South East Asian J. Math. & Math. Sc.
Vol.6 No.2(2008), pp.45–50

ON CERTAIN ETA-FUNCTIONS IDENTITIES

R.Y. Denis, S.N. Singh* and S.P. Singh*

Department of Mathematics
University of Gorakhpur, Gorakhpur-273009, India
E-mail: ddry@sancharnet.in

*Department of Mathematics
T.D.P.G. College, Jaunpur-222002, India

(*Received: February 07, 2008*)

Dedicated to Professor G.E. Andrews on his seventieth birthday

Abstract: In this paper, we establish certain Eta-function identities.

Keywords: Eta function, basic hypergeometric functions, basic bilateral hypergeometric functions
AMS Subject Classification: 33D15, 33E20

1. Introduction

For α and q real or complex ($|q| < 1$), we define

$$[\alpha]_n \equiv [\alpha;q]_n = (1-\alpha)(1-\alpha q)\dots(1-\alpha q^{n-1}), \quad n > 0, \quad [\alpha]_0 = 1$$

$$[\alpha]_\infty \equiv [\alpha;q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n).$$

With the help of above notations, we define a basic hypergeometric function

$$ {}_r\Phi_s\left[\begin{matrix} a_1, a_2, \dots, a_r & ;q;z \\ b_1, b_2, \dots, b_s & ; \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{[a_1]_n\,[a_2]_n \dots [a_r]_n\, z^n}{[q]_n\,[b_1]_n\,[b_2]_n \dots [b_s]_n}, \qquad (1.1)$$

valid for $|z| < 1$.

We also define a basic bilateral hypergeometric function

$$ {}_r\Psi_r\left[\begin{matrix} a_1, a_2, \cdots, a_r & ;q;z \\ b_1, b_2, \cdots, b_r & ; \end{matrix}\right] = \sum_{n=-\infty}^{\infty} \frac{[a_1]_n\,[a_2]_n \dots [a_r]_n\, z^n}{[b_1]_n\,[b_2]_n \dots [b_r]_n} \qquad (1.2)$$

valid for $|b_1 b_2 \dots / a_1 a_2 \dots a_r| < |z| < 1$.

(1.2) reduces to (1.1) if any of the denominator parameters tends to q.