ON EULER TYPE INTEGRALS

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Dedicated to Professor G.E. Andrews on his seventieth birthday

Abstract: In this paper, we establish a theorem connecting Euler type single and double integrals. We derive a number of new results as application of the theorem.

1. Introduction

We recall the Euler integral which defines the beta function

$$B(\alpha, \beta) \ = \ \int_0^1 u^{\alpha-1}(1-v)^{\beta-1}du \ = \ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \ \ \text{Re}(\alpha), \ \text{Re}(\beta) > 0 \quad (1.1)$$

and a connection between single and double Eulerian integral

$$\int_0^1 \int_0^1 f(xy)(1-x)^{\alpha-1}y^{\alpha}(1-y)^{\beta-1} \, dxdy \ = \ B(\alpha, \beta)\int_0^1 f(t)(1-t)^{\alpha+\beta-1}dt \ (1.2)$$

which is a special case of the result [5, p.379,4.2.4(1)] for $c = 0$.

Recently Ismail and Pitman [3] obtained explicit evaluations of some integrals of Euler's type

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}f(u)du$$

for some particular functions f, specially in the symmetric case $\alpha = \beta$. Khan *et al.* in [4] extended further these results to generalize the evaluations of certain Euler type integrals.

Motivated and inspired by the result (1.2), the work of Ismail and Pitman [3] and Khan, Agarwal, Pathan and Mohammad [4], in this paper, we obtain a