand years old. Its exact date is 556 960 A.D and its writers name is still unknown. But its use was for

preparing calendar. It has been consist nearly134 leafs. Similarly other manuscripts in distinct forms are found in different collections focus Nepali mathematical developments. These manuscripts show the indigenous mathematical developments in Nepal.

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