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A NEW CLASS OF BINARY γ-PRE OPEN SETS

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Abstract: In this paper, we introduce the open set namely binary γ-pre open sets in binary topological space. Also we define new class of sets called binary γ-pre generalized closed set. Several theoretical results of these notions are discussed with an example.

Keywords and Phrases: b_γ-p-open set, b_γ-p-closed set, b_γ-p-$int((E, S))$, b_γ-p-$cl((E, S))$, b_γ-p-g-closed set.

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1. Introduction

In [8], binary topology concept was formulated by S. Nithyanantha Jothi and P. Thangavelu. In [2], S. Kasahara imposed the operation γ of a topology τ concept. H. Ogata [10] developed a new concept of γ-open set . G. S. S. Krishnan and K. Balachandran [3] proposed the γ-preopen sets. G. S. S. Krishnan and K. Balachandran [4] investigated the new γ-open sets. In [5], K. Muthulakshmi and M. Gilbert Rani introduced Binary γ-open sets in Binary topological space and definition of Binary γ-pre-open set is introduced in [6].

In this paper, we write some notations: BTS, $b_\gamma O$, $b_\gamma C$, b_γ-p-open set, b_γ-p-closed set, $B_\gamma O(\mathbb{N},\mathbb{Q})$, bPO, b_γ-p-g-closed set, $b_\gamma R$, bRO, O-γ, (binary topological space, binary gamma open set, binary gamma closed set, binary gamma pre open set, binary gamma pre closed set, binary pre open set, binary gamma pre generalized closed set, binary γ-regular, binary regular operation, operation γ).

2. Preliminaries

Definition 2.1. [6] *Let $(X, Y, \mathcal{M})$ be a binary topological space. An operation γ on $\mathcal{M}$ is a mapping $\gamma : \mathcal{M} \to \mathbb{P}(X) \times \mathbb{P}(Y)$ such that $(U, V) \subseteq ((U, V))$ for every $(U, V) \in \mathcal{M}$ where $\gamma((U, V))$ denotes the value of γ at (U, V) and $\mathbb{P}(X)$ and $\mathbb{P}(Y)$ are power sets of X and Y respectively.*

Definition 2.2. [6] *Let a nonempty set $(A, B) \subseteq (X, Y)$. A point $(x, y) \in (A, B)$ is said to be binary γ-interior of (A, B) iff there exists an binary neighborhood (M, N) of (x, y) such that $\gamma((M, N)) \subseteq (A, B)$. The set of all such binary points is denoted by b_γ-int(A, B).*

(ie) b_γ-int$(A, B) = \{(x, y) \in (A, B)/(x, y) \in (M, N) \in \mathcal{M}$ and $\gamma((M, N)) \subseteq (A, B)\} \subseteq (A, B)$.

Definition 2.3. [6] *Let (A, B) be a subset of (X, Y). Then (A, B) is binary γ-open in $(X, Y, \mathcal{M})$ if and only if $(A, B) = b_\gamma$-int(A, B).*

Result 2.4. [[7], Result 2.19] *If a BTS $(H, R, \mathcal{M})$ is binary γ-regular, then $b_\gamma = \mathcal{M}$ and implies b_γ-int$((T, L)) = b$-int$((T, L))$.*

Proposition 2.5. [[7], Proposition 2.21] *Let $(H, R, \mathcal{M})$ be a BTS with an OP-γ on $\mathcal{M}$ and (T, L) be a subset of (H, R), Then*

1. b_γ-$cl(b_\gamma$-$cl(T, L)) = b_\gamma$-$cl((T, L))$
2. b_γ-$int(b_\gamma$-$int(T, L)) = b_\gamma$-$int((T, L))$
3. b_γ-$cl((T, L)) = (H, R) - b_\gamma$-$int((H, R) - (T, L))$
4. b_γ-$int((T, L)) = (H, R) - b_\gamma$-$cl((H, R) - (T, L))$

Proposition 2.6. [[7], Proposition 2.22] *Let $(H, R, \mathcal{M})$ be a BTS and γ be a binary regular operation on $\mathcal{M}$ and let $(T, L) \subseteq (H, R)$. Then*

1. b_γ-$cl((T, L)) \cap (U, V) \subseteq b_\gamma$-$cl((T, L) \cap (U, V))$ *for every* $b_\gamma O(U, V)$.
2. b_γ-$int((T, L) \cup (G, H)) \subseteq b_\gamma$-$int((T, L)) \cup (G, H)$ *for every* $b_\gamma C(G, H)$.

3. Binary gamma pre open set (b_γ-p-open set)

Definition 3.1. *Let $(N, Q, \mathcal{M})$ be a BTS with an O-γ on $\mathcal{M}$. Then a subset (E, S) of (N, Q) is said to be a b_γ-p-open set if $(E, S) \subseteq (b_\gamma\text{-}int(b_\gamma\text{-}cl(E, S))$. The family of all b_γ-p-open set is denoted by $B_\gamma PO(N, Q)$.*

The complement of b_γ-p-open set is said to be b_γ-p-closed set.

This definition is given in [7]. Using this definition, we discuss some results with an example.

Example 3.2. $N = \{k, t\}$, $Q = \{1, 2, 3\}$ Binary topology $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{k\}, \{1\}), (\{k\}, \{1, 2\}), (\{t\}, \phi), (\{t\}, \{1\}), (\{t\}, \{3\}), (\{t\}, \{1, 3\}), (N, \{1\}), (N, \{1, 2\}), (N, \{1, 3\}), (N, Q)\}$

Define an O-γ on $\mathcal{M}$ such that for every $(E, S) \in \mathcal{M}$,

$$\gamma((E, S)) = \begin{cases} (E, S) & \text{if } (E, S) = (\{t\}, \{1\}) \\ (E, S) \cup (\{t\}, \{3\}) & \text{if } (E, S) \neq (\{t\}, \{1\}) \end{cases}$$

$B_\gamma O(N, Q) = \{(\phi, \phi), (N, Q), (\{t\}, \{1\}), (\{t\}, \{3\}), (\{t\}, \{1, 3\}), (N, \{1, 3\})\}$

$B_\gamma PO(N, Q) = \{(N, Q), (\phi, \phi), (\{t\}, \phi), (\{t\}, \{1\}), (\{t\}, \{2\}), (\{t\}, \{3\}), (\{k, t\}, \{1\}), (\{k, t\}, \{2\}), (\{k, t\}, \{3\}), (\{t\}, \{1, 2\}), (\{t\}, \{1, 3\}), (\{t\}, \{2, 3\}), (N, \{1, 3\}), (N, \phi), (N, \{2, 3\}), (N, \{1, 2\}), (\{k\}, Q), (\{t\}, Q), (\phi, Q), (\phi, \{1, 3\})\}$.

Remark 3.3. *Let $(N, Q, \mathcal{M})$ be a BTS with an O-γ on $\mathcal{M}$. Then every $b_\gamma O$ is $b_\gamma PO$.*

Proof. By definition of 3.1, we can proved the result.

Remark 3.4. *The converse of the above result need not be true.*

Example 3.5. In Example 3.2, $(\{t\}, \{2\})$ is $b_\gamma PO$. But it is not a $b_\gamma O$.

Theorem 3.6. *The concept of bPO and $b_\gamma PO$ are independent.*

It is showing by the following example.

Example 3.7. In Example 3.2, $(\{k\}, \{1\})$ is bPO. But it is not a $b_\gamma PO$. $(\{t\}, \{2\})$ $b_\gamma PO$. But it is not a bPO. Hence $b_\gamma PO$ and bPO are independent.

Definition 3.8. *Let (E, S) be a subset of a BTS $(N, Q, \mathcal{M})$ with an O-γ on $\mathcal{M}$. Then the intersection of all b_γ-p-closed sets containing (E, S) is binary gamma pre closure of (E, S) (briefly b_γ-pcl$((E, S))$).*

Definition 3.9. *Let $(N, Q, \mathcal{M})$ be a BTS with an O-γ on $\mathcal{M}$. A subset (Y, L) of (N, Q) is a binary γ-pre-neighbourhood of a binary point $(\{x\}, \{y\}) \in (N, Q)$ if there exists a b_γ-p-open set (G, H) such that $(\{x\}, \{y\}) \in (G, H) \subseteq (Y, L)$.*

Theorem 3.10. *Let $(N, Q, \mathcal{M})$ be a BTS with an O-γ on $\mathcal{M}$. Then the following results hold for the two b_γ-pcl of subsets (E, S), (C, D):*

1. *If* $(E, S) \subseteq (C, D)$, *then* b_γ*-pcl*$(E, S) \subseteq b_\gamma$*-pcl*$(C, D)$
2. $(E, S) = b_\gamma$*-pcl*(E, S) *if and only if* (E, S) *is* b_γ*-p-closed set in* $(N, Q, \mathcal{M})$.
3. $(\{x\}, \{y\}) \in b_\gamma$*-pcl*$(E, S)$ *if and only if* $(E, S) \cap (G, H) \neq (\phi, \phi)$, *for every* b_γ*-p-open set* (G, H) *of* (N, Q) *containing* $(\{x\}, \{y\})$.

Proof.

1. Given $(E, S) \subseteq (C, D)$. Clearly b_γ-*pcl*(C, D) is the b_γ-*p*-closed set containing (E, S). But b_γ-*pcl*(E, S) is the smallest b_γ-*p*-closed set containing (E, S). Hence b_γ-*pcl*$(E, S) \subseteq b_\gamma$-*pcl*$(C, D)$.

2. Suppose (E, S) is b_γ-*p*-closed set. Then the smallest b_γ-*p*-closed set containing (E, S) is (E, S) itself. Therefore $(E, S) = b_\gamma$-*pcl*(E, S).

   Conversely, assume that $(E, S) = b_\gamma$-*pcl*(E, S). We know that b_γ-*pcl*(E, S) is b_γ-*p*-closed set. Therefore (E, S) is b_γ-*p*-closed set.

3. Suppose $(\{x\}, \{y\}) \in b_\gamma$-*pcl*$(E, S)$. Let (G, H) be a b_γ-*p*-open set containing $(\{x\}, \{y\})$. Since $(\{x\}, \{y\}) \in b_\gamma$-*pcl*$(E, S)$, binary γ-pre-neighbourhood of a binary point $(\{x\}, \{y\}) \cap (E, S) \neq (\phi, \phi)$. $(E, S) \cap (G, H) \neq (\phi, \phi)$.

   Conversely, suppose $(E, S) \cap (G, H) \neq (\phi, \phi)$ for every b_γ-*p*-open set (G, H) of (N, Q) containing $(\{x\}, \{y\})$. Since binary γ-pre-neighbourhood of a binary point $(\{x\}, \{y\})$ containing $(\{x\}, \{y\})$, we have binary γ-pre-neighbourhood of a binary point $(\{x\}, \{y\}) \cap (E, S) \neq (\phi, \phi)$. Hence $(\{x\}, \{y\}) \in b_\gamma$-*pcl*$(E, S)$.

Theorem 3.11. *Let* $(N, Q, \mathcal{M})$ *be a BTS with an O-*γ *on* $\mathcal{M}$. *Then the following properties hold for a family* $\{(A_\alpha, B_\alpha)/\alpha\triangle\}$

1. $\cup_{\alpha\in\triangle}(b_\gamma\text{-}pcl((A_\alpha, B_\alpha))) \subseteq b_\gamma\text{-}pcl(\cup_{\alpha\in\triangle}((A_\alpha, B_\alpha)))$.
2. $\cap_{\alpha\in\triangle}(b_\gamma\text{-}pcl(A_\alpha, B_\alpha)) \supseteq b_\gamma\text{-}pcl(\cap_{\alpha\in\triangle}((A_\alpha, B_\alpha)))$

Proof. 1) We know that $(A_\alpha, B_\alpha) \subseteq \cup_{\alpha\in\triangle}(A_\alpha, B_\alpha)$ for every $\alpha \in \triangle$. By Theorem 3.10, (1), $b_\gamma\text{-}pcl((A_\alpha, B_\alpha)) \subseteq b_\gamma\text{-}pcl(\cup_{\alpha\in\triangle}(A_\alpha, B_\alpha))$ for every $\alpha \in \triangle$, it follows that $\cup_{\alpha\in\triangle}(b_\alpha\text{-}pcl((A_\alpha, B_\alpha))) \subseteq b_\gamma\text{-}pcl(\cup_{\alpha\in\triangle}(A_\alpha, B_\alpha))$.

Theorem 3.12. *The arbitrary union of* b_γ*-p-open sets in the BTS* $(N, Q, \mathcal{M})$ *with an O-*γ *on* $\mathcal{M}$ *is* b_γ*-p-open set.*

Proof. Let $\{(A_k, B_k)/k \in \triangle\}$ be the family of is b_γ-*p*-open sets. Then the every

$k(A_k, B_k) \subseteq b_\gamma\text{-}int(b_\gamma\text{-}cl((A_k, B_k)))$. $\Rightarrow \cup_{k\in\triangle}(A_k, B_k) \subseteq (b_\gamma\text{-}int(b_\gamma\text{-}cl(A_k, B_k))) \subseteq (b_\gamma\text{-}int(\cup_{k\in\triangle}(b_\gamma\text{-}cl((A_k, B_k)) \subseteq (b_\gamma\text{-}int(b_\gamma\text{-}cl(\cup_{k\in\triangle}(A_k, B_k)))$.

$\cup_{k\in\triangle}(A_k, B_k)$ is a b_γ-p-open set.

Result 3.13. *The intersection of two b_γ-p-open sets need not be b_γ-p-open set.*

Example 3.14. Let $N = \{i, j, k\}$, $Q = \{1, 2, 3\}$ and Binary topology $\mathcal{M} = \{(\phi, \phi), (N, Q), (\{i\}, \{1\}), (\{k\}, \{3\}), (\{i, k\}, \{1, 3\}), (\{i, j\}, \{1, 2\})\}$.

Define an O-γ on $\mathcal{M}$ such that for every $(E, S) \subseteq \mathcal{M}$,

$$\gamma((E,S)) = \begin{cases} (E,S) & \text{if } (E,S) = (\{i\},\{1\}) \\ (E,S) \cup (\{c\},\{3\}) & \text{if } (E,S) \neq (\{i\},\{1\}) \end{cases}$$

$B_\gamma O(N, Q) = \{(\phi, \phi), (N, Q), (\{i\}, \{1\}), (\{k\}, \{3\}), (\{i, k\}, \{1, 3\})\}$.

Here $(\{i, j\}, \{1, 3\})$ and $(\{j, k\}, \{1, 2\})$ are b_γ-p-open sets.

Now $(\{i, j\}, \{1, 3\}) \cap (\{j, k\}, \{1, 2\}) = (\{j\}, \{1\})$

Here $(\{j\}, \{1\})$ is not b_γ-p-open set.

Remark 3.15. *If (N, Q, M) is $b_\gamma R$, then the concept of b_γ-p-open set and bPO are coincide.*

Proof. Using the result 2.19 [7], we get the proof.

Theorem 3.16. *Let (N, Q, M) be a BTS, γ be a bRO on $\mathcal{M}$ and $(E, S) \subseteq (N, Q)$. If (E, S) is b_γ-p-open set and (Y, L) is $b_\gamma O$, then $(E, S) \cap (Y, L)$ is also b_γ-p-open set.*

Proof. Using proposition 2.21 [7], we get the proof.

Definition 3.17. *Let (E, S) be a subset of a BTS $(N, Q, \mathcal{M})$ with an O-γ on $\mathcal{M}$. Then union of all b_γ-p-open sets contained in (E, S) is b_γ-pre interior of (E, S) (briefly b_γ-pint$((E, S))$).*

Proposition 3.18. *Let (E, S) be a subset of a BTS (N, Q, M) with an O-γ on $\mathcal{M}$. Then,*

1. *b_γ-pint$((E, S))$ is a b_γ-p-open set which is contained in (E, S).*
2. *b_γ-pint$((E, S)) = (E, S)$ if and only if (E, S) is b_γ-p-open set.*

Proof.

1. Using the Definition 3.17 and Theorem 3.12, we get the proof.
2. Follows from the definition of b_γ-p-open set.

Proposition 3.19. *Let (E, S) be a subset of a BTS (N, Q, M) with a bRO-γ on $\mathcal{M}$. Then for any subset (E, S) of (N, Q),*

1. b_γ-$pcl((E,S)) = (E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int(E,S)))$.
2. b_γ-$pint((E,S)) = (E,S) \cap (b_\gamma$-$int(b_\gamma$-$cl(E,S)))$.

Proof.

1. Given $(E,S) \subseteq (N,Q)$. Consider b_γ-$int((E,S)) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S)))$. Then by Proposition 2.22(b) [7], b_γ-$int((E,S) \cup b_\gamma$-$cl(b_\gamma$-$int((E,S))) \subseteq b_\gamma$-$int((E,S))$ $\cup(b_\gamma$-$cl(b_\gamma$-$int((E,S)))$. b_γ-$cl(b_\gamma$-$int((E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S)))) \subseteq b_\gamma$-$cl(b_\gamma$-$int((E,S)) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S)))) \subseteq b_\gamma$-$cl(b_\gamma$-$int(E,S)) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S)))) \subseteq b_\gamma$-$cl(b_\gamma$-$int(E,S)) \subseteq (E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int(E,S)))$. From this, $(E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S))))$ is b_γ-p-closed set. Hence b_γ-$pcl((E,S)) \subseteq (E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S))))$. Now b_γ-$cl(b_\gamma$-$int((E,S))) \subseteq b_\gamma$-$cl(b_\gamma$-$int(b_\gamma$-$pcl((E,S))))$. We know that b_γ-$pcl((E,S))$ is b_γ-p-closed set. If follows that b_γ-$cl(b_\gamma$-$int(E,S))) \subseteq b_\gamma$-$pcl((E,S))$. This implies $(E,S) \cup b_\gamma$-$cl(b_\gamma$-$int((E,S))) \subseteq b_\gamma$-$pcl((E,S))$. Hence b_γ-$pcl((E,S)) = (E,S) \cup (b_\gamma$-$cl(b_\gamma$-$int((E,S)))$.

2. Using proposition 2.22(a) [7] and follows the proof of (1), we can get the result.

Proposition 3.20. *Let (E,S) be a subset of a BTS $(N,Q, athcalM)$ with a O-γ on $\mathcal{M}$. Then $(E,S) \subseteq b_\gamma$-pcl$((E,S))$ and b_γ-pcl$((E,S))$ is b_γ-p-closed set in $(N,Q,\mathcal{M})$.*

Proof. Let $\{(A_k,B_k)/k \in \Omega\}$ be the family of is b_γ-p-closed sets in $(N,Q,\mathcal{M})$ containing (E,S). Then by the definition of b_γ-pre closure, b_γ-$pcl((E,S)) = \cap_{k\in\Omega}(A_k,B_k)$. Since $(E,S) \subseteq (A_k,B_k)$ for each $k \in \Omega$, $(E,S) \subseteq \cap_{k\in\Omega}(A_k,B_k)$. Hence $(E,S) \subseteq b_\gamma$-$pcl((E,S))$.

Proposition 3.21. *Let (N,Q,M) be a BTS and γ be a bRO on $\mathcal{M}$. Then for any subset (E,S) of (N,Q),*

1. b_γ-$pcl((b_\gamma$-$int((E,S)))) = b_\gamma$-$cl(b_\gamma$-$int((E,S)))$
2. b_γ-$pint(b_\gamma$-$cl((E,S))) = b_\gamma$-$int(b_\gamma$-$cl((E,S)))$
3. b_γ-$cl(b_\gamma$-$pint((E,S))) = b_\gamma$-$cl(b_\gamma$-$int(b_\gamma$-$cl((E,S))))$
4. b_γ-$int((b_\gamma$-$pcl((E,S)))) = b_\gamma$-$int(b_\gamma$-$cl(b_\gamma$-$int((E,S))))$.

Proof.

1. b_γ-$pcl(b_\gamma$-$int((E,S))) = b_\gamma$-$int(E,S)) \cup b_\gamma$-$cl(b_\gamma$-$int(b_\gamma$-$int((E,S))))$, (by Proposition 3.19(1)) $= b_\gamma$-$cl(b_\gamma$-$int((E,S)))$.

2. b_γ-$pint((b_\gamma$-$cl(E,S))) = b_\gamma$-$cl(E,S) \cap b_\gamma$-$int(b_\gamma$-$cl(b_\gamma$-$cl(E,S)))$ (by Proposition 3.19(2)) $= b_\gamma$-$cl((E,S)) \cap b_\gamma$-$int(b_\gamma$-$cl((E,S))) = b_\gamma$-$int(b_\gamma$-$cl((E,S)))$.

3. b_γ-$cl(b_\gamma$-$pint((E,S))) = b_\gamma$-$cl(b_\gamma$-$int(b_\gamma$-$cl((E,S))))$ (by (2))

4. b_γ-$int((b_\gamma$-$pcl((E,S)))) = b_\gamma$-$int(b_\gamma$-$cl(b_\gamma$-$int((E,S))))$. (by (1))

Proposition 3.22. *Let (E,S) be a subset of a BTS (N,Q,M) with bRO-γ on $\mathcal{M}$. Then b_γ-$pcl(b_\gamma$-$pint((E,S))) = b_\gamma$-$pint(E,S) \cup b_\gamma$-$cl(b_\gamma$-$int((E,S)))$.*

Proof. Since every $b_\gamma O$ is $b_\gamma PO$, it follows as b_γ-$int(E,S) \subseteq b_\gamma$-$pint(E,S)$. Clearly b_γ-$pint(E,S) \subseteq (E,S)$. So b_γ-$int(E,S) \subseteq (E,S)$. This implies b_γ-$int(b_\gamma$-$pint(E,S)) = b_\gamma$-$int(E,S)$. .... (i)

Now b_γ-$pcl(b_\gamma$-$pint((E,S))) = b_\gamma$-$pint((E,S)) \cup (b_\gamma$-$cl(b_\gamma$-$int(b_\gamma$-$pint((E,S))))$ (by Proposition 3.19 (1)) $= b_\gamma$-$pint((E,S)) \cup (b_\gamma$-$cl(b_\gamma$-$int(E,S)))$ (by (i))

4. b_γ-p-g-closed set

Definition 4.1. *Let $(N,Q,\mathcal{M})$ be a BTS with an O-γ on $\mathcal{M}$. Then a subset (E,S) of (N,Q) is called b_γ-p-g-closed set ($b_\gamma PGC$) if b_γ-$pcl(E,S) \subseteq (Y,L)$ whenever $(E,S) \subseteq (Y,L)$ and (Y,L) is b_γ-p-open set.*

Example 4.2. Let $N = \{r,w\}$, $Q = \{1,2,3\}$ and $\mathcal{M} = \{(\phi,\phi), (\phi,\{1\}), (\{r\},\{1\}), (\{r\},\{1,2\}), (\{w\},\phi), (\{w\},\{1\}), (\{w\},\{3\}), (\{w\},\{1,3\}), (N,\{1\}), (N,\{1,2\}), (N,\{1,3\}), (N,Q)\}$.

Define an O-γ on $\mathcal{M}$ such that for every $(E,S) \in \mathcal{M}$,

$$\gamma((E,S)) = \begin{cases} (E,S) & \text{if } (E,S) = (\{r\},\{1\}) \\ (E,S) \cup (\{w\},\{3\}) & \text{if } (E,S) \neq (\{r\},\{1\}) \end{cases}$$

$b_\gamma O(N,Q) = \{(N,Q), (\phi,\phi), (\{r\},\{1\}), (\{w\},\{3\}), (\{r,w\},\{1,3\})\}$

$b_\gamma PO(N,Q) = \{(\{r\},\{1\}), (\{r\},\{3\}), (\{w\},\{1\}), (\{w\},\{3\}), (\{w\},\{1,3\}), (\{r\},\{2,3\}), (\{w\},\{1,2\}), (\{w\},\{1,3\}), (\{w\},\phi), (\{w\},\phi), (\{r\},\{1,2,3\}), (\{w\},\{1,2,3\}), (\{r,w\},\{1\}), (\{r,w\},\{2\}), (\{r,w\},\{3\}), (\{r,w\},\{1,2\}), (\{r,w\},\{1,3\}), (\{r,w\},\{2,3\}), (\{r,w\},\{1,2,3\})\}$.

b_γ-p-closed sets are $(\{w\},\{2,3\})$, $(\{w\},\{1,2\})$, $(\{r\},\{2,3\})$, $(\{r\},\{1,2\})$, $(\{w\},\{1\})$, $(\{w\},\{2\})$, $(\{r\},\phi)$, $(\{r\},\{2\})$, $(\{r\},\{1,2,3\})$, $(\{w\},\{1,2,3\})$, $(\{w\},\phi)$, $(\{r\},\phi)$, $(\phi,\{2,3\})$, $(\phi,\{1,3\})$, $(\phi,\{1,2\})$, $(\phi,\{3\})$, $(\phi,\{2\})$, $(\phi,\{1\})$, (ϕ,ϕ).

Here all the b_γ-pre closed sets are b_γ-p-g-closed sets.

Theorem 4.3. *Let (E,S) be a b_γ-p-closed subset of a BTS $(N,Q,\mathcal{M})$ with a O-γ*

on $\mathcal{M}$. Then (E, S) is b_γ-p-g-closed set.

Proof. Let (E, S) be an b_γ-p-closed set in $(N, Q, \mathcal{M})$. And let $(E, S) \subseteq (G, H)$ where (G, H) is b_γ-p-open set in $(N, Q, \mathcal{M})$. Since (E, S) is b_γ-p-closed set, we have b_γ-$pcl((E, S)) = (E, S)$. Hence b_γ-$pcl((E, S)) \subseteq (G, H)$. Hence (E, S) is b_γ-p-g-closed set.

Remark 4.4. *The converse of the above theorem is not true.*

Example 4.5. Let $N = \{k, r, m, n\}$, $Q = \{1, 2, 3, 4\}$ and $\mathcal{M} = \{(\phi, \phi), (\{k\}, \{1\}), (\{r\}, \{2\}), (\{m\}, \{3\}), (\{k, r\}, \{1, 2\}), (\{k, m\}, \{1, 3\}), (\{r, m\}, \{2, 3\}), (\{k, r, m\}, \{1, 2, 3\}), (\{k, r, n\}, \{1, 2, 4\}), (N, Q)\}$. Define an O-γ on $\mathcal{M}$ such that for every $(E, S) \in \mathcal{M}$,

$$\gamma((E, S)) = \begin{cases} b\text{-}int(b\text{-}cl(E, S) & \text{if } (E, S) = (\{k\}, \{1\}) \\ b\text{-}cl(E, S) & \text{if } (E, S) \neq (\{k\}, \{1\}) \end{cases}$$

$b_\gamma PO(N, Q) = \{(\{k\}, \{1\}), (\{m\}, \{3\}), (\{k, m\}, \{1, 3\}), (\{k, r\}, \{1, 2\}), (\{k, n\}, \{1, 4\}), (\{k, r, m\}, \{1, 2, 3\}), (\{k, r, n\}, \{1, 2, 4\}), (\{k, m, n\}, \{1, 3, 4\}), (\phi, \phi), (N, Q)\}$

b_γ-p-closed sets are $(\{r, m, n\}, \{2, 3, 4\}), (\{r, n\}, \{2, 4\}), (\{m, n\}, \{3, 4\}), (\{r, m\}, \{2, 3\}), (\{n\}, \{4\}), (\{m\}, \{3\}), (\{r\}, \{2\}), (N, Q), (\phi, \phi)$.

Here $(\{r\}, \{3\})$ is $b_\gamma PGC$ but it is not b_γ-p-closed set.

Theorem 4.6. *The union of any two b_γ-p-g-closed sets need not be b_γ-p-g-closed set.*

Example 4.7. Let $N = \{u, t\}$, $Q = \{1, 2, 3\}$ and Binary topology $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{u\}, \{1\}), (\{u\}, \{1, 2\}), (\{t\}, \phi), (\{t\}, \{1\}), (\{t\}, \{3\}), (\{t\}, \{1, 3\}), (N, \{1\}), (N, \{1, 2\}), (N, \{1, 3\}), (N, Q)\}$. Let $\gamma : M \to \mathbb{P}(X) \times \mathbb{P}(Y)$ be defined as follows: For every $(E, S) \in \mathcal{M}$,

$$\gamma((E, S)) = \begin{cases} (E, S) & \text{if } (E, S) = (\{u\}, \{1\}) \\ (E, S) \cup (\{t\}, \{3\}) & \text{if } (E, S) \neq (\{u\}, \{1\}) \end{cases}$$

$b_\gamma PO(N, Q) = \{(\{u\}, \{1\}), (\{u\}, \{3\}), (\{u\}, \{1, 3\}), (\{u\}, \{1, 2, 3\}), (\{t\}, \{1\}), (\{t\}, \{3\}), (\{t\}, \{1, 2, 3\}), (\{u, t\}, \{1\}), (\{u, t\}, \{1, 2\}), (\{u, t\}, \{3\}), (\{u\}, \{2, 3\}), (\{t\}, \{1, 2\}), (\{t\}, \{1, 3\}), (\{u, t\}, \{1, 2\}), (\{u, t\}, \{1, 3\}), (\{u, t\}, \{2, 3\}), (\{u\}, \phi), (\{t\}, \phi), (\{u, t\}, \phi), (\phi, \{1\}), (\phi, \{1, 2\}), (\phi, \{1, 3\}), (\phi, \phi), (N, Q)\}$.

Here $(\{u\}, \{3\})$ and $(\{t\}, \{1\})$ are b_γ-p-g-closed sets, but $(\{u\}, \{3\}) \cup (\{t\}, \{1\}) = (\{u, t\}, \{1, 3\})$ is not $b_\gamma PGC$.

Theorem 4.8. *The intersection of any two b_γ-p-g-closed sets need not be b_γ-p-g-closed set.*

Example 4.9. In Example 4.5, $(\{k, n\}, \{1, 2\})$ and $(\{k, r\}, \{2, 4\})$ are b_γ-p-g-closed sets, but $(\{k, n\}, \{1, 2\}) \cap (\{k, r\}, \{2, 4\}) = (\{k\}, \{2\})$ is not $b_\gamma PGC$.

5. Conclusion

In this paper, we presented concepts of b_γ-pre open sets and b_γ-pre closure of a subset in binary topological space. As a result of these findings, we are able to apply them to other areas of general topology, including fuzzy topology, intuitionistic topology, for example.

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