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N-THETA FUNCTION IDENTITIES

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Dedicated to Professor G. E. Andrews on his seventieth birthday

Abstract: Ramanujan develops, in Chapter 16 of his second notebook, the theory of theta-function and recorded several identities without proofs. All these have been proved by Adiga, Berndt, Bhargava and Watson. In this paper, we establish several results of N-theta function which are analogous to the results in the Entries in Chapter 16 of Ramanujan's second notebook.

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1. Introduction

Ramanujan develops, in Chapter 16 of his second notebook [3], the theory of theta-function and his theta-function is defined by

$$f(a,b) \;=\; \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}, \quad |ab|<1.$$

Following Ramanujan, we define

$$\varphi(q) := f(q,q) \;=\; \sum_{n=-\infty}^{\infty} q^{n^2} \tag{1.1}$$

$$\psi(q) := f(q,q^3) \;=\; \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} \tag{1.2}$$

$$f(-q) := f(-q,-q^2) \;=\; \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}}. \tag{1.3}$$

Following Ramanujan, we define a new N- Theta function by

$$f_N(a,b) \;=\; \sum_{k=-\infty}^{\infty} a^{\frac{k^N(k^N+1)}{2}} b^{\frac{k^N(k^N-1)}{2}}, \; |ab|<1. \tag{1.4}$$