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CHEBYSHEV POLYNOMIALS AND BI-UNIVALENT FUNCTIONS ASSOCIATING WITH q-DERIVATIVE OPERATOR

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Abstract: In this paper, we introduce and investigate a new subclass of Bi-univalent functions defined in the open unit disk, associated with Chebyshev polynomials by applying q-derivative operator. Furthermore, We find estimates for the general Taylor-Maclaurin coefficients of the functions in this class and also we obtain an estimation for Fekete-Szegö problem for this class.

Keywords and Phrases: Analytic functions, Univalent and Bi-univalent functions, Fekete-Szegö inequality, Chebyshev polynomials and q-derivative operator.

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1. Introduction

We indicate by $\mathcal{A}$ the collection of functions, which are analytic in the open unit disk $\mathbb{D}$ given by

$$\mathbb{D} = \{z \in \mathbb{C} \quad and \quad |z| < 1\}$$

and have the following normalized form :

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

We also denote by $\mathcal{S}$ the sub-collection of the set $\mathcal{A}$ consisting of functions which are also univalent in $\mathbb{D}$. The Koebe one-quarter theorem [6] asserts that the image of $\mathbb{D}$ under each univalent function f in $\mathcal{S}$ contains a disk of radius $1/4$. According to this, every function $f \in \mathcal{S}$ has an inverse map f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{D})$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right),$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \ldots \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{D}$ if both f and its inverse f^{-1} are univalent in $\mathbb{D}$. Let Σ stand for the class of bi-univalent functions in $\mathbb{D}$ given by (1.1). For more basic results one may refer Srivastava et al. [17] and references there in.

With a view to recalling the principle of subordination between analytic functions, let the functions f and g be analytic in $\mathbb{D}$. We say that the function f is subordinate to g if there exists a Schwarz function w, which is analytic in $\mathbb{D}$ with

$$w(0) = 0 \quad and \quad |w(z)| < 1 \quad (z \in \mathbb{D}),$$

such that

$$f(z) = g(w(z)).$$

This subordination is denoted by

$$f \prec g \quad or \quad f(z) \prec g(z) \quad (z \in \mathbb{D}).$$

It is well known that, if the function g is univalent in $\mathbb{D}$, then (see [13])

$$f \prec g \quad (z \in \mathbb{D}) \iff f(0) = g(0) \quad and \quad f(\mathbb{D}) \subset g(\mathbb{D}).$$

For $q \in (0, 1)$, the Jackson q-derivative of a function $f \in \mathcal{A}$ is given by (see [10, 11])

$$D_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z} & for \quad z \neq 0, \\ f'(0) & for \quad z = 0. \end{cases} \tag{1.3}$$

From (1.3), we have

$$D_q f(z) = 1 + \sum_{n=2}^{\infty} [n]_q a_n z^{n-1} \tag{1.4}$$

where

$$[n]_q = \frac{1-q^n}{1-q},$$

is sometimes called the basic number n. In particular If $q \to 1^-$, $[n]_q \to n$.

$$[2]_q = \frac{1-q^2}{1-q} = 1 + q, \quad [3]_q = \frac{1-q^3}{1-q} = 1 + q + q^2. \tag{1.5}$$

Chebyshev polynomials, which are used by us in this paper, play a considerable role in numerical analysis. We know that the Chebyshev polynomials are four kinds. The most of books and research articles related to specific orthogonal polynomials of Chebyshev family contain essentially results of Chebyshev polynomials of first and second kinds $T_n(t)$ and $U_n(t)$ and their numerous uses in different applications; see [5, 7, 12, 16].

The Chebyshev polynomials of the first and second kinds are orthogonal for $t \in [-1, 1]$ and defined as follows:

Definition 1.1. *The Chebyshev polynomials of the first kind are defined by the following three-terms recurrence relation:*

$$\begin{aligned} T_0(t) &= 1, \\ T_1(t) &= t, \\ T_{n+1}(t) &= 2tT_n(t) - T_{n-1}(t). \end{aligned}$$

The first few of the Chebyshev polynomials of the first kind are

$$T_2(t) = 2t^2 - 1, \quad T_3(t) = 4t^3 - 3t, \quad T_4(t) = 8t^4 - 8t^2 + 1, ... \tag{1.6}$$

The generating function for the Chebyshev polynomials of the first kind, $T_n(t)$, is given by:

$$F(z, t) = \sum_{n=0}^{\infty} T_n(t) z^n = \frac{1 - tz}{1 - 2tz + z^2}, \quad (z \in \mathbb{D}).$$

Definition 1.2. *The Chebyshev polynomials of the second kind are defined by the following three-terms recurrence relation:*

$$U_0(t) = 1,$$

$$U_1(t) = 2t,$$

$$U_{n+1}(t) = 2tU_n(t) - U_{n-1}(t).$$

The first few of the Chebyshev polynomials of the second kind are

$$U_2(t) = 4t^2 - 1, \quad U_3(t) = 8t^3 - 4t, \quad U_4(t) = 16t^4 - 12t^2 + 1, ... \tag{1.7}$$

The generating function for the Chebyshev polynomials of the second kind, $U_n(t)$, is given by:

$$H(z,t) = \sum_{n=0}^{\infty} U_n(t) z^n = \frac{1}{1 - 2tz + z^2}, \quad (z \in \mathbb{D}).$$

The Chebyshev polynomials of the first and second kinds are connected by the following relations:

$$\frac{dT_n(t)}{dt} = nU_{n-1}(t); \quad T_n(t) = U_n(t) - tU_{n-1}(t); \quad 2T_n(t) = U_n(t) - U_{n-2}(t).$$

Motivated by aforementioned study on bi-univalent functions [8, 19, 20] and present investigation of bi-univalent functions associated with various polynomials as well as by many recent works on the Fekete-Szegö functional and other coefficient estimates (see [4, 14]), in the present paper we introduce a new subclass $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ of the function class Σ involving q-derivative operator related with Chebyshev polynomials as given in Definition 1.3.

Definition 1.3. *For $0 \leq \wp \leq 1$, $0 \leq \vartheta \leq 1$, $t \in (0,1]$ and $0 < q < 1$, a function $f \in \Sigma$ is said to be in the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ if it satisfies the subordinations*

$$\left[(1-\vartheta)\frac{zD_qf(z)}{f(z)} + \vartheta\frac{D_q(zD_qf(z))}{D_qf(z)}\right]^{\wp} \prec H(z,t) := \frac{1}{1-2tz+z^2}$$

and

$$\left[(1-\vartheta)\frac{wD_qg(w)}{g(w)} + \vartheta\frac{D_q(wD_qg(w))}{D_qg(w)}\right]^{\wp} \prec H(w,t) := \frac{1}{1-2tw+w^2}$$

where the function $g = f^{-1}$ is given by (1.2).

Suitably fixing the parameter $\vartheta = 0$ or $\vartheta = 1$ we state following new subclasses:

Example 1.4. For $0 \leq \wp \leq 1$, $t \in (0,1]$ and $0 < q < 1$, a function $f \in \Sigma$ is said to be in the class $\mathfrak{L}_\Sigma(\wp, t, q)$ if it satisfies the subordinations

$$\left[\frac{zD_qf(z)}{f(z)}\right]^\wp \prec H(z,t) := \frac{1}{1 - 2tz + z^2}$$

and

$$\left[\frac{wD_qg(w)}{g(w)}\right]^\wp \prec H(w,t) := \frac{1}{1 - 2tw + w^2}$$

where the function $g = f^{-1}$ is given by (1.2).

Example 1.5. For $0 \leq \wp \leq 1$, , $t \in (0,1]$ and $0 < q < 1$, a function $f \in \Sigma$ is said to be in the class $\mathfrak{M}_\Sigma(\wp, t, q)$ if it satisfies the subordinations

$$\left[\frac{D_q(zD_qf(z))}{D_qf(z)}\right]^\wp \prec H(z,t) := \frac{1}{1 - 2tz + z^2}$$

and

$$\left[\frac{D_q(wD_qg(w))}{D_qg(w)}\right]^\wp \prec H(w,t) := \frac{1}{1 - 2tw + w^2}$$

where the function $g = f^{-1}$ is given by (1.2).

Remark 1.6. *It should be remarked that the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ is a generalization of well-known classes considered earlier. These classes are:*

(i) *For $q \to 1$ the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ reduces to the class $H_\Sigma(\vartheta, t, \wp)$ which was studied by Girgaonkar and Joshi* [9].

(ii) *For $q \to 1$ and $\wp = 1$ the clas $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ reduces to the class $H_\Sigma(\vartheta, t)$, which was introduced and studied by Sahsene Altinkaya and Sibel Yalcin* [1].

(iii) *For $q \to 1$, $\vartheta = 0$ and $\wp = 1$ the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ reduces to the class $S_\Sigma(t)$ which was introduced by Sahsene Altinkaya and Sibel Yalcin* [3].

(iv) *For $\vartheta = 0$ and $\wp = 1$ the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ reduces to the class $\mathcal{S}^*_\Sigma(t, q)$, which was studied by Nandini and Latha* [15].

(v) *For $\vartheta = 1$ and $\wp = 1$ the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ reduces to the class $\mathcal{K}_\Sigma(t, q)$, which was studied by Nandini and Latha* [15].

Now we investigate the optimal bounds for the Taylor- Maclaurin coefficients $|a_2|$ and $|a_3|$ and Furthermore we establish Fekete-Szegö inequality for the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$ in the following sections.

2. Initial Coefficient Results

Theorem 2.1. *For $0 \leq \wp \leq 1$, $0 \leq \vartheta \leq 1$, $t \in (0, 1]$ and $0 < q < 1$, let $f \in \mathcal{A}$ be in the class $\mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$. Then*

$$|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{|\left\{\wp(q+q^2)[1+\vartheta(q+q^2)] - \phi(\vartheta,\wp)\right\}4t^2 + \wp^2q^2[1+\vartheta q]^2|}} \tag{2.1}$$

and

$$|a_3| \leq \frac{2t}{\wp(q+q^2)[1+\vartheta(q+q^2)]} + \frac{4t^2}{\wp^2q^2[1+\vartheta q]^2}. \tag{2.2}$$

where

$$\phi(\vartheta,\wp) = \frac{\wp(\wp+1)}{2}q^2[1+\vartheta q]^2 + \wp q[1+\vartheta(q^2+2q)]^2. \tag{2.3}$$

Proof. Let $f \in \mathfrak{G}_\Sigma(\wp, \vartheta, t, q)$. Then there are two analytic functions $u, v : \mathbb{D} \to \mathbb{D}$ given by

$$u(z) = u_1z + u_2z^2 + u_3z^3 + ... \qquad (z \in \mathbb{D}) \tag{2.4}$$

and

$$v(w) = v_1w + v_2w^2 + v_3w^3 + ... \qquad (w \in \mathbb{D}), \tag{2.5}$$

with $u(0) = v(0) = 0$ and $\max\{|u(z)|, |v(w)|\} < 1 \quad (z, w \in \mathbb{D})$, such that

$$\left[(1-\vartheta)\frac{zD_qf(z)}{f(z)} + \vartheta\frac{D_q(zD_qf(z))}{D_qf(z)}\right]^\wp = H(u(z), t)$$

and

$$\left[(1-\vartheta)\frac{wD_qg(w)}{g(w)} + \vartheta\frac{D_q(wD_qg(w))}{D_qg(w)}\right]^\wp = H(v(w), t)$$

or, equivalently, that

$$\left[(1-\vartheta)\frac{zD_qf(z)}{f(z)} + \vartheta\frac{D_q(zD_qf(z))}{D_qf(z)}\right]^\wp = 1 + U_1(t)u(z) + U_2(t)u^2(z) + ... \tag{2.6}$$

$$\left[(1-\vartheta)\frac{wD_qg(w)}{g(w)} + \vartheta\frac{D_q(wD_qg(w))}{D_qg(w)}\right]^\wp = 1 + U_1(t)v(w) + U_2(t)v^2(w) + ... \tag{2.7}$$

Combining (2.4), (2.5), (2.6) and (2.7) we find that

$$\left[(1-\vartheta)\frac{zD_qf(z)}{f(z)}+\vartheta\frac{D_q(zD_qf(z))}{D_qf(z)}\right]^{\wp}=1+U_1(t)u_1z+[U_1(t)u_2+U_2(t)u_1^2]z^2... \quad (2.8)$$

and

$$\left[(1-\vartheta)\frac{wD_qg(w)}{g(w)}+\vartheta\frac{D_q(wD_qg(w))}{D_qg(w)}\right]^{\wp}=1+U_1(t)v_1w+[U_1(t)v_2+U_2(t)v_1^2]w^2+... \quad (2.9)$$

It is well known that, if

$$\max\{|u(z)|,|v(w)|\}<1,(z,w\in\mathbb{D}),$$

then

$$|u_j|\leq 1 \qquad and \qquad |v_j|\leq 1 \quad (\forall j\in\mathbb{N}) \quad (2.10)$$

Now, by comparing the corresponding coefficients in (2.8) and (2.9) and after some simplification, we have

$$\wp q[1+\vartheta q]a_2=U_1(t)u_1, \quad (2.11)$$

$$\left\{\frac{\wp(\wp-1)}{2}q^2[1+\vartheta q]^2-\wp q[1+\vartheta(q^2+2q)]\right\}a_2^2+\wp(q+q^2)\left(1+\vartheta(q+q^2)\right)a_3$$
$$=U_1(t)u_2+U_2(t)u_1^2, \quad (2.12)$$

$$-\wp q[1+\vartheta q]a_2=U_1(t)v_1 \quad (2.13)$$

$$\left\{2\wp(q+q^2)[1+\vartheta(q+q^2)]-\wp q[1+\vartheta(q^2+2q)]+\frac{\wp(\wp-1)}{2}q^2[1+\vartheta q]^2\right\}a_2^2$$
$$-\wp(q+q^2)[1+\vartheta(q+q^2)a_3$$
$$=U_1(t)v_2+U_2(t)v_1^2. \quad (2.14)$$

It follows from (2.11) and (2.13) that

$$u_1=-v_1 \quad (2.15)$$

and

$$2\wp^2q^2[1+\vartheta q]^2a_2^2=(U_1(t))^2(u_1^2+v_1^2). \quad (2.16)$$

If we add (2.12) and (2.14), we find that

$$\left\{2\wp\left((q+q^2)[1+\vartheta(q+q^2)]-q[1+\vartheta(q^2+2q)]\right)+\wp(\wp-1)q^2[1+\vartheta q]^2\right\}a_2^2$$
$$=U_1(t)(u_2+v_2)+U_2(t)(u_1^2+v_1^2). \quad (2.17)$$

Upon substituting the value of $u_1^2+v_1^2$ from (2.16) into the right-hand side of (2.17), we deduce that

$$a_2^2 = \left[\frac{(U_1(t))^3(u_2+v_2)}{\begin{array}{r}\{2[\wp(q+q^2)[1+\vartheta(q+q^2)] - \wp q[1+\vartheta(q^2+2q)]] - \wp(\wp+1)q^2[1+\vartheta q]^2\} \\ (U_1(t))^2 + 2\wp^2q^2[1+\vartheta q]^2 U_2(t)\end{array}} \right] \tag{2.18}$$

By further computations using (1.7), (2.10) and (2.18), we obtain

$$|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{|\left\{\wp(q+q^2)[1+\vartheta(q+q^2)] - \phi(\vartheta,\wp)\right\} 4t^2 + \wp^2q^2[1+\vartheta q]^2|}} \tag{2.19}$$

where $\phi(\vartheta,\wp)$ is given by (2.3).
Next, if we subtract (2.14) from (2.12), we can easily see that

$$2\wp(q+q^2)[1+\vartheta(q+q^2)](a_3-a_2^2) = U_1(t)(u_2-v_2) + U_2(t)(u_1^2-v_1^2). \tag{2.20}$$

In view of (2.15) and (2.16), we find from (2.20) that

$$a_3 = \frac{U_1(t)(u_2-v_2)}{2\wp(q+q^2)[1+\vartheta(q+q^2)]} + \frac{(U_1(t))^2(u_1^2+v_1^2)}{2\wp^2q^2[1+\vartheta q]^2}.$$

Thus by applying (1.7), we obtain

$$|a_3| \leq \frac{2t}{\wp(q+q^2)[1+\vartheta(q+q^2)]} + \frac{4t^2}{\wp^2q^2[1+\vartheta q]^2}. \tag{2.21}$$

Remark 2.2. *Theorem 2.1, provides the known results for the classes which are mentioned in Remark 1.6.*

3. Fekete-Szegö inequality

As discussed in [21], in the next theorem, we present the Fekete-Szegö inequality, for the class $\mathfrak{G}_\Sigma(\wp,\vartheta,t,q)$.

Theorem 3.1. *For $0 \leq \wp \leq 1$, $0 \leq \vartheta \leq 1$, $t \in (0,1]$, $0 < q < 1$ and $\mu \in \mathbb{R}$, let $f \in \mathcal{A}$ be in the class $\mathfrak{G}_\Sigma(\wp,\vartheta,t,q)$. Then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{2t}{\wp(q+q^2)[1+\vartheta(q+q^2)]}; & |\mu-1| \leq \aleph \\ \frac{8t^3|\mu-1|}{|\{\wp(q+q^2)[1+\vartheta(q+q^2)]-\phi(\vartheta,\wp)\}4t^2+\wp^2q^2[1+\vartheta q]^2|}; & |\mu-1| \geq \aleph. \end{cases}$$

where

$$\aleph = \frac{|\frac{\wp^2q^2[1+\vartheta q]^2}{4t^2} + \wp(q+q^2)[1+\vartheta(q+q^2)] - \phi(\vartheta,\wp)|}{\wp(q+q^2)[1+\vartheta(q+q^2)]}$$

Proof. It follows from (2.18) and (2.20) that

$$\begin{aligned}
a_3-\mu a_2^2 &= \frac{U_1(t)(u_2-v_2)}{2\wp(q+q^2)[1+\vartheta(q+q^2)]}+(1-\mu)a_2^2\\
&= \frac{U_1(t)(u_2-v_2)}{2\wp(q+q^2)[1+\vartheta(q+q^2)]}\\
&+ \left[\frac{(U_1(t))^3(u_2+v_2)(1-\mu)}{\begin{array}{r}\left\{2[\wp(q+q^2)[1+\vartheta(q+q^2)]-\wp q[1+\vartheta(q^2+2q)]]+\wp(\wp-1)q^2[1+\vartheta q]^2\right\}\\ (U_1(t))^2-2\wp^2q^2[1+\vartheta q]^2U_2(t)\end{array}}\right]\\
&= \frac{U_1(t)}{2}\left[\left(\eta(\mu,t)+\frac{1}{\wp(q+q^2)[1+\vartheta(q+q^2)]}\right)u_2\right.\\
&+ \left.\left(\eta(\mu,t)-\frac{1}{\wp(q+q^2)[1+\vartheta(q+q^2)]}\right)v_2\right],
\end{aligned}$$

where

$$\eta(\mu,t)=\frac{(1-\mu)4t^2}{\{\wp(q+q^2)[1+\vartheta(q+q^2)]-\phi(\vartheta,\wp)\}\,4t^2+\wp^2q^2[1+\vartheta q]^2}.$$

Thus, according to (1.7), we have

$$|a_3-\mu a_2^2|\le\begin{cases}\frac{2t}{\wp(q+q^2)[1+\vartheta(q+q^2)]}; & 0\le|\eta(\mu,t)|\le\frac{1}{\wp(q+q^2)[1+\vartheta(q+q^2)]}\\ 2t|\eta(\mu,t)|; & |\eta(\mu,t)|\ge\frac{1}{\wp(q+q^2)[1+\vartheta(q+q^2)]}.\end{cases}$$

Taking $\mu=1$ in Theorem 3.1, we led to the following corollary.

Corollary 3.2. *For $0\le\wp\le1$, $0\le\vartheta\le1$, $t\in(0,1]$ and $0<q<1$, let $f\in\mathcal{A}$ be in the class $\mathfrak{G}_\Sigma(\wp,\vartheta,t,q)$. Then*

$$|a_3-a_2^2|\le\frac{2t}{\wp(q+q^2)[1+\vartheta(q+q^2)]}.$$

Remark 3.3. *Theorem 3.1, provides the known Fekete-Szegö inequality results for the classes which are mentioned in Remark 1.6.*

4. Concluding Remarks and Observations

In the present paper, we mainly get upper bounds of initial Taylor coefficients of new class of bi-univalent functions connected with the Chebyshev polynomial . Also, we can discuss the related research of the coefficient problem and Fekete-Szegö inequality. Further by fixing $\vartheta=0$ and $\vartheta=1$ we can state the above results for functions classes given in Examples 1.4 and 1.5. For motivating further researches on the subject-matter of this, we have chosen to draw the attention of the interested readers towards a considerably large number of results related Hankel determinants. In conclusion, with an opinion mostly to encouraging and inspiring further researches on applications of the basic (or $q-$) analysis and the basic (or $q-$) calculus in Geometric Function Theory of Complex Analysis along the lines, considering our present investigation and based on recently-published works on the Fekete-Szegö and Hankel determinant problem (see, for details, [18] and references cited therein.

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