

A FRAGMENT ON EULER'S CONSTANT IN RAMANUJAN'S LOST NOTEBOOK

Bruce C. Berndt¹ and Tim Huber²

Department of Mathematics
University of Illinois, 1409 West Green Street, Urbana, IL 61801, USA

²Department of Mathematics
396 Carver Hall, Iowa State University, Ames, IA 50011, USA
E-mails: berndt@math.uiuc.edu; huber@iastate.edu

Dedicated to our close friend George Andrews on his 70th birthday

(Received: January 10, 2008)

Abstract: A formula for Euler's constant found in Ramanujan's lost notebook and also in a problem he submitted to the *Journal of the Indian Mathematical Society* is proved and discussed.

Keywords and Phrases: Euler's constant, Ramanujan's lost notebook, series approximations

2000 AMS Subject Classification: 11J17, 40A25

1. Introduction

Like many mathematicians, Ramanujan was evidently fascinated with Euler's constant γ . He wrote only one paper on Euler's constant [10], [11], but published with his lost notebook [13, pp. 274–277] are two partial manuscripts devoted to γ . The first author and D.C. Bowman [4] previously examined one of these partial manuscripts [13, pp.274–275], and in this paper the remaining fragment is examined. Both partial manuscripts are related to early interests of Ramanujan. The first partial manuscript is related to Frullani integrals, which were featured in Ramanujan's *Quarterly Reports* [1,2], [3, pp.295–336], written in the year prior to Ramanujan's departure for Cambridge. The second partial manuscript is related to the first problem that Ramanujan submitted to the *Journal of the Indian Mathematical Society* [8], [11, p. 322] and to the first six entries of Chapter 2 in his second notebook [11], [3, pp.25–35]. Moreover the second partial manuscript gives Ramanujan's solution to another problem [9,] [11, p. 325] that he submitted to the *Journal of the Indian Mathematical Society*. No solution to this problem

¹Research partially supported by grant MDA904-00-1-0015 from the National Security Agency.