

## CONTROLLED $K$ -FRAMES IN 2-HILBERT SPACES

Neha Pauriyal and Mahesh C. Joshi\*

Department of Applied Sciences and Humanities,

Birla Institute of Applied Sciences,

Bhimtal - 263136, Uttarakhand, INDIA

E-mail : nehapauriyal1996@gmail.com

\*Department of Mathematics,

D. S. B. Campus, Kumaun University,

Nainital - 263002, Uttarakhand, INDIA

E-mail : mcjoshi69@gmail.com

(Received: Aug. 21, 2025 Accepted: Dec. 28, 2025 Published: Dec. 30, 2025)

**Abstract:** In this paper, we introduce a new generalization of controlled  $K$ -frames to the context of 2-Hilbert spaces, thereby extending beyond classical Hilbert space theory. We develop foundational results by examining the operator-theoretic properties of controlled  $K$ -frames in this setting, establishing equivalent conditions that characterize them, and exploring their stability under suitable transformations. This builds directly on prior work introducing controlled  $K$ -frames in Hilbert  $C^*$ -modules, where the concept was first defined, equivalent conditions were established, relationships between  $K$ -frames and controlled  $K$ -frames were revealed, and invariance and perturbation properties were analyzed. Our work elevates these ideas by adapting them to the richer structure of 2-Hilbert spaces-a framework extending Hilbert spaces through inner products valued in  $C^*$ -algebras.

**Keywords and Phrases:** Frame, Controlled  $K$ -frame, Controlled  $K$ -frame operator, Controlled  $K$ - Bessel sequence.

**2020 Mathematics Subject Classification:** 42C15, 46C50.